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Multivariable Adaptive Predictive Control for Stochastic Systems with Time Delays

by

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Kirthi Sarachchandra Walgama

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
OF Master of Science
IN
Process Control

Department of Chemical Engineering

EDMONTON, ALBERTA
Fall, 1986

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Abstract

This thesis documents the development of SISO and MIMO predictive control and time delay compensation schemes which include a Kalman filter to provide minimum variance estimates of the future outputs of a stochastic process. The Kalman Filter Predictors (KFP) are first developed for applications where the process parameters are known and then extended to adaptive versions which include an extended least squares estimator for on-line identification of the necessary parameters in the state space model of the process.

A key step is the formulation of a state space model which includes the current and the undelayed values of the process output as state variables. The only non zero, non unity elements in the state space matrices are the parameters of the input-output ARMA model of the given process. Proof of the observability and stabilizability of the state space model leads to a proof of the stability of the KFP and the convergence of the time varying KFP to the steady state KFP. The sparseness and the known structure of the state space matrices leads to a very efficient implementation of the KFP.

The extension of the KFP to the MIMO case is based on the interactor factorization of Wolovich and Falb which factors out the "natural" time delays from the multivariable transfer function matrix and guarantees important properties such as the invertibility of the residual matrix. This KFP

is used as a basis for a state space formulation of a multivariable predictive controller which gives minimum variance controller performance.

It is shown by using the innovation model approach that the KFP can be expressed in ARMAX model form and that it represents a natural extension of the Smith Predictor(SP) to multivariable stochastic systems. The ARMAX model form of the KFP provides the basis for a 'self-tuning' KFP, and also provides a link between the process and measurement noise concepts in state space representation and the formulation of the noise term in ARMAX process models.

A MIMO adaptive predictive control system is developed, using an adaptive KFP, which provides minimum variance control performance once the parameter estimation has converged. The precompensator approach developed by Singh and Narendra is used to convert triangular interactor matrices into diagonal form, and it is then shown that a multivariable process with p outputs can be partitioned in to p-independent KFPs. This leads to an implementation which is computationally efficient and which requires only the MISO parameters of the MIMO process. The formulation of the KFP is modified by including the disturbance dynamics in the state space formulation to give ~~unbiased~~ estimates due to sustained disturbances, offset or model-process mismatch. The end result is a practical, multivariable, adaptive predictive control scheme which can be applied to stochastic processes with time delays and non-zero mean disturbances.

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Table of Contents

Chapter Page

1. Introduction 1

2. Kalman Filter Predictor 3

  2.1 Introduction 3

  2.2 State Space Representation 6

  2.3 Kalman Filter Predictor 10

    2.3.1 Kalman Filter 11

      2.3.1.1 Time varying Kalman Filter Algorithm ..12

      2.3.1.2 Steady State Kalman Filter 13

    2.3.2 Stability and convergence of the Kalman Filter Predictor 15

  2.4 Innovation Model Approach for the Kalman Filter Predictor 20

    2.4.1 Innovation Model for KFP 21

    2.4.2 Configuration of the KFP 24

    2.4.3 Input output model for the KFP 27

  2.5 Feedback control using the KFP 27

    2.5.1 PID Control 27

    2.5.2 Predictive Control 29

  2.6 Modified Kalman Filter Predictor for deterministic disturbances 33

    2.6.1 KFP in presence of Deterministic Disturbances 33

    2.6.2 Modified Kalman Filter Predictor 35

    2.6.3 Incremental Predictive control for the Modified KFP 39

  2.7 Discussion 40

    2.7.1 Kalman Filter Predictor vs Internal Model Control 40

2.7.2 Kalman Filter Predictor vs Smith Predictor 41

2.7.3 Implementation of the KFP 47

2.7.4 Predictive Control with SP and KFP 48

2.7.5 Variable time-delay processes 50

2.7.6 Input Output model vs State Space Model ..51

2.8 Simulation Results and Discussion 53

2.9 Conclusions 60

Bibliography 87

3. Multivariable Kalman Filter Predictor 89

3.1 Introduction 89

3.2 MIMO systems with Multiple Time Delays 92

3.2.1 Introduction 92

3.2.2 Interactor Factorization 93

3.3 Predictors for MIMO systems with Time delays ...98

3.3.1 Introduction 98

3.3.2 Predictor Concept for MIMO systems 100

3.3.3 Relationship between ARMA and TFM models for the MIMO system with Interactor Factorization 101

3.4 State Space Formulation 105

3.4.1 Observable State Space Realization for Discrete Time MIMO Systems 105

3.4.2 State Space formulation for the Kalman Filter Predictor 110

3.5 Multivariable Kalman Filter Predictor 118

3.5.1 Stability and convergence of the Multivariable Kalman Filter Predictor ...119

3.6 Innovation Model approach for the Multivariable Kalman Filter Predictor 121

3.7 Comparison of MIMO KFP with MIMO Interactor Predictor 128

3.8 Feedback Control System using the Kalman Filter Predictor 130

3.8.1 PID dead time compensator 130

3.8.2 Predictive Control 132

3.9 Discussion 137

3.10 Simulation Results and Discussion 142

3.11 Conclusions 153

Bibliography 186

4. An Adaptive Kalman Filter Predictor 188

4.1 Introduction and Overview 188

4.2 Adaptive Kalman Filter Predictor (AKFP) 191

4.2.1 State Space Formulation 191

4.2.2 Adaptive Kalman Filter Predictor 192

4.2.3 Parameter Estimation 195

4.3 Innovation Model Approach for the Adaptive Kalman Filter Predictor 198

4.3.1 Comments on the stability and the convergence of the Adaptive Kalman Filter Predictor 201

4.4 A 'Self-Tuning' Kalman Filter Predictor 204

4.4.1 Comments on the "Self-Tuning" Kalman Filter Predictor 206

4.5 Feedback Control using the Adaptive Kalman Filter Predictor 208

4.5.1 PID Control 208

4.5.2 Adaptive Predictive Control 209

4.5.3 Comments on the Adaptive Kalman Filter Predictive Control (AKFPC) scheme 210

4.6 An Adaptive Kalman Filter Predictor for Deterministic Disturbances 212

4.6.1 Modified Kalman Filter Predictor 212

4.6.2 Modified Adaptive Kalman Filter Predictor 214

4.6.3 Incremental Adaptive Predictive Control for the modified Adaptive Kalman Filter Predictor 215

4.7 Simulation Results and Discussion 216

4.8 Conclusions 221

Bibliography 241

5. Multivariable Adaptive Kalman Filter Predictor 243

5.1 Introduction and Overview 243

5.2 Interactor Factorization 245

5.3 Predictor Concept for MIMO Systems 247

5.4 ARMA model for a MIMO system using interactor factorization 248

5.5 State Space Formulation 251

5.6 Adaptive Kalman Filter Predictor 253

5.6.1 Precompensator Design 256

5.6.2 Parameter Identification 258

5.6.2.1 Multivariable Identification Methods 258

5.6.2.2 Parameter Identification for KFP 259

5.6.3 Kalman Filter 262

5.7 A "Self-Tuning" Multivariable Kalman Filter Predictor ....26[illegible]

5.8 Adaptive Predictive Control System using AKFP .265

5.9 Multivariable Modified Adaptive Kalman Filter Predictor 266

5.9.1 State Space Formulation 267

5.9.2 Modified Adaptive Kalman Filter Predictor 268

5.9.3 Incremental Adaptive Predictive Control Scheme 269

5.10 Simulation Results and Discussion 269

5.11 Conclusions 273

Bibliography 274

6. Overall Conclusions and Future Work 307

6.1 Conclusions 307

6.2 Future Work 312

6.3 Bibliography 314

Appendix A - Kalman Filter (KF) 315

Appendix B - Stability & Convergence of Kalman Filter ...323

Appendix C - Innovation Model of the Kalman Filter 328

Appendix D - A Lemma for KFP 330

Appendix E - Simplified implementation of the Kalman Filter Algorithm for KFP 332

Appendix F - Procedure to Obtain the Interactor Matrix ..335

Appendix G - Ogunnaike and Ray Multivariable Predictor ..341

Appendix H - Interactor Predictor 344

Appendix I 349

Appendix J - A Predictive Controller Using the Interactor Predictor 351

Appendix K - Procedure for Precompensator Design 353

Appendix L - Comments on Simulation Results 358

List of Tables

Table Page

2.1 Process and Measurement Noise Variances used in Simulations 62

3.1 Process and Measurement Noise Variances Used in Simulations 154

3.2 Controller Parameters for Different Predictor Schemes 154

4.1 Parameter Estimation in the AKFP after 800 sampling instants (Stochastic Process) 223

4.2 Parameter Estimation in the STKFP after 800 sampling instants (Stochastic Process) 223

5.1 Parameter Estimation for the Distillation Column Model 275

5.2 Parameter Estimation for the Process with a Precompensator 276

E.1 Computation Load in Direct and Simplified Klaman Filter Algorithms 334

List of Figures

Figure Page

2.1 Schematic Block Diagram of the State Space Representation for a Process with Time Delays. 63

2.2 Response of Unit Delay Blocks for a Step input to the Process. 63

2.3 Feedback Control Scheme Using a Predictor. 64

2.4 Structure of the Kalman Filter Predictor. 64

2.5 Structure of the Internal Model Control System. 65

2.6 Structure of the Smith Predictor. 65

2.7 Elaborated Structure of the Kalman Filter Predictor. Bank of m_3 Integrators are added to Modify the KFP for Deterministic Disturbances. 66

2.8a PI Control of 1st Order Deterministic Process Using KFP and SP 67

2.9b PI Control of 1st Order Deterministic Process using KFP and SP. 68

2.9a Predictive Control of 1st Order Deterministic Process, using KFP and SP. 69

2.9b Predictive Control of 1st Order Deterministic Process, using KFP and SP. 70

2.10a PI control of a 1st Order Stochastic Process using KFP and SP. 71

2.10b PI Control of a 1st Order Stochastic Process using KFP and SP. 72

2.10c Variance of Output Error for PI Control of 1st OrderStochastic Process using KFP and SP. 73

2.11a Predictive Control of a 1st Order Stochastic Process using KFP and SP. 74

2.11b Predictive Control of a 1st Order Stochastic Process using KFP and SP. 75

Figure Page

2.11c Variance of the Output Error for Predictive Control of the 1st Order Stochastic Process using KFP and SP. 76

2.12a Predictive Control of a 2nd Order Stochastic Process using KFP and SP. 77

2.12b Predictive Control of a 2nd Order Stochastic Process using KFP and SP. 78

2.12c Variance of the Output Error for Predictive Control of the 2nd Order Stochastic Process using KFP and SP. 79

2.13a PI Control of 1st Order Process in Presence of Deterministic Disturbances using KFP, SP and MKFP. 80

2.13b PI Control of 1st Order Process in Presence of Deterministic Disturbances using KFP, SP and MKFP. 81

2.14 Predictive Control of 1st Order Process in Presence of Deterministic Disturbances using KFP, SP and MKFP. 82

2.15 Predictive Control of 2nd Order Process in Presence of Deterministic Disturbances using MKFP and SP. 83

2.16 Open Loop Prediction in the SP and MKFP under Deterministic disturbances, for 1st and 2nd Order processes. 84

2.17a Predictive Control of SP and KFP with Unmodelled Dynamics. 85

2.17b Predictive Control of MKFP with Unmodelled Dynamics. 86

3.1 Schematic block diagram of a MIMO process with time delays, based on the interactor factorization 155

3.2 Schematic block diagram of a MIMO process, decomposed into a p MIMO processes, based on interactor factorization 155

3.3 Schematic block diagram of the multivariable Kalman Filter Predictor 156

Figure Page

3.4 Serially connected p MISO Kalman Filter Predictors 157

3.5 Structure of the Multivariable Kalman Filter Predictor 158

3.6a PI control of the Top Composition using KFP, IP and ORP (Deterministic Case) 159

3.6b PI Control of the Bottom Composition using KFP, IP and ORP (Deteministic Case) 160

3.6c PI Control of Top Composition using KFP, IP and ORP (Deterministic Case) 161

3.6d PI Control of Bottom Composition using KFP, IP and ORP (Deterministic Case) 162

3.7a PI Control of Top Composition using KFP, IP and ORP (Stochastic Case) 163

3.7b PI Control of Bottom Composition using KFP, IP and ORP (Stochastic Case) 164

3.7c PI Control of Top Composition using KFP, IP and ORP 165

3.7d PI Control of Bottom Composition using KFP, IP and ORP 166

3.7e Variance of tracking error for KFP, IP and ORP 167

3.8a Predictive Control of the Top Composition using KFP and IP (Deterministic Case) 168

3.8b Predictive Control of the Bottom Composition using KFP and IP (Deterministic Case) 169

3.8c Predictive Control of the Top Composition using KFP and IP (Deterministic Case) 170

3.8d Predictive Control of the Bottom Composition using KFP and IP (Deterministic Case) 171

3.9a Predictive Control of the Top Composition using KFP and IP (Stochastic Case) 172

3.9b Predictive Control of the Bottom Composition using KFP and IP (Stochastic Case) 173

Figure Page

3.9c Predictive Control of the Top Composition using KFP and IP (Stochastic Case) 174

3.9d Predictive Control of the Bottom Composition using KFP and IP (Stochastic Case) 175

3.9d Variance of Tracking error for KFP and IP with an Exponential Filter 176

3.10a Predictive Control of y_1 using KFP and IP (Deterministic Process with a Triangular Interactor) 177

3.10b Predictive Control of y_2 using KFP and IP (Deterministic process with a Triangular Interactor) 178

3.10c Predictive Control of a Deterministic process with a Triangular Interactor, using KFP and IP 179

3.10d Predictive Control of a Deterministic process with a Triangular Interactor, using KFP and IP 180

3.11a Predictive Control of y_1 using KFP and IP (Stochastic process with Triangular Interactor) 181

3.11b Predictive Control of y_2 using KFP and IP (Stochastic process with Triangular Interactor) 182

3.11c Predictive Control of the stochastic process with Triangular Interactor, using KFP and IP 183

3.11d Predictive Control of the stochastic process with Triangular Interactor using KFP and IP 184

3.11e Variance of Tracking error for kFP and IP with an Exponential Filter 185

4.1 Schematic Block Diagram of the State Space Representation fora process with time delays. 224

4.2 The Schematic Block Diagram of the Adaptive Kalman Filter Predictor. 224

Figure Page

4.3 The structure of the Adaptive Kalman Filter Predictor. 225

4.4 The structure of the Adaptive Smith Predictor. 226

4.5 The Schematic Block diagram of the Adaptive Predictive Control system based on the AKFP. 227

4.6a Adaptive predictive Control using AKFP and ASP (Deterministic Process). 228

4.6b Adaptive predictive Control using AKFP and ASP (Deterministic Process). 229

4.7a Adaptive Predictive Control using AKFP, and ASP with an Exponential Filter(Stochastic Process). 230

4.7b Adaptive Predictive Control using AKFP, and ASP with an Exponential Filter(Stochastic Process). 231

4.7c Variance of the Output Tracking Error for AKFP, and ASP with an Exponential Filter. 232

4.7d Parameter Estimation in AKFP (using Extended Least Squares Identification). 233

4.8a Adaptive Predictive Control with STKFP (Stochastic Process). 234

4.8b Variance of the Output Tracking Error for AKFP, STKFP, and ASP with an Exponential Filter. 235

4.8c Parameter Estimation in STKFP using Extended Least Squares Identification. 236

4.8d Kalman Gains in the STKFP. 237

4.9a Incremental Adaptive Predictive Control using AMKFP and ASP in presence of deterministic disturbances. 238

4.9b Incremental Adaptive Predictive Control using AMKFP and ASP in presence of deterministic disturbances. 239

4.9c Parameter Estimation in the Incremental RLS Identification Scheme. 240

Figure Page

5.1 Schematic block diagram of a MIMO process with time delays, based on the interactor factorization. 277

5.2 Multivariable Kalman Filter Predictor Configuration for a Diagonal Interactor (p Independent MISO Kalman Filter Predictors). 277

5.3 Schematic Block Diagram of the Multivariable Adaptive Kalman Filter Predictor. 278

5.4 Schematic Block Diagram of the Multivariable Adaptive Predictive Control Scheme. 278

5.5a Adaptive Predictive Control of the Top Composition Using AKFP and AIP (Deterministic Case). 279

5.5b Adaptive Predictive Control of the Bottom Composition Using AKFP and AIP (Deterministic Case). 280

5.5c Adaptive Predictive Control of the Top Composition Using AKFP and AIP (Deterministic Case). 281

5.5d Adaptive Predictive Control of the Bottom Composition Using AKFP and AIP (Deterministic Case). 282

5.5e Parameter Estimation in AKFP for the First MISO Subsystem (using Extended Least Squares Identification). 283

5.5f Parameter Estimation in AKFP for the Second MISO Subsystem (using Extended Least Squares Identification). 284

5.6a Adaptive Predictive Control of the Top Composition Using AKFP and AIP (Stochastic Case). 285

5.6b Adaptive Predictive Control of the Bottom Composition Using AKFP and AIP (Stochastic Case). 286

5.6c Adaptive Predictive Control of the Top Composition Using AKFP and AIP (Stochastic Case). 287

Figure Page

5.6d Adaptive Predictive Control of the Bottom Composition Using AKFP and AIP (Stochastic Case). 288

5.6e Variance of Tracking Error for AKFP and AIP. 289

5.6f Parameter Estimation in AKFP for the First MISO Subsystem (using Extended Least Squares Identification). 290

5.6g Parameter Estimation in AKFP for the Second MISO Subsystem (using Extended Least Squares Identification). 291

5.7a Adaptive Predictive Control of y_1 using AKFP and AIP (Deterministic Process with a Precompensator). 292

5.7b Adaptive Predictive Control of y_2 using AKFP and AIP (Deterministic Process with a Precompensator). 293

5.7c Adaptive Predictive Control of y_1 using AKFP and AIP (Deterministic Process with a Precompensator). 294

5.7d Adaptive Predictive Control of y_2 using AKFP and AIP (Deterministic Process with a Precompensator). 295

5.7e Parameter Estimation in AKFP for the First MISO Subsystem (using Extended Least Squares Identification). 296

5.7f Parameter Estimation in AKFP for the Second MISO Subsystem (using Extended Least Squares Identification). 297

5.8a Adaptive Predictive Control of y_1 using AKFP and AIP (Stochastic Process with a Precompensator). 298

5.8b Adaptive Predictive Control of y_2 using AKFP and AIP (Stochastic Process with a Precompensator). 299

5.8c Adaptive Predictive Control of y_1 using AKFP and AIP (Stochastic Process with a Precompensator). 300

Figure Page

5.8d Adaptive Predictive Control of y_2 using AKFP and AIP (Stochastic Process with a Precompensator). 301

5.8e Variance of Tracking Error for AKFP and AIP. 302

5.8f Parameter Estimation in AKFP for the First MISO Subsystem (using Extended Least Squares Identification). 303

5.8g Parameter Estimation in AKFP for the Second MISO Subsystem (using Extended Least Squares Identification). 304

G.1 Schematic Block Diagram of the Ogunnaike and Ray Predictor (ORP). 343

H.1 Schematic Block Diagram of the Interactor Predictor. 347

H.2 Schematic Block Diagram of the Interactor Predictor Based on Equation H.15. 348

Notation and Nomenclature

Only the symbols used in more than one place are listed here.

Notation

For Chapter 2

$y(t)$	Process output
$u(t)$	Process input
$A(q^{-1})$ $B(q^{-1})$ $B'(q^{-1})$	ARMA model of the process, equations (2.2.2, 3 and 4)
n	Order of the process
$y_1(t)$	Output of the process without time delays, equation (2.2.9)
$a_1 \cdots a_n$	Coefficients of $A(q^{-1})$ polynomial, equation (2.2.2)
$b_1 \cdots b_n$	Coefficients of $B(q^{-1})$ polynomial, equation (2.2.3)
d	Time delay of the process without the delay due to ZOH, equation (2.2.3)
x^1	State vector of the process without time delays, order n, equation (2.2.10)
Φ_1 Λ_1 Θ_1	State transition matrix, Input coefficient vector and output coefficient vector of the process without time delays, equations (2.2.10 and 11)
z	State vector due to unit delays, order d, equation (2.2.12)

Φ_2 Λ_2 Θ_2	State transition matrix, Input coefficient vector and output coefficient vector of the state space model of z, equations (2.2.12 and 13)
x	State vector of the process with time delays, order n+d, equations (2.2.14 and 15)
Φ Λ Θ	State transition matrix, input coefficient matrix, output coefficient matrix and disturbance coefficient matrix of the process with time delays equations (2.2.14 and 15)
$\omega(t)$	Process noise
$v(t)$	Measurement noise
$L(t)$	Kalman gain vector, equation (2.3.4)
R_{ω}	Covariance of process noise
R_v	Covariance of measurement noise
$\hat{x}(t)$	A posteriori estimation of x(t), section (2.3.11)
$\bar{x}(t)$	A priori estimation of x(t), section (2.3.11)
$P(t)$	A posteriori estimation error covariance of x(t), order (n+d)x(n+d), section (2.3.1.1)
$M(t)$	A priori estimation error covariance of x(t), order (n+d)x(n+d), section (2.3.1.1)
R_2	$= R_v$
R_1	Covariance matrix of process noise order (n+d)x(n+d), equation (2.3.8)

$\bar{\Phi}$	Steady state filter transition matrix equation (2.3.11)
E	See equation (2.3.13)
M	Steady state matrix of M(t)
W	Observability matrix of the state space model (2.2.14 and 15), equation (2.3.18)
ω	Innovation sequence, equation (2.4.4)
$K_1(t,q^{-1})$	Time varying polynomial of order n+1, equation (2.4.8)
$K_2(t,q^{-1})$	Time varying polynomial of order d-2, equation (2.4.12)
$C(t,q^{-1})$	Time varying polynomial of order n+d-1, equation (2.4.17)
$G_p(q^{-1})$	Transfer function of the process model without time delays, equation (2.4.23)
$G(q^{-1})$	Transfer function of the process
$G_m(q^{-1})$	Transfer function of the process model
$G_F(t,q^{-1})$	Transfer function of the filter in the KFP, equation (2.4.24)
$G_c(q^{-1})$	Transfer function of the controller
G_{Fss}	Steady state gain of $G_F(t)$, equation (2.4.33)
$y_e(t)$	Error between the process output and the model output
$y^*(t)$	Desired output (set point)
$\xi(t)$	Represents process noise, measurement noise and any type of disturbances to the process

$E[\cdot]$	Statistical expectation of a random variable
σ_y^2 σ^2	Noise variances, equations (2.5.11 and 15)
$\bar{y}(t)$	See equation (2.5.17)
$A_s(q^{-1})$ $B_s(q^{-1})$	Actual ARMA polynomials of the process, equation (2.5.33)
$G_{\gamma 1}$	See figure (2.7) and equation (2.6.4)
$G_{\gamma 2}$	See figure (2.7) and equation (2.6.5)
$\bar{\omega}(t)$ $\omega'(t)$	See figure (2.7) and equations (2.6.6 and 7)
x_d	State due to disturbance, equation (2.6.14)
L_ρ	Kalman gain of state x_ρ
$D(q^{-1})$	Polynomial of degree n, equation (2.6.17)
$\hat{y}(t)$	Estimated output, equation (2.4.2)
Δ	Difference operator equation (2.6.11)
$\delta y^*(t)$	See equation (2.6.11)
$\hat{y}(t+d\|t)$	Prediction of future output at time t+d based on data at time t

For Chapter 3

$T(q)$	TFM of the process
$\xi_T(q)$	Interactor matrix, equation (3.2.9)
$\xi^{-1}(q)$	Inverse-interactor, equation (3.2.18)
$\left.\begin{matrix} R_T(q) \\ R(q) \end{matrix}\right\}$	Residual matrices, equations (3.2.11 and 16)

λ_j	j=1,p, time delays in the inverse interactor, equation (3.3.9)
p	No. of outputs
m	No. of inputs
y	Output vector - order p
u	Input vector - order m
D_T	See equation (3.2.15)
H_T	See equation (3.2.14)
$\bar{y}$	Filtered output, equation (3.3.3)
y^0	See equation (3.3.4)
R_{ij}	For i=1,p, j=1,m, see equation (3.3.6)
a^i	See equation (3.3.7)
b^{ij}	See equation (3.3.8)
d_{ij}	See equation (3.3.6)
g_{ij}	See equation (3.3.11)
α_j	See equation (3.3.13)
β^j	See equation (3.3.14)
$\left.\begin{matrix}A(q^{-1})\\B(q^{-1})\end{matrix}\right\}$	ARMA model for the TFM R(q), see equation (3.3.16)
D	See equation (3.3.20)
G_H	
$\left.\begin{matrix}A_T(q^{-1})\\B_T(q^{-1})\end{matrix}\right\}$	ARMA model of T(q), see equation (3.3.23)
n_i	i=1,p order of the ith MISO system - see equation (3.4.6)

Φ_1 Λ_1 Θ_1	State transition matrix, input coefficient matrix, and output coefficient matrix of the state space representation of $R(q)$, equations (3.4.28 and 29)
Φ_2 Λ_2 Θ_2	State transition matrix, input coefficient matrix, and output coefficient matrix of the state space representation of $\xi^{-1}(q)$, equations (3.4.37 and 38)
Φ Λ Γ Θ	State transition matrix, input coefficient matrix, disturbance coefficient matrix and output coefficient matrix of the state space representation of the augmented system, equations (3.4.47 and 48)
Φ_j Λ_j Γ_j Θ_j	State transition matrix, input coefficient matrix, disturbance coefficient matrix and output coefficient matrix of the j^{th} MISO system, equations (3.4.53 and 54)
x^j	State vector of the j^{th} MISO system - order $nj+\lambda j$, see equation (3.4.60)
u^j	Input vector of the order $m+j-1$, equation (3.4.61)
R_{wj}	Covariance of the process noise wj in j^{th} MISO system
R_{vj}	Covariance of the measurement noise vj in j^{th} MISO system

$\left.\begin{matrix} w_j \\ v_j \end{matrix}\right\}$	Process noise and measurement noise of the j^{th} MISO system
L_j	Kalman Gains of the j^{th} MISO Kalman filter
$\hat{x}_j$	A posteriori state estimation of the j^{th} KFP
ω_j	Innovation sequence of the j^{th} KFP, see equation (3.6.6)
$\hat{y}_j$	Output prediction in j^{th} KFP, see equation (3.6.5)
N^j	See equation (3.6.3)
F_j	See equation (3.6.19)
$\hat{y}_j^0$	Prediction of y_j^0 , see equation (3.6.10)
$\hat{y}^0$	Prediction of y^0, see equation (3.6.23)
G_F	See equation (3.6.24)
g_{Fj}	See equation (3.6.25)
ξ_s^{-1}	Steady state compensator equation (3.7.6)
G_s	See equation (3.7.1)
y^*	Desired output vector order p
$\bar{y}^*$	Filtered desired output
y_d	Represents noise or any disturbances to the process
y_e	Error between the actual output and the model output
$E[\cdot]$	Expectation of any random variable
$F_j(t)(q^{-1})$	See equation (3.6.19)
$K_i^j\,(t)(q^{-1})$	See equation (3.6.9)

$K_2^j(t)(q^{-1})$	See equation (3.6.15)
$K_1(t)(q^{-1})$	See equation (3.6.21)
$F(t)(q^{-1})$	See equation (3.6.28)

For Chapter 4

The symbols used are as same as in chapter 2. Only the additional notation is given below:

θ_0	True process parameter vector
$\hat{\theta}$	Estimated parameter vector
ψ	Regressior vector
μ	Forgetting factor
$G(t)$	Gain of the Identification scheme
$P(t)$	Covariance of the Identification scheme
$e(t)$	Apriori or a posteriori prediction error

For Chapter 5

The symbols used are as same as in chapter 3. Only the additional notation is given below:

$W(q)$	Precompensator TFM
$\xi_{TW}(q)$	Diagonal Interactor of the Precompensated Process
$E(\cdot)$	See equation (5.6.3)
$T_p(q)$	Precompensated process
$\psi_j(t)$	Regressor vector of the jth MISO ystem, equation(5.6.3)

$\hat{\theta}_j$	Parameter vector of the j^{th} MISO system, equation (5.6.14)
$\epsilon_j(t)$	See equation (5.9.1)
$\Delta\hat{y}^{*}(t)$	See equation (5.9.1)
$\Delta\hat{y}^{0}(t)$	See equation (5.9.14)

Nomenclature

AIP	Adaptive Interactor Predictor
AKFP	Adaptive Kalman Filter Predictor
AMKFP	Adaptive MKFP
ARIMA	Auto Regressive Integrated Moving Average
ARMA	Auto Regressive Moving Average
ARMAX	Auto Regressive Moving Average with auxliary input
ASP	Adaptive Smith Predictor
DARMA	Deterministic ARMA
EKF	Extended Kalman Filter
ERLS	Extended Recursive Least Squares
IMC	Internal Model Control
IP	Interactor Predictor
KFP	Kalman Filter Predictor
LQG	Linear Quadratic Gaussian
LQOC	Linear Quadratic Optimal Control
MIMO	Multi Input Multi Output
MISO	Multi Input Single Output
MKFP	Modified Kalman Filter Predictor
ORP	Ogunnaike and Ray Predictor

PID or PI	Proportional, Integral and Derivative control
SISO	Single Input Single Output
SP	Smith Predictor
STKFP	Self-tuning KFP
TFM	Transfer Function Matrix
Transfer Function	used to refer to both time invariant e.g $G_p(q^{-1})$, and time varying function e.g. $G_F(t,q^{-1})$

1. Introduction

Time-delays and noise are two of the most common problems in industrial process control applications. The original objectives of this thesis were therefore to use a Kalman filter approach to develop a time-delay compensation scheme which could be applied to stochastic procsses. However, as the work progressed it became obvious that the Kalman filter approach could also be used as the basis for developing predictive controllers and, still later, that these systems could be made adaptive by incorporating an on-line parameter estimation algorithm. In its final form then, this thesis deals with:

1) The formulation of state space models for multivariable processes with time delays
2) Kalman filtering
3) predictive controllers
4) adaptive systems.

For convenience, chapters 2 through 5, are presented as four relatively independent sections in accordance with the "paper format" option for theses at the University of Alberta. These chapters represent the approximate chronological order of the work and present the results in the logical order of:

1) SISO known parameter systems
2) MIMO known parameter systems
3) SISO adaptive systems

4) MIMO adaptive systems.

Each chapter builds on the results of previous chapters and numerous cross references have been included. However, key results from earlier chapters have been summarized in each chapter to make them relatively independent.

Chapters 2 through 5, each contain their own introduction, literature survey, theoretical development, discussion, simulated applications and conclusions. However, the overall conclusions and some suggestions for further work have been included in chapter 6 to provide a more global perspective on the contributions of this thesis.

2. Kalman Filter Predictor

2.1 Introduction

Time delays are often encountered in chemical processes, and significantly degrade the performance of conventional control loops. Therefore, a large volume of literature has been published on dead time compensation. One of the most common approaches is based on the predictor technique, first proposed by Smith (1957, 1959). The Smith predictor (SP), predicts the future process output, based on a priori knowledge of the process dynamics, and uses this predicted value to improve the performance of the feedback control system.

Another practical situation where the conventional PID controller tends to fail or requires detuning, is in the presence of process and/or measurement noise. When both random *measurement* noise and *process* noise are present in the process output, only the *process* noise can be compensated using a feedback control scheme. Thus the ideal thing to do is to filter out the measurement noise, and pass only the process noise and the process information to the controller. Although a filter can be designed to reduce the measurement noise there is always a compromise between noise elimination and degradation of closed loop stability. Stronger filtering usually requires detuning the controller and hence results in sluggish control performance.

The Smith perdictor, which was developed for use with deterministic (noise free) systems, does not take into account the statistical properties of the noise. It simply includes the noise in the calculated predicted output. This can be improved by introducing a filter as in Internal Model Control (IMC) due to Garcia and Morari (1982), but the improvement depends on the relative amount of process and measurement noise present.

A predictor scheme, based on a steady state Kalman filter (Kalman, 1960) was proposed by Bialkowski (1978, 1983). The Kalman filter, since it is an optimal stochastic state estimator, gives minimum variance estimates of the future outputs. An appropriate state space representation of the processes is used to predict the future outputs. The Kalman filter takes into account the apriori knowledge of the statistical properties of the process and measurement noise, and performs optimal filtering. Satisfactory performance of this Kalman filter scheme, in pulp and paper industry applications is reported by Bialkowski (1978).

In this chapter the theoretical aspects of the Kalman Filter Predictor (KFP) are investigated further. For example, the observability of the general state space formulation of the Kalman filter perdictor is proved, which guarantees the stability and the convergence of the KFP. Then by introducing the innovation model concept for the Kalman filter perdictor, it is shown that the Kalman filter perdictor is equivalent to a Smith predictor with a filter, as in internal model control. The innovation model also leads to a simplified implementation of the KFP.

The predicted future process output values from the KFP can be used in several different ways, e.g. feedback control systems such as LQG, PID, Predictive etc. PID and predictive control schemes are investigated in this report. It is shown that the predictive controller gives minimum variance control.

Satisfactory performance of feedback control schemes based on KFP, are demonstrated by means of simulations. If the noise statistics are not known a priori, the ratio of the process and measurement noise covariances can be used as a tuning parameter, to obtain the minimum variance control.

In the presence of deterministic disturbances, e.g. steps etc., the KFP gives biased output predictions, due to inappropriate formulation of the state space model. This problem is overcome by augmenting the state space equations with an additional state corresponding to the noise, as used in Balchan et al (1971, 1973) and Bialkowski (1983). The additional state in the KF acts as an integrator. Simulated results show that this modified KFP when used along with an incremental predictive controller gives good disturbance rejection.

2.2 State Space Representation

The basis for the KFP lies in the specific state space formulation it uses to represent the process. A linear model for an n^{th} order discretized process with time delays is shown, in block diagram form, in figure 2.1.

In this formulation, the output of the block representing the process without time delay (except the delay due to ZOH), and the output of each delay block are considered to be state variables. If there are d-delay blocks, the last d states of the state vector are due to unit delays. There will be an additional n (order of the process) state variables due to the process itself. The states corresponding to the unit delays are simply delayed values of the output $y_1(t)$ of the process without time delays (see figure 2.1). This is also demonstrated in figure 2.2, using a first order process with time delays.

Initially the process noise and the measurement noise present in the system are neglected for convenience. They are introduced later in the final formulation. Assume that the process of interest can be adequately modelled by a n^{th} order ARMA representation,

$$y(t) = A^{-1}(q^{-1})B(q^{-1})u(t) \tag{2.2.1}$$

where

$$A(q^{-1}) = 1+a_1q^{-1}+ \cdot \cdot \cdot + a_nq^{-n} \tag{2.2.2}$$

$$B(q^{-1}) = q^{-(1+d)}[\ b_1+b_2q^{-1}+ \cdot \cdot \cdot +b_nq^{-n+1}\ ] \tag{2.2.3}$$

d - (an integer) is the time delay in the process, excluding the discretization delay.

The ARMA representation for the process without time delays is given by,

$$y_1(t) = q^{-1}A^{-1}(q^{-1})B'(q^{-1})u(t) \tag{2.2.4}$$

where

$$B'(q^{-1}) = b_1+b_2q^{-1}+ \cdot \cdot \cdot +b_nq^{-n+1} \tag{2.2.5}$$

Instead of the ARMA representation a state space formulation in observable canonical form can be derived via a suitable linear transformation. The observable canonacal state space model is important to express y_1 as a state.

Let the standard state space formulation for the process be,

$$\mathbf{x}_0(t+1) = \Phi_0\mathbf{x}_0(t) + \Lambda_0\ u(t-d) \tag{2.2.6}$$

$$y(t) = \Theta_0\mathbf{x}_0(t) \tag{2.2.7}$$

The state space formulation of the process without time delay is given by,

$$\mathbf{x}_0^1\ (t+1) = \Phi_0\mathbf{x}_0^1\ (t) + \Lambda_0\ u(t) \tag{2.2.8}$$

$$y_1(t) = \Theta_0\mathbf{x}_0^1\ (t) \tag{2.2.9}$$

An observable state space formulation, corresponding to the ARMA model (2.2.4) or, by a linear transformation to (2.2.8, and 9) is given by,

$$\mathbf{x}^1(t+1) = \Phi_1\mathbf{x}^1(t) + \Lambda_1\ u(t) \tag{2.2.10}$$

$$y_1(t) = \Theta_1\mathbf{x}^1(t) \tag{2.2.11}$$

where,

$$\Phi_1 = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & -a_n \\ 1 & 0 & \cdot & \cdot & \cdot & -a_{n-1} \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ 0 & 0 & \cdot & 1 & 0 & -a_2 \\ 0 & 0 & \cdot & 0 & 1 & -a_1 \end{bmatrix}_{n\times n} \qquad \Lambda_1 = \begin{bmatrix} b_n \\ b_{n-1} \\ \cdot \\ \cdot \\ \cdot \\ b_2 \\ b_1 \end{bmatrix}_{n\times 1}$$

$$\Theta_1 = [\ 0\ 0\ \cdot\ \cdot\ \cdot\ 0\ 1\ ]^T_{1\times n}$$

$$x = [\ x_1^1\ , x_2^1\ , \cdot\ \cdot\ \cdot x_n^1\ ]^T$$

Considering $y_1(t)$ as the input, the state space representation corresponding to the delay states is given by,

$$z(t+1) = \Phi_2 z(t) + \Lambda_2\ y_1(t) \tag{2.2.12}$$

$$y(t) = \Theta_2 z(t) \tag{2.2.13}$$

where,

$$\Phi_2 = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ 0 & 0 & \cdot & \cdot & 1 & 0 \end{bmatrix}_{d\times d} = \begin{bmatrix} 0 \\ e_1^T \\ e_2^T \\ \cdot \\ \cdot \\ e_{d-1}^T \end{bmatrix}_{d\times d}$$

$$\Lambda_2 = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}_{d\times 1}$$

$$e_j^T = [\ 0\ 0\ \cdot\ \cdot\ \cdot\ 1\ \cdot\ \cdot\ 0\ ]_{1\times d}$$

$$\Theta_2 = [\ 0\ 0\ \cdot\ \cdot\ \cdot\ 0\ 1\ ]^T{}_{1\times d}$$

$$z = [\ z_1, z_2, \cdot\ \cdot\ \cdot z_d\ ]^T$$

Assuming that only the states of the system (2.2.10 and 11) are influenced by the process noise, and also introducing measurement noise at the output, the above two systems can be augmented to give the following state space formulation, which represents the process with time delays.

$$x(t+1) = \Phi x(t) + \Lambda u(t) + \Gamma w(t) \qquad (2.2.14)$$

$$y(t) = \Theta x(t) + v(t) \qquad (2.2.15)$$

where

$$\Phi = \begin{bmatrix} \Phi_1 & 0 \\ \Theta_x & \Phi_2 \end{bmatrix}_{(n+d)x(n+d)} \qquad \Lambda = \begin{bmatrix} \Lambda_1 \\ 0 \end{bmatrix}_{(n+d)x1} \qquad \Gamma = \begin{bmatrix} \Gamma_1 \\ 0 \end{bmatrix}_{(n+d)x1}$$

$$\Theta_x = \begin{bmatrix} \Theta_1 \\ 0 \end{bmatrix} \qquad \Gamma_1 = \begin{bmatrix} \gamma_1 \\ \gamma_n \end{bmatrix} \qquad \Theta = [\; 0 \quad \Theta_2 \;]_{1x(n+d)}$$

$w(t)$ - process noise or state excitation noise

$v(t)$ - measurement or observation noise

$w(t)$ and $v(t)$ are assumed to be white noise sequences with zero mean and have covariances defined by,

$$E\{w(s)\ w(s)^T\} = R_w \text{ and } E\{v(s)\ v(s)^T\} = R_v$$

respectively.

$x(t) = [x^1(t)^T : z(t)^T]^T$ where for convenience x is defined as follows:

$$x(t) = [x_1,\ x_2,\ \ldots\ x_n,\ x_{n+1},\ \ldots\ x_{n+d}]^T.$$

The states x_n to x_{n+d-1} are the future outputs of the process, and states x_{n+1} to x_{n+d} are just the delayed versions of x_n, ie y_1.

If (2.2.14 and 15) are obtained from the ARMA representation, the only non-zero, non-unity elements in the matrices Φ and Λ are the ARMA parameters.

The dimension of the state vector depends on the order of the process and on the time delay. Thus it depends on the sampling time used for the discretization. It is also assumed that the time delay in the process is an integer multiple of the sampling time.

Notice that the state transition matrix Φ is a singular matrix.

2.3 Kalman Filter Predictor

The states $x_n(t)$ to $x_{n+d-1}(t)$ of the state space formulation (equations 2.2.14 and 15) are the future outputs y(t+d) to y(t+1) respectively. Thus estimating the states of the process at time t is equivalent to predicting the future outputs y(t+1) to y(t+d). In principle any state estimator can be used to estimate these states. However, since it is assumed that both process and measurement noise are present, a Kalman filter is used to produce optimal predictions of the future outputs, i.e.,

$$\hat{x}_n(t) = \hat{y}(t+d|t),\ \hat{x}_{n+1}(t) = \hat{y}(t+d-1|t),\ \cdots,$$
$$\hat{x}_{n+d-1}(t) = \hat{y}(t+1|t).$$

This predictor scheme, based on the state space formulation given by equations (2.2.14 and 15), and the Kalman filter, is called the Kalman filter perdictor. A schematic block diagram of a feedback control scheme employing a predictor, e.g. KFP, SP, etc., is shown in figure 2.3. The predicted outputs $\hat{y}(t+1|t)$ to $\hat{y}(t+d|t)$ can be used with any type of control scheme. Two types of control schemes are discussed later in section 2.5.

The Kalman filter perdictor can be implemented using either a steady state Kalman filter or a time varying Kalman filter, as discussed in the next section.

2.3.1 Kalman Filter

The Kalman filter first presented by Kalman (1960) and Kalman and Bucy (1961) can be used as a noise filter an estimator or a predictor, for stochastic processes. In this work the Kalman filter is employed as an estimator. The prediction property of the KFP is due to the specific state space formulation.

If the process noise and the measurement noise present in the system are Gaussian, then the Kalman filter gives the minimum variance estimates of the states, that is it estimates the conditional mean of x(t), given the data,

$$Y_t^T = [y(t), y(t-1), \cdots]$$

$$\hat{\mathbf{x}}(t) = E[\ \mathbf{x}(t) | Y_t] \qquad (2.3.1)$$

$\hat{\mathbf{x}}(t)$ is the minimum variance estimate of $\mathbf{x}(t)$, that is,

$$E[\ \mathbf{x}(t) - \hat{\mathbf{x}}(t) | Y_t] = 0 \qquad (2.3.2)$$

and the covariance of the estimation error,

$$\mathbf{P} = E\{[\mathbf{x}(t)-\hat{\mathbf{x}}(t)][\mathbf{x}(t)-\hat{\mathbf{x}}(t)]^T | Y_t\} \leq \mathbf{P}_F \qquad (2.3.3)$$

where $\mathbf{P}_F$ is the error covariance given by any other filter.

There are a number of different ways that one could express the Kalman filter algorithm (Astrom, 1970, Goodwin and Sin, 1984).

The Kalman filter algorithm used in the KFP, is a 2-step algorithm in which the states and the covariances are updated, when a measurement is available, and before the next measurement is made. However this algorithm can also be implemented as a single algorithm step. Some of the relevant theory leading to the 2-step algorithm is given in Appendix A.

2.3.1.1 Time varying Kalman Filter Algorithm

The time varying Kalman filter algorithm (Franklin the Powell ,1980), and which is discussed in Appendix A, is as follows:

a. Gain Calculation:

$$L(t) = M(t)\Theta^T[\Theta M(t)\Theta^T+R_2]^{-1} \qquad (2.3.4)$$

where, $R_2 = R_v$

b. Measurement Update (at the time of measurement):

b1. State Update:

$$\hat{x}(t) = \bar{x}(t) + L(t)[y(t)-\Theta\bar{x}(t)] \qquad (2.3.5)$$

b2. Covariance Update:

$$P(t) = M(t) - L(t)\Theta M(t) \qquad (2.3.6)$$

c. Time Update (between measurements):

c1. State Update:

$$\bar{x}(t+1) = \Phi\hat{x}(t) + \Lambda u(t) \qquad (2.3.7)$$

c2. Covariance Update:

$$M(t+1) = \Phi P(t)\Phi^T + R_1 \qquad (2.3.8)$$

where, $R_1 = \Gamma R_w \Gamma^T$

where,

$\hat{\mathbf{x}}(t) = \hat{\mathbf{x}}(t|t) = \mathbf{E}\,[\mathbf{x}(t)|Y_t]$

= estimation of $\mathbf{x}(t)$ using data up to time t.

$\bar{\mathbf{x}}(t) = \bar{\mathbf{x}}(t|t-1) = \mathbf{E}\,[\mathbf{x}(t)|Y_{t-1}]$

=estimation of $\mathbf{x}(t)$ using data up to time t-1.

$\mathbf{P}(t) = \mathbf{E}\,[(\mathbf{x}(t)-\hat{\mathbf{x}}(t))(\mathbf{x}(t)-\hat{\mathbf{x}}(t))^T]$

=error covariance of estimate $\hat{\mathbf{x}}(t)$

$\mathbf{M}(t) = \mathbf{E}\,[(\mathbf{x}(t)-\bar{\mathbf{x}}(t))(\mathbf{x}(t)-\bar{\mathbf{x}}(t))^T]$

=error covariance of estimate $\bar{\mathbf{x}}(t)$.

$\mathbf{L}(t)$ = Kalman gain

From equations (2.3.4, 6 and 8) it is clear that the Kalman gain and the covariances **P** and **M** are not functions of the measurements. Hence the gains and the covariances can be calculated off line to reduce the computational burden.

2.3.1.2 Steady State Kalman Filter

The algorithm given above can be used to calculate the time varying Kalman gains and the error covariances. In many cases the error covariances $\mathbf{M}(t)$ and $\mathbf{P}(t)$, and hence the Kalman gain $\mathbf{L}(t)$ converge to steady state values as t-->∞.

If $\mathbf{M}(t)$ and $\mathbf{P}(t)$ converge to **M** and **P**, the limiting solution of **M** will satisfy the following Algebraic Riccati Equation (ARE), obtained by combining equations (2.3.6 and 8) and substituting $\mathbf{M}(t+1) = \mathbf{M}$.

The algebraic Riccati equation is given by,

$$M - \Phi M \Phi^T + \Phi L \Theta M \Phi^T - R_1 = 0 \qquad (2.3.9)$$

where

L - steady state Kalman filter gain

$$L = M\Theta^T[\Theta M \Theta^T + R_2]^{-1} \qquad (2.3.10)$$

Also define the steady state filter state transition matrix $\overline{\Phi}$, and $\overline{\Theta}$,

$$\overline{\Phi} = \Phi - L\Theta\Phi \qquad (2.3.11)$$

$$\overline{\Theta} = R_2^{-1/2}\,\Theta \qquad (2.3.12)$$

and factorize R_1 and R_2 as,

$$R_1 = EE^T \qquad (2.3.13)$$

$$R_2 = R_2^{1/2}\, R_2^{1/2} \qquad (2.3.14)$$

Then the ARE is given by,

$$M - \Phi M \Phi^T + \Phi M \overline{\Theta}^T(\overline{\Theta} M \overline{\Theta}^T + I)^{-1}\,\overline{\Theta} M \Phi^T - EE^T = 0 \qquad (2.3.15)$$

There are a number of different methods that can be used to solve the ARE. Some of these methods are given in Potter (1966), Vaughan (1970) and Kleinman (1968). Two methods that are computationally efficient, and have less numerical difficulties are given in Laub (1979) and Pappas et al (1980).

Since the state space formulation used in the KFP has a singular state transition matrix, it belongs to a special class of systems, with respect to the calculation of the steady state solution of the ARE. The singularity of the state transition matrix causes difficulties in using some of the methods to solve ARE, because it is necessary to invert the matrix Φ (Vaughan, 1970). However, the singularity of Φ does not create problems in recursive estimation because it is not necessary to invert the matrix Φ.

When Φ is singular, the two methods proposed by Pappas et al (1980) can be used to solve ARE. The first of these two methods is related to Potter's method, and the second method is related to Laub's method which utilizes generalized Schur vectors.

2.3.2 Stability and convergence of the Kalman Filter Predictor

The stability of the KFP depends on the stability of the Kalman filter, which in turn depends on the solution of the ARE (2.3.15). For stability analysis we are interested in the solutions of ARE, which are real, symmetric, positive semi-definite and which give a steady state filter having its roots on or inside the unit circle, i.e. eigenvalues of $\bar{\Phi}$.

A summary of these properties as discussed in Goodwin and Sin (1984) is given in Appendix B. Further details and proofs of the theorems are given in Martensson (1971), Kucera (1972a, 1972b) and Anderson & Moore (1979).

From Lemma B1 in Appendix B, the properties of the steady state solution of the ARE depend on the pairs (Θ, Φ)

and (Φ, E). E as given by equation (2.3.13).

Condition 1. (Θ, Φ) is detectable.

(Θ, Φ) is detectable if all unobservable modes have corresponding eigenvalues strictly inside the unit circle.

Condition 2. (Φ, E) is stabilizable.

(Φ, E) is stabilizable if all uncontrollable modes of (Φ, E) have corresponding eigenvalues strictly inside the unit circle.

These two conditions lead to the following important results:

a. Stability of the Kalman filter predictor

If condition 1 and 2 are satisfied, from Lemma B.1 in Appendix B, there will be a stabilizing solution for the ARE, which guarantees that the corresponding filter state transition matrix $\bar{\Phi}$ has all its eigenvalues inside the unit circle. Thus the stability of the Kalman filter predictor is guaranteed.

b. Convergence of Kalman Filter Predictor

If conditions 1 and 2 are satisfied, theorem B.1 in Appendix B, guarantees the convergence of the time varying Kalman filter predictor to the steady state KFP. According to theorem B1, the error covariance $M(t)$, Kalman gain $L(t)$ and $\bar{\Phi}(t)$ in the KFP converge exponentially to their corresponding steady state values M, L and $\bar{\Phi}$, if M_0 (initial covariance) ≥ 0.

Condition 1 is proved by proving the observability of the pair (Θ, Φ), of the augmented state space formulation

(2.2.14 and 15).

The stabilizability condition for the pair (Φ, **E**) can be verified by testing for the controllability of the pair (Φ, **E**) and then the eigenvalues corresponding to the uncontrollable modes. Obviously the number of uncontrollable modes depends on the matrix **E**. Under deterministic conditions **E**=0, and this will result in n+d uncontrollable modes in pair (Φ, **E**). Hence it is necessary to prove that all the eigenvalues of the augmented system lie within the unit circle.

Conditions 1 and 2 are proved in following two Lemmas.

Lemma 2.3.1

If the state space representation given by equations (2.2.10 and 11) is observable then the augmented state space representation given by equation (2.2.14 and 15) is also observable.

Proof.

Since the state space formulation given by (equations 2.2.10 and 11) and (2.2.12 and 13) are observable, the observability matrices given by,

$$W_1 = [\ \Theta_1\ \Theta_1\Phi_1\ \Theta_1\Phi_1^2\ \cdots\ \Theta_1\Phi_1^{n-1}\ ]^T \qquad (2.3.16)$$

and,

$$W_2 = [\ \Theta_2\ \Theta_2\Phi_2\ \Theta_2\Phi_2^2\ \cdots\ \Theta_2\Phi_2^{d-1}\ ]^T \qquad (2.3.17)$$

have the ranks n and d respectively.

For the augmented system given by equations (2.2.14 and 15) with,

$$\Phi = \begin{bmatrix} \Phi_1 & 0 \\ \Theta_1 & 0 \\ 0 & e_1^T \\ \cdot & \cdot \\ 0 & e_{d-1}^T \end{bmatrix}_{(n+d)\times(n+d)} \qquad \Theta = [0 \; \cdot \; \cdot \; 1]_{1\times(n+d)}$$

The observability matrix is given by,

$$W = [\; \Theta \;\; \Theta\Phi \;\; \Theta\Phi^2 \;\cdot\;\cdot\; \Theta\Phi^{n+d-1} \;]^T \qquad (2.3.18)$$

To prove the observability, it is necessary to show that the rank of W is n+d.

For r≤d,

$$\Phi^r = \left[\begin{array}{c|c} \Phi_1^r & 0 \\ \hline \Theta_1\Phi_1^{r-1} & 0 \\ \cdot & \cdot \\ \cdot & \cdot \\ \Theta_1\Phi_1 & 0 \\ \Theta_1 & e_1^T \\ 0 & \cdot \\ \cdot & \cdot \\ 0 & e_{d-r}^T \end{array}\right] \qquad (2.3.19)$$

and,

$$\Theta\Phi^r = [\; 0 \quad e_{d-r}^T \;]_{1\times(n+d)} \qquad (2.3.20)$$

For n+d-1 ≥ r > d

$$\Phi^r = \left[\begin{array}{c|c} \Phi_1^r & 0 \\ \hline \Theta_1\Phi_1^{r-1} & \\ \Theta_1\Phi_1^{r-2} & 0 \\ \cdot & \\ \cdot & \\ \Theta_1\Phi_1^{r-d} & \end{array}\right] \qquad (2.3.21)$$

and,

$$\Theta\Phi_r = [\ \Theta_1\Phi_1^{r-d} \quad 0\ ]_{1\times(n+d)} \tag{2.3.22}$$

Substituting (2.3.20 and 22) in (2.3.18),

$$W = \left[\begin{array}{c|c} & e_d^T \\ 0 & e_{d-1}^T \\ & \cdot \\ & \cdot \\ & e_1^T \\ \hline \Theta_1 & \\ \Theta_1\Phi_1 & 0 \\ \cdot & \\ \cdot & \\ \Theta_1\Phi_1^{n-1} & \end{array}\right] \tag{2.3.23}$$

Obviously the rank of W is n+d, thus the augmented system is observable.

Lemma 2.3.2

If the system given by (2.3.10 and 11) is stable, then the eigenvalues of the augmented system lie within the unit circle.

Proof:-

Since

$$\Phi = \left[\begin{array}{c|c} \Phi_1 & 0 \\ \hline e_n^T & \\ 0 & \Phi_2 \\ \cdot & \\ 0 & \end{array}\right] \tag{2.3.24}$$

$$[qI-\Phi] = \begin{bmatrix} qI-\Phi_1 & 0 \\ & \\ -e_n^T & \\ 0 & qI-\Phi_2 \\ \cdot & \\ \cdot & \\ 0 & \end{bmatrix} \tag{2.3.25}$$

Eigenvalues of the augmented system are given by the roots of the equation,

$$\det[qI-\Phi] = 0$$

$$= \det[qI-\Phi_2]\det[qI-\Phi_2] = 0 \qquad (2.3.26)$$

Since,

$$\Phi_2 = \begin{bmatrix} 0 \\ e_1^T \\ \cdot \\ \cdot \\ e_{d-1}^T \end{bmatrix}_{dxd}$$

$$\text{then } \det[qI-\Phi_2] = q^d \qquad (2.3.27)$$

which means that all the eigenvalues of Φ_2 are at $q=0$. Also,

$$\det[qI-\Phi_1] = (q-p_1)(q-p_2) \cdot \cdot (q-p_n). \qquad (2.3.28)$$

Since the process without time delays is stable the eigenvalues $p_1 \cdots p_n$ are stable. Thus for any value of E the augmented state space system is (Θ, E) stabilizable.

2.4 Innovation Model Approach for the Kalman Filter Predictor

The Kalman filter predictor gives the minimum variance estimates of the states i.e. predicted outputs, but its structure and internal behaviour are not apparent. The internal operation of the KFP can be better understood by deriving the transfer function of the Kalman filter. Owing to the large dimensions of the matrices involved, a general derivation is cumbersome. Hence the innovation model concept as given in Appendix C is used to investigate the configuration of the KFP.

2.4.1 Innovation Model for KFP

The innovation model of the KFP is obtained by applying equation (C.1) to the state space formulation given by equations (2.2.14 and 15).

$$\hat{x}(t+1) = \Phi\hat{x}(t) + \Lambda u(t) + L(t+1)[\ y(t+1) - \Theta\Phi\hat{x}(t) - \Theta\Lambda u(t)\ ] \qquad (2.4.1)$$

with,

$$\Theta\Phi = [\ 0\ 0\ \cdot\ \cdot\ 1\ 0\ ] \text{ if } d\geq 1$$

$$\Theta\Lambda = [\ 0\ ]$$

From equation (C.2) the estimated current output is,

$$\hat{y}(t) = \hat{y}(t|t-1) = \Theta\Phi\hat{x}(t-1) + \Theta\Lambda u(t-1) = \hat{x}_{n+d-1}(t-1) \qquad (2.4.2)$$

Equation (2.4.1) can also be written as,

$$\hat{x}(t+1) = I'\hat{x}(t) + \Phi'\hat{x}_n(t) + \Lambda u(t) + L(t+1)\omega(t) \qquad (2.4.3)$$

where,

$$I' = \begin{bmatrix} 0 & 0 & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \cdot & 0 \\ \cdot & 1 & & & \cdot \\ \cdot & \cdot & & & \cdot \\ 0 & 0 & \cdot & 1 & 0 \end{bmatrix}_{(n+d)x(n+d)} \qquad \Phi' = \begin{bmatrix} -a_n \\ -a_{n-1} \\ \cdot \\ -a_1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}_{(n+d)x1}$$

and,

$$\omega = y(t) - \hat{y}(t) \qquad (2.4.4)$$

where, $\omega(t)$, the innovation sequence - is a random signal with zero mean.

Applying Lemma D.1 to equation (2.4.3) to solve for $\hat{x}_n(t)$, gives

$$\hat{x}_n(t) = [\ 1-A(q^{-1})\ ]\hat{x}_n(t) + q^{-1}B'(q^{-1})u(t) + K_1(t,q^{-1})\omega(t) \tag{2.4.5}$$

where,

$$A(q^{-1}) = 1+a_1q^{-1}+ \cdot \cdot \cdot + a_nq^{-n} \tag{2.4.6}$$

$$B'(q^{-1}) = b_1+b_2q^{-1}+ \cdot \cdot \cdot +b_nq^{-n+1} \tag{2.4.7}$$

$$K_1(t,q^{-1}) = L_n(t) + L_{n-1}(t-1)q^{1} \cdot \cdot + L_1(t-n+1)q^{-n+1} \tag{2.4.8}$$

Equation (2.4.5) can be re-written as,

$$\hat{x}_n(t) = q^{-1}A^{-1}(q^{-1})B'(q^{-1})u(t) + A^{-1}(q^{-1})K_1(t,q^{-1})\omega(t) \tag{2.4.9}$$

Using Lemma D.1 in equation (2.4.3) to solve for $\hat{x}_{n+d-1}(t)$,

$$\hat{x}_{n+d-1}(t) = [\ 1-A(q^{-1})\ ]q^{-d+1}\hat{x}_n(t) + q^{-d}B'(q^{-1})u(t) + K(t)\omega(t) \tag{2.4.10}$$

where,

$$K(t,q^{-1}) = K_2(t,q^{-1}) + q^{-d+1}K_1(t,q^{-1}) \tag{2.4.11}$$

with

$$K_2(t,q^{-1}) = L_{n+d-1}(t) + L_{n+d-2}(t-1)q^{-1} + \cdot \cdot \cdot \cdot + L_{n+1}(t-d+2)q^{-d+2} \tag{2.4.12}$$

From (2.4.9,10 and 11),

$$\hat{x}_{n+d-1} = q^{-d}B'(q^{-1})A^{-1}(q^{-1})u(t) + A^{-1}(q^{-1})K_1(t-d+1)(q^{-1})q^{-d+1}\omega(t) +K_2(t,q^{-1})\omega(t) \tag{2.4.13}$$

Since,

$$y(t) = \hat{x}_{n+d-1}(t-1) + \omega(t) \tag{2.4.14}$$

substituting (2.4.14) in (2.4.13) we get,

$$A(q^{-1})y(t) = B(q^{-1})u(t) + A(q^{-1})[1+q^{-1}K_2(t-1,q^{-1}) + q^{-d}A^{-1}(q^{-1})K_1(t-d,q^{-1})]\omega(t) \qquad (2.4.15)$$

where,

$$B(q^{-1}) = q^{-(d+1)}B'(q^{-1}) \qquad (2.4.16)$$

Defining,

$$\begin{aligned} C(t,q^{-1}) &= A(q^{-1})[1+q^{-1}K_2(t-1,q^{-1}) + q^{-d}A^{-1}(q^{-1})K_1(t-d,q^{-1})] \\ &= 1+c_1(t)q^{-1}+c_2(t)q^{-2} + \cdot\cdot\ c_{n+d-1}(t)q^{-d-n+1} \end{aligned} \qquad (2.4.17)$$

From equation (2.4.1 and 17),

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(t,q^{-1})\omega(t) \qquad (2.4.18)$$

Equation (2.4.18) shows that the Kalman filter predictor can be expressed in terms of an input/output model. The time varying nature of the C polynomial is due to the time varying Kalman gains. As shown in section (2.3.2), the time varying KFP will converge to the steady state Kalman filter, and hence the polynomial $C(t,q^{-1})$ will converge to $C(q^{-1})$.

From equation (2.4.2 and 13) we get,

$$\hat{y}(t) = A^{-1}(q^{-1})B(q^{-1})u(t)\ [1+q^{-1}K_2(t-1,q^{-1}) + q^{-d}A^{-1}(q^{-1})K_1(t-d,q^{-1})]\omega(t) \qquad (2.4.19)$$

The predicted value of the process without time delays, i.e. $\hat{y}_1(t) = \hat{y}(t+d)|t) = \hat{x}_n(t)$ is given from equation (2.4.9).

$$\hat{y}(t+d|t) = q^{-1}A^{-1}(q^{-1})B'(q^{-1})u(t) + A^{-1}(q^{-1})K_1(t,q^{-1})\omega(t) \tag{2.4.20}$$

From equations (2.4.18 and 20),

$$\hat{y}(t+d|t) = q^{-1}A^{-1}(q^{-1})B'(q^{-1})u(t)$$

$$\pm \frac{K_1(t,q^{-1})}{C(t,q^{-1})}[\ y(t) - q^{-(d+1)}A^{-1}(q^{-1})B'(q^{-1})u(t)\ ] \tag{2.4.21}$$

2.4.2 Configuration of the KFP

Equation (2.4.21) can be used to obtain the configuration of the KFP and to interpret its behaviour.

The following transfer functions are defined for convenience,

$$\begin{aligned} y_m(q^{-1}) &= q^{-(d+1)}A^{-1}(q^{-1})B'(q^{-1})u(t) \\ &= A^{-1}(q^{-1})B(q^{-1})u(t) \\ &= G_m(q^{-1})u(t) \end{aligned} \tag{2.4.22}$$

$$\begin{aligned} y_p(q^{-1}) &= q^{-1}A^{-1}(q^{-1})B'(q^{-1})u(t) \\ &= G_p(q^{-1})u(t) \end{aligned} \tag{2.4.23}$$

and,

$$G_r(t,q^{-1}) = \frac{K_1(t,q^{-1})}{C(t,q^{-1})} \tag{2.4.24}$$

Substituting equations (2.4.22, 23 and 24) in (2.4.21) we get,

$$\begin{aligned} \hat{y}(t+d|t) &= G_p(q^{-1})u(t) \\ &\quad + G_r(t,q^{-1})\ [\ y(t) - G_m(q^{-1})u(t)\ ] \end{aligned} \tag{2.4.25}$$

or,

$$\hat{y}(t+d|t) = y_p(t) + G_r(t,q^{-1})\ [\ y(t) - y_m(t)\ ] \tag{2.4.26}$$

A schematic block diagram of the internal structure of the KFP, based on equation (2.4.26) is shown in figure 2.4. This gives a simple interpretation to the KFP. This structure is similar to the Internal Model Control (IMC) due to Garcia and Morari (1982) and to Smith predictor (SP) (Smith, 1957, 1959), as shown in figure 2.5 and 6 respectively. Also see the discussion in section 2.7.

In the KFP the error $y_e(t)$, between the actual output y and the model output y_m, is filtered by the time varying filter $G_F(t,q^{-1})$, before being added to the output of the model G_p, i.e. y_p, to obtain the predicted output $\hat{y}(t+d|t)$. $y_e(t)$ represents the noise, disturbance and/or any unmodelled dynamics present in the system.

The time varying nature of the filter $G_F(t,q^{-1})$ is due to the time varying Kalman gains. This time varying filter will asymptotically converge to a steady state filter if the Kalman gains converge to their steady state value (convergence of the KF is discussed in section 2.3).

The stability of the KFP, i.e. the bounded prediction $\hat{y}(t+d|t)$, discussed in section 2.3, can now be interpreted as the stability of filter $G_F(t,q^{-1})$ if the model G_m is a stable system. Since the polynomials C and K_1 are asymptotically time invariant, the time invariant filter $G_F(q^{-1})$ is considered for stability. The stability of the steady state filter $G_F(q^{-1})$ depends on the roots of the polynomial $C(q^{-1})$. If the roots of the polynomial $C(q^{-1})$ are inside the unit circle, then $G_F(q^{-1})$ is a stable filter.

Consider the equation (2.4.1 and 4),

$$\hat{x}(t+1) = \Phi\hat{x}(t) + \Lambda u(t) + L(t+1)[\ y(t+1) - \Theta\Phi\hat{x}(t) - \Theta\Lambda u(t)\ ] \qquad (2.4.27)$$

$$\omega(t) = y(t) - \Theta\Phi\hat{x}(t-1) - \Theta\Lambda u(t-1) \qquad (2.4.28)$$

Writing equations (2.4.25 and 28), in transfer function form and eliminating $\hat{x}(t)$ we get,

$$\omega(t) = \{\ 1 - \Theta\Phi[\ qI - \Phi + L\Theta\Phi\ ]^{-1}L\}\ y(t) + \{\ \Theta\Phi[\ qI - \Phi + L\Theta\Phi\ ]^{-1}[\ \Lambda - L\Theta\Lambda\ ] - \Theta\Lambda\ \}q^{-1}u(t) \qquad (2.4.29)$$

Consider the equation (2.4.18),

$$\omega(t) = \frac{A(q^{-1})}{C(q^{-1})}y(t) - \frac{B(q^{-1})}{C(q^{-1})}u(t) \qquad (2.4.30)$$

Comparing equations (2.4.29 and 30),

$$C(q^{-1}) = \det [\ qI - \Phi + L\Theta\Phi\ ] \qquad (2.4.31)$$

Since $\bar{\Phi} = \Phi - L\Theta\Phi$, see equation (2.3.11),

$$C(q^{-1}) = \det [\ qI - \bar{\Phi}\ ] \qquad (2.4.32)$$

It was shown in section (2.3.2) that for the KFP, the steady state filter transition matrix $\bar{\Phi}$ has its eigenvalues inside the unit circle. Hence $G_F(q^{-1})$ is a stable filter.

The steady state gain of the time invariant filter $G_F(q^{-1})$ is given by,

$$G_{Fss} = \lim_{t \to \infty} G_F(q^{-1}) = \frac{\sum_{i=1}^{n} L_i}{\sum_{i=1}^{n+d-1} C_i} \qquad (2.4.33)$$

If $G_{FSS} \neq 1$ then the KFP will cause problems in the presence of deterministic (e.g. step) disturbances. Further discussion of this problem is given in section (2.6).

2.4.3 Input output model for the KFP

The equations (2.4.4 and 18-20) obtained in the previous section provide a means of interpreting the KFP scheme from an input-output model point of view. If the input-output model given by equation (2.4.18), with steady state $C(q^{-1})$ polynomial, is known the coefficients of $K_1(q^{-1})$ and $K_2(q^{-1})$ polynomials can be calculated using the relationship given by equation (2.4.17). Thus $\hat{y}(t)$, $\hat{y}(t+d|t)$ and $\omega(t)$ can be evaluated from the KFP innovation model equations (2.4.19, 20 and 24) respectively.

2.5 Feedback control using the KFP

The predicted future outputs, and if necessary the other estimated states of the KFP, can be employed with any type of controller design. Bialkowski (1978) used the steady state KFP with PID and LQG controller designs. In this chapter we investigate the PID and predictive control schemes using the KFP.

2.5.1 PID Control

The conventional PID control algorithm can be employed with the KFP as shown in figure 2.3. The tracking error in this case is calculated not with the measured output $y(t)$, but with the predicted output $\hat{y}(t+d|t)$. The PID controller parameters are tuned for the model without delays G_p, rather than for the actual process.

The tracking error is given by,

$$e(t) = y^*(t) - \hat{y}(t+d|t) \qquad (2.5.1)$$

$y^*(t)$ - the desired output (set point).

and the PI control algorithm as,

$$u(t) = K_c[\ e(t) + \frac{T}{\tau_I} \sum_{i=0}^{t} e_i(t)\ ] \qquad (2.5.2)$$

where

T - sampling time

K_c - proportional gain

τ_I - integral reset time

In figure 2.4, $\xi(t)$ represents all types of disturbances (process noise, measurement noise and deterministic disturbances expressed as an additive signal to the output y(t)). The closed loop transfer function of the KFP with controller G_c, expressed in terms of the transfer functions G_m, G_p, G, and G_F in figure 2.4 is given by,

$$y(t) = \frac{GG_c}{1+G_cG_p+G_cG_F(t)(G-G_m)} y^*(t)$$

$$- \frac{G_FGG_c}{1+G_cG_p+G_cG_F(t)(G-G_m)} \xi(t) + \xi(t) \qquad (2.5.3)$$

The controller output u(t) is given by,

$$u(t) = \frac{G_c}{1+G_cG_p+G_cG_F(t)(G-G_m)} y^*(t)$$

$$- \frac{G_FG_c}{1+G_cG_p+G_cG_F(t)(G-G_m)} \xi(t) \qquad (2.5.4)$$

From equation (2.5.3), it is clear that the stability of the closed loop system depends on the roots of the characteristic equation,

$$1 + G_cG_p + G_c(G - G_m)\ G_F = 0 \tag{2.5.5}$$

When the KFP is used in an error driven feedback control system, stability of filter G_F, i.e. stability of the KFP, is not essential. However a stable KFP is preferred because it would provide a greater flexibility in selecting the controller parameters.

2.5.2 Predictive Control

The KFP is a model based predictor scheme and in particular it can be derived from the ARMA model of the process. This knowledge of the process parameters can be incorporated into a control algorithm by using a predictive control scheme.

The predictive control law to be used with the KFP is derived such that the mean-square error (or variance) between the measured output $y(t+d+1)$ and the desired output $y^*(t+d+1)$ is minimized.

Define the cost function,

$$J(t+d+1) = E\ \{\ [\ y(t+d+1) - y^*(t+d+1)\ ]^2\ \} \tag{2.5.6}$$

Choose $u(t)$ to minimize $J(t+d+1)$,

$$J(t+d+1) = E\ \{\ E\ \{\ [\ y(t+d+1) - y^*(t+d+1)\ ]^2 | Y_t\ \}\ \} \tag{2.5.7}$$

If $J^*(t+d+1)$ is the optimal value of $J(t+d+1)$, then,

$$J^*(t+d+1) = E\ \{\ \min E\ \{\ [\ y(t+d+1) - y^*(t+d+1)\ ]^2 | Y_t\ \}\ \} \tag{2.5.8}$$

Consider the ARMAX model of the process obtained by the innovation model approach, and which is given by equation (2.4.18),

$$A(q^{-1})\, y(t) = q^{-(d+1)}\, B'(q^{-1})u(t) + C(q^{-1})\omega(t) \qquad (2.5.9)$$

where, $\{\omega(t)\}$ is a white noise sequence satisfying,

$$E\ \{\ \omega(t) | Y_t\ \} = 0 \qquad (2.5.10)$$

$$E\ \{\ \omega(t)^2 | Y_t\ \} = \sigma^2 \qquad (2.5.11)$$

From equation (2.5.9),

$$y(t+d+1) = [\ 1 - A(q^{-1})\ ]\ y(t+d+1) + B'(q^{-1})\ u(t) + C(q^{-1})\ \omega(t+d+1) \qquad (2.5.12)$$

Using the Kalman filter it is possible to obtain the predictive terms,

$$[\ \hat{y}(t+d|t),\ \hat{y}(t+d-1|t), \cdots,\ \hat{y}(t+1|t)\ ]$$

or

$$[\ \hat{y}(t+d|t),\ \hat{y}(t+d-1|t-1), \cdots,\ \hat{y}(t+1/t-d+1)\ ] \qquad (2.5.13)$$

Since these predictive terms are obtained as the estimated states of the KF, let σ_y^2 be the error covariance given by,

$$\sigma_y^2 = E\ \{\ [\ y(t+d+1) - \hat{y}(t+d+1)\ ]^2 | Y_t\ \} \qquad (2.5.14)$$

$$= E\ \{\ [\ x_n(t+1) - \hat{y}(t+d-1) + v(t+d+1)\ ]^2 | Y_t\ \}$$

The second equation in (2.5.14) follow since,

$$y(t+d+1) = x_n(t+1) + v(t+d+1).$$

Since $[\ x_n(t+1) - \hat{y}(t+d-1)\ ]$ and $v(t+d+1)$ are independent,

$$\sigma_y^2 = \sigma_{nn}^2 + R_v \qquad (2.5.15)$$

where, σ_{nn}^2 - the (n,n)th element of the steady state error covariance matrix $P(\cdot)$ that satisfies the Riccati difference equation.

Taking the expectation of equation (2.5.12),

$$E\,[\ y(t+d+1|t)\ ] = E\ \{\ [\ 1 - A(q^{-1})\ ]\ y(t+d+1|t) + B'(q^{-1})u(t)\ \} \qquad (2.5.16)$$

Define

$$\bar{y}(t+d+1) = E\,[\ y(t+d+1|t)\ ] \qquad (2.5.17)$$

Consider the term

$$E\ \{\ [\ 1-A(q^{-1})\ ]\ y(t+d+1|t)\ \}$$
$$= E\,[\ - a_1 y(t+d|t) - a_2 y(t+d-1|t) \cdots - a_n y(t+d-n|t)\ ] \qquad (2.5.18)$$

Since the Kalman filter prediction is given by,

$$\hat{y}(t+d|t) = E\,[\ y(t+d|t)\ ] = \hat{y}(t+d) \qquad (2.5.19)$$

it follows from equation (2.5.16, 17 and 9) that,

$$\bar{y}(t+d+1) = [\ 1-A(q^{-1})\ ]\ \hat{y}(t+d+1) + B'(q^{-1})u(t) \qquad (2.5.20)$$

Equation (2.5.20) can be interpreted as the predictor equation for time t+d+1, based on the predictions made up to time t+d by the KFP.

From the cost function (2.5.8),

$$J^*(t+d+1) = E\ \{\ \min E\ \{\ [\ (y(t+d+1) - \bar{y}(t+d+1)) + (\bar{y}(t+d+1) - y^*(t+d+1))\ ]^2\ \}\ \} \qquad (2.5.21)$$

$$J^*(t+d+1) = E\ \{\ \min E\ \{\ [\ y(t+d+1) - \bar{y}(t+d+1)\ ]^2 + [\ \bar{y}(t+d+1) - y^*(t+d+1)\ ]^2\ \}\ \} \qquad (2.5.22)$$

Subtracting (2.5.20) from (2.5.12),

$$y(t+d+1) - \bar{y}(t+d+1) = [\ 1-A(q^{-1})\ ]\ [\ y(t+d+1) - \hat{y}(t+d+1)\ ] + C(q^{-1})\ \omega(t+d+1) \qquad (2.5.23)$$

Hence,

$$E \{ [y(t+d+1) - \hat{y}(t+d+1)]^2 \}$$

$$= E \{ [1-A(q^{-1})] (y(t+d+1) - \hat{y}(t+d+1))^2 \}$$

$$+ (E [(C(q^{-1})\omega(t+d+1))^2] \qquad (2.5.24)$$

From (2.5.15, 11 and 24),

$$E \{ [y(t+d+1) - \bar{y}(t+d+1)]^2 \}$$

$$= \sum_{j=1}^{n} a_j^2 \sigma_y^2 + \sum_{j=0}^{n+d-1} c_i^2 \sigma^2 \text{ with } c_0 = 1 \qquad (2.5.25)$$

From equation (2.5.25 and 22),

$$J^{*}(t+d+1) = E \{ \min E \{ [\bar{y}(t+d+1) - y^{*}(t+d+1)]^2 \} \}$$

$$+ \sum_{j=1}^{n} a_j^2 \sigma_y^2 + \sum_{i=i}^{n+d-1} c_i^2 \sigma^2 \qquad (2.5.26)$$

Thus the minimum $y^{*}(t+d+1)$ is obtained when,

$$\bar{y}(t+d+1) = y^{*}(t+d+1) \qquad (2.5.27)$$

with a minimum variance of,

$$J^{*}(t+d+1) = \sum_{j=1}^{n} a_j^2 \sigma_y^2 + \sum_{i=i}^{n+d-1} c_i^2 \sigma^2 \qquad (2.5.28)$$

The minimum variance control law using the KFP is obtained by substituting (2.5.27) in (2.5.20).

The control law is,

$$u(t) = \frac{1}{b_1} \{ y^{*}(t+d+1) - [1-A(q^{-1})] \hat{y}(t+d+1)$$

$$+ [b_1-B'(q^{-1})] u(t) \} \qquad (2.5.29)$$

with $b_1 \neq 0$

2.6 Modified Kalman Filter Predictor for deterministic disturbances

2.6.1 KFP in presence of Deterministic Disturbances

It was mentioned in section (2.4.2) that the filter $G_r(q^{-1})$ in the KFP causes problems in the presence of deterministic disturbances, because they violates the assumption of white noise made in the Kalman filter formulation. A remedy for this non-unity gain problem can be achieved by a further expansion of of the KFP. For convenience consider only the steady state Kalman filter. From the innovation model for the KFP in section (2.4) we have the following equations. From (2.4.19),

$$\hat{y}(t) = A^{-1}(q^{-1})B(q^{-1})u(t) + [\ q^{-1}K_2(q^{-1}) + q^{-d}A^{-1}(q^{-1})K_1(q^{-1})\ ]\omega(t) \qquad (2.6.1)$$

and from (2.4.20),

$$\hat{y}(t+d|t) = q^{-1}A^{-1}(q^{-1})B'(q^{-1})u(t) + A^{-1}(q^{-1})K_1(q^{-1})\omega(t) \qquad (2.6.2)$$

Consider the following transfer functions,

from (2.6.1)

$$\frac{\hat{y}(t)}{\omega(t)} = q^{-1}\,\frac{A(q^{-1})K_2(q^{-1}) + q^{-d+1}K_1(q^{-1})}{A(q^{-1})} = \frac{h_1q^{-1} + h_2q^{-2} + \cdots + h_{n+d-1}q^{-n-d+1}}{A(q^{-1})} \qquad (2.6.3)$$

$$G_{r1}(q^{-1}) = h_1q^{-1} + h_2q^{-2} + \cdot\cdot\cdot\cdot + h_{n+d-1}q^{-n-d+1} \qquad (2.6.4)$$

$$= q^{-1}(h_1+h_2q^{-1}) + \ .\ .\ .\ = q^{-1}[h_1+h_2(1-q^{-1})] + .\ .\ .$$

and from (2.6.2),

$$\frac{K_1(q^{-1})}{A(q^{-1})} = \frac{L_n + L_{n+1}q^{-1} + \cdots + L_1q^{-n+1}}{A(q^{-1})} = \frac{G_{r2}(q^{-1})}{A(q^{-1})} \qquad (2.6.5)$$

$G_{F1}(q^{-1})$ can be interpreted as a set of m_1 parallel proportional plus derivative(PD) controllers, where,

$m_1 = (n+d-1)/2$ (if n+d-1 is even), and

$m_1 = (n+d)/2$ (if n+d-1 is odd).

Similarly $G_{F2}(q^{-1})$ can be interpreted as a set of m_2 parallel PD controllers, where

$m_2 = (n-1)/2$ (if n-1 is even), and $m_2 = n/2$ (if n-1 is odd).

The schematic diagram of the KFP given in figure 2.4 can now be expanded using equations (2.6.1 and 2) and the interpretation for G_{F1} and G_{F2}, as shown in figure 2.7 (only the part with solid lines).

To compensate for the disturbances, the full information regarding the disturbances should be transmitted to, and included in the prediction $\hat{y}(t+d|t)$. That is, $\omega'(t)$ in figure 2.7 should be equal to $\xi(t)$ if perfect modelling is assumed. But the closed loop configuration of G_{F1} with a set of PD controllers in the feedback loop, as shown in figure 2.7 would lead to the following transfer function:

$$\bar{\omega}(t) = \frac{1}{A(q^{-1}) + G_{F1}(q^{-1})}\xi(t) \tag{2.6.6}$$

If $\xi(t)$ is a step disturbance with magnitude ξ_{ss} then the steady state value of $\bar{\omega}(t)$ is,

$$\bar{\omega}_{ss} = \frac{1}{\sum_{r=1}^{n} a_r + \sum_{r=1}^{n+d-1} h_r} \tag{2.6.7}$$

from (2.6.7),

$$\bar{\omega}_{ss} = \xi_{ss} \quad \text{only if}$$

$$\sum_{r=1}^{n} a_r + \sum_{r=1}^{n+d-1} h_r = 1 \qquad (2.6.8)$$

From figure 2.7,

$$\omega'(t) = K_1(q^{-1})\bar{\omega}(t) \qquad (2.6.9)$$

The steady state value of $\omega'(t)$ is,

$$\omega'_{ss} = \frac{\sum_{r=1}^{n} L_r}{\sum_{r=1}^{n} a_r + \sum_{r=1}^{n+d-1} h_r} \qquad (2.6.10)$$

From equation (2.6.10) it is clear that the KFP configuration does not guarantee that $\omega'_{ss} = \xi_{ss}$. That is it does not guarantee that it would pass all the information of the deterministic disturbances to the predicted output $\hat{y}(t+d|t)$. This results in a bias in the predicted output $\hat{y}(t+d|t)$ and hence in the control or regulation of $y(t)$ even if G_c is a PID controller .

This bias in the predictor can be interpreted as resulting from the PD estimation nature of the KFP.

2.6.2 Modified Kalman Filter Predictor

Based upon the interpretation given in the above section regarding the bias problem in the KFP, an intuitive approach to overcome this problem is to change the PD estimation of the KFP to a PID estimation scheme, by introducing a set of integrators in parallel with the PD controllers of G_{F1} and G_{F2}, as shown by the dotted lines in figure 2.7. Assume for simplicity of explanation that the disturbance ξ is a step of magnitude ξ_{ss}. Then the PID system in the feedback loop will make the steady state value of $\bar{\omega}(t)$ go to zero, by making the output of the integrator go to ξ_{ss} At steady state $\omega'(t)$ will also reach zero. and since the output of the integrator is added to the predicted output $\hat{y}(t+d|t)$ the bias is removed.

This adhoc way of modifying the KFP can be achieved more formally by modifying the state space formulation as suggested by Balchan et al (1970, 1973). This idea was used by Bialkowski (1983) in his steady state KFP.

As presented in section (2.3.3), when the process noise is not white noise, the state space model of the process can be augmented by the states corresponding to the noise dynamics.

By interpreting the step disturbances $\xi(t)$ as a random signal which changes at random time instants, by random step sizes, the dynamics of the disturbances can be represented by integrated white noise (see Balchan et al (1970, 1973) and Tuff et al (1985), as follows,

$$\xi(t) = \frac{w(t)}{\Delta} \qquad (2.6.11)$$

where,

$\Delta = 1-q^{-1}$ differencing operator,

$w(t)$ - random signal, generally zero but may attain values p_i at arbitrary time instants i,

$\xi(t)$ - random steps of height p_i starting at time i.

Thus the augmented state space equation given by (2.2.14 and 15) can be augmented with an additional state having an integrator as noise dynamics.

$$x(t+1) = \begin{bmatrix} 1 & 0 & 0 \\ \gamma & \Phi_1 & 0 \\ 0 & 0 & \Phi_2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \Lambda \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w(t) \qquad (2.6.12)$$

$$y(t) = [\ 0:0:\Theta_1\ ]x(t) + v(t) \qquad (2.6.13)$$

where

$$x(t) = [x_\rho,\ x_1,\ x_2,\ \cdots,\ x_n,\ x_{n+1},\ \cdot\ \cdot\ x_{n+d}] \qquad (2.6.14)$$

Now using the innovation model for the above state space model and applying Lemma D.1 (consider the steady state KF),

$$\begin{aligned} \hat{y}(t+d|t) &= \hat{x}_n(t) \\ &= q^{-1}A^{-1}(q^{-1})B'(q^{-1})u(t) + A^{-1}(q^{-1})\Delta^{-1}L_\rho(\gamma_n q^{-1} + \\ &\quad \cdots + \gamma_1 q^{-n})\omega(t) + A^{-1}(q^{-1})K'(q^{-1})\omega(t) \qquad (2.6.15) \end{aligned}$$

and

$$\begin{aligned} \hat{y}(t) &= \hat{x}_{n+d-1}(t-1) \\ &= A^{-1}(q^{-1})B(q^{-1})u(t) + [\ K_1(q^{-1})q^{-d}A^{-1}(q^{-1}) + \\ &\quad q^{-1}K_2(q^{-1})\ ]\omega(t) + A^{-1}(q^{-1})\Delta^{-1}L_\rho\ (\gamma_n q^{-1} + \cdots \\ &\quad + \gamma_1 q^{-n})q^{-d}\omega(t) \qquad (2.6.16) \end{aligned}$$

L_ρ is the Kalman gain corresponding to the noise state $x_\rho(t)$.

Define,

$$D(q^{-1}) = L_\rho\ [\ \gamma_n q^{-1} + \cdots + \gamma_1 q^{-n}\ ] \qquad (2.6.17)$$

Since,

$$\hat{y}(t) = y(t) - \omega(t) \qquad (2.6.18)$$

from (2.6.16) and (2.6.18),

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})\omega(t) + D(q^{-1})q^{-d}\frac{\omega(t)}{\Delta} \qquad (2.6.19)$$

for $C(q^{-1})$ as defined by equation (2.4.17).

From equations (2.6.15 and 19),

$$\hat{y}(t+d|t) = q^{-1}A^{-1}(q^{-1})B'(q^{-1})u(t)$$

$$+ \frac{\Delta K_1(q^{-1}) + D(q^{-1})}{\Delta C(q^{-1}) + D(q^{-1})q^{-d}}[y(t) - A^{-1}(q^{-1})B(q^{-1})u(t)]$$

or

$$\hat{y}(t+d|t) = G_p(q^{-1})u(t) + G_F(q^{-1})[y(t) - G_m(q^{-1})u(t)] \tag{2.6.20}$$

where

$$G_F(q^{-1}) = \frac{\Delta K_1(q^{-1}) + D(q^{-1})}{\Delta C(q^{-1}) + D(q^{-1})q^{-d}} \tag{2.6.21}$$

and G_p and G_m are defined by (2.4.22 and 23).

Comparing equations (2.6.1 and 2) with equations (2.6.15 and 16) respectively, it is clear that the additional terms $\Delta^{-1}D(q^{-1})$ in (2.6.15) and $\Delta^{-1}q^{-d}D(q^{-1})$ in (2.6.16) contribute a set of integrators connected in parallel to the existing PD controllers in G_{F1} and G_{F2}, in the same way as it is shown in figure 2.7.

Thus by augmenting the state space formulation of the KFP by an additional state, a bank of m_2 ($m_2 = n/2$ if n is even, $m_2 = (n+1)/2$ if n is odd) integrators are connected in parallel to the existing PD controllers of the KFP. The integral gains can be selected by the ratio R_w/R_v because the gains are functions of L_ρ. The MKFP is stable despite the added poles on the unit circle (cf. theorem B3).

The same configuration of the KFP given in figure 2.4 can be obtained for the modified KFP, using the equation

(2.6.20). The only difference is the definition of the filter G_F.

2.6.3 Incremental Predictive control for the Modified KFP

The predicted outputs of the modified KFP can be used in PID control as in section (2.5). Since the prediction $\hat{y}(t+d|t)$ is unbiased, the PID control will result in zero offset steady state set point tracking or regulation. But to use predictive control when the disturbance is deterministic type, it is necessary to modify the predictive control algorithm by including the disturbance term in the predictive control law or to use the incremental predictive control law, where incremental variables are used in the controller design.

Consider the ARIMA model (Tuff et al, 1985) for the process with deterministic disturbances.

$$A(q^{-1})y(t) = q^{-(d+1)}B'u(t) + C(q^{-1}) \frac{w(t)}{\Delta} \qquad (2.6.22)$$

w(t) is as defined in equation (2.6.11), and has a zero mean, and $C(q^{-1}) = 1$.

Equation (2.6.22) can also be written as,

$$\begin{aligned} &\Delta y(t+d+1) + [A(q^{-1}) - 1] \Delta y(t) \\ &= B'(q^{-1})\Delta u(t) + w(t+d+1) \end{aligned} \qquad (2.6.23)$$

Taking the conditional expectation of (2.6.23) yields,

$$\Delta\hat{y}(t+d+1|t) + a_1\Delta\hat{y}(t+d|t)\cdots + a_n\Delta\hat{y}(t|t) = B'\Delta u(t) \qquad (2.6.24)$$

The minimum variance estimates for $\Delta\hat{y}(t+d|t)$, $\Delta\hat{y}(t+d-1|t-1)$, $\cdots$, $\Delta\hat{y}(t/t-d)$ can be obtained from the KFP, and

$$\Delta\hat{y}(t+d|t) = \hat{y}(t+d|t) - \hat{y}(t+d-1|t-1) \qquad (2.6.25)$$

If the control law is calculated such that the predicted output $\hat{y}(t+d|t)$ will reach the set point in one sample time instant, the incremental control law $\Delta u(t)$ is calculated by defining,

$$\begin{aligned}\Delta\hat{y}(t+d+1) &= y^*(t+d+1) - \hat{y}(t+d|t)\\ &= \delta y^*(t+d+1) \qquad (2.6.26)\end{aligned}$$

Thus the incremental control law is given by,

$$\begin{aligned}\Delta u(t) = \frac{1}{b_1}\,[\,\delta y^*(t+d+1) &- [\ 1 - A(q^{-1})\ ]\ \Delta\hat{y}(t+d|t)\\ &+ [\ b_1 - B'(q^{-1})\ ]\ \Delta u(t)\ ] \qquad (2.6.27)\end{aligned}$$

2.7 Discussion

2.7.1 Kalman Filter Predictor vs Internal Model Control

The configuration of the Internal Model Control (IMC) scheme due to Garcia and Morari (1982) is shown in figure 2.5. They have shown that the Smith predictor (Smith, 1957, 1959) is a subset of IMC, by approximating the 2nd model block of the IMC, i.e. by removing the time delay from in the model, as shown in figure 2.6.

The structural similarity of KFP and IMC is clearly shown by figure 2.4 and 2.5.

The introduction of the filter in the IMC was justified by showing the equivalent structure for a Linear Quadratic Optimal Control (LQOC) system, which is capable of handling stochastic disturbances. Their claim on the superiority of the IMC, and the justification for using the filter is as follows:

a1. The LQOC scheme cannot handle deterministic disturbances.

a2. The filter in the LQOC (or as in KFP) configuration cannot be easily tuned or designed.

a3. The filter used in IMC is easy to design and robustness analysis is possible.

It is important to note that as shown by Balchan et al (1970, 1973), and in section 2.6, deterministic disturbances can be handled by using a state space representation that takes into account the stochastic model of the disturbance dynamics. As Balchan and others clearly stated, it is not a problem of the LQOC or the KF, but the inappropriate formulation of the state space representation. It appears that the authors of IMC were not aware of the work of Balchan and others.

The filter in the IMC is mainly used to improve the robustness of the system. In IMC the noise filtering is done in an adhoc manner, but the Kalman filter is designed to give minimum variance estimates by taking into account the statistical properties of the noise and the model of the process.

2.7.2 Kalman Filter Predictor vs Smith Predictor

From figure 2.4 and figure 2.6, it is clear that the KFP and the SP have the same configuration. They have exactly the same functional blocks except for the filter G_F in the KFP and it would appear that the KFP reduces to the SP if $G_F \longrightarrow 1$.

To compare the behaviour of the KFP and SP, the closed loop equation and the predictor equations are derived for both of these schemes.

For the SP illustrated in figure 2.6 the predictor equation is,

$$\hat{y}(t+d|t) = G_p u(t) + [\ y(t) - G_m u(t)\ ] \tag{2.7.1}$$

and the closed loop equation for an error driven feedback control system is,

$$y(t) = \frac{GG_c}{1+G_cG_p+G_c(G-G_m)}\, y^*(t) - \frac{GG_c}{1+G_cG_p+G_c(G-G_m)}\,\xi(t) + \xi(t) \tag{2.7.2}$$

For the KFP, the predictor equation is given by (2.4.25),

$$\hat{y}(t+d|t) = G_p u(t) + G_F(t)[y(t) - G_m u(t)] \tag{2.7.3}$$

and the closed loop equation is given by (2.5.3),

$$y(t) = \frac{GG_c}{1+G_cG_p+G_c(G-G_m)G_F(t)}\, y^*(t) - \frac{G_F(t)G_cG}{1+G_cG_p+G_c(G-G_m)G_F(t)}\,\cancel{\xi(t) + \xi(t)} \tag{2.7.4}$$

Remarks

1. The presence of noise in the feedback path causes undesirable fluctuations in the control output u(t). Using a filter to remove the noise is a solution to this problem. Considering the noise as process noise and measurement noise, ideally we would like to remove all

measurement noise which cannot be controlled, and pass the process noise to the controller for compensation.

There are a number of different locations the noise filtering can be performed. Common approaches are to filter the measurement y(t) or the controller output u(t). But a filter placed at these locations, would also filter the important process dynamics information, along with the noise.

The location of the filter in the KFP is the most suitable location for the filter, because the signal filtered in this case carries only the noise, if the model is perfect.

2. The SP is designed to work with deterministic process. When the process is corrupted by process and/or measurement noise, the SP handles it by simply adding the error $y_e(t)$, to $y_p(t)$ to obtain the prediction $\hat{y}(t+d|t)$. The error $y_e(t)$ represents the noise present in the process and the unmodelled dynamics in the SP. This could result in very noisy control. The performance of the SP can be improved for applications with stochastic noise, by introducing a filter (e.g. the familiar exponential filter) in the same location as in the KFP . Now the structure of the KFP is exactly the same as the SP, but the filter G_F in the KFP is an optimal filter based on the optimal gains of the KF. This filter assures the minimum error variance achieved through the filter G_F is less than or equal to the error

variance of any other filter. The KFP is thus the preferred method for applications with noise and is based on a sound theoretical formulation rather than heuristic arguments.

3. When there is no mismatch between the process and the model, and the system is free of measurement and process noise then $G_m = G$ and $\xi(t) = 0$.

   For both SP and KFP,

   $$\hat{y}(t+d|t) = G_p(q^{-1})u(t) \tag{2.7.5}$$

   and

   $$y(t) = \frac{GG_c}{1 + G_cG_p} u(t) \tag{2.7.6}$$

   Under perfect modelling and noise free conditions the SP and KFP give the same output prediction and output control assuming that they both use the same controller which is tuned to G_p. When the KFP is implemented using the KF algorithm, G_p is not explicitly present in the scheme. The tuning of G_c to control G_p in the KFP can be done by selecting $R_w/R_v = 0$, which would make $G_F = 0$, which is appropriate for the noise free case.

4. In the presence of process and measurement noise ($\xi(t) \neq 0$), but under perfect modelling the prediction and the output control of the SP and KFP are given by the following equations.

   For the SP from (2.7.1 and 2),

   $$\hat{y}(t+d|t) = G_pu(t) + [y(t) - G_mu(t)] \tag{2.7.7}$$

$$y(t) = \frac{GG_c}{1+G_cG_p} \; y^*(t) - \frac{GG_c}{1+G_cG_p} \xi(t) + \xi(t) \qquad (2.7.8)$$

For the KFP from (2.7.3 and 4),

$$\hat{y}(t+d|t) = G_pu(t) + G_r \,[y(t) - G_mu(t)\,] \qquad (2.7.9)$$

$$y(t) = \frac{GG_c}{1+G_cG_p} \; y^*(t) - \frac{G_cGG_r}{1+G_cG_p} \xi(t) + \xi(t) \qquad (2.7.10)$$

It is clear from equations (2.7.8 and 10) that the closed loop characteristic equations for set point tracking, are the same for both SP and KFP. But for noise $\xi(t)$, the closed loop characteristic equation of the KFP contains the poles of the filter G_r as its roots. Since G_r is proved to be stable, the introduction of the filter G_r does not effect the stability of the closed loop system.

5. When there are unmodelled dynamics i.e. $G \neq G_m$, the closed loop characteristic equations are given by,

   for SP

$$1 + G_cG_p + G_c(G-G_m) = 0 \qquad (2.7.11)$$

   and for KFP

$$1 + G_cG_p + G_rG_c(G-G_m) = 0 \qquad (2.7.12)$$

   In this case the closed loop characteristic equations for set point tracking and noise reduction are the same. However the interesting point is that now the poles of G_r are not the roots of the characteristic equation. Hence closed loop stability does not demand the stability of the filter G_r.

The unmodelled dynamics can be taken into account in the KFP by proper selection of R_w/R_v. The Kalman filter processes the unmodelled dynamics like a disturbance component. Changing R_w/R_v would change the roots of the closed loop characteristic equation. This would provide a means of obtaining a more robust control scheme in the presence of unmodelled dynamics. However, the manipulation of the poles of G_r, using R_w/R_v is not straightforward. This is one of the shortcomings of the Kalman filter.

It is difficult to generalize the influence of unmodelled dynamics on the stability of the control scheme. However if the signal due to unmodelled dynamics is bounded then one can expect that the closed loop system remains stable.

6. It is interesting to investigate the conditions under which the KFP will reduce to the SP. If $\xi(t) \neq 0$, KFP will reduce to a SP when $G_r=1$.

Consider the filter G_r for the KFP and the modified KFP.

For the normal KFP, from equation (2.4.24)

$$G_r(t,q^{-1}) = \frac{\Delta K_1(q^{-1}) + D(q^{-1})}{\Delta C(q^{-1}) + D(q^{-1})q^{-d}} \qquad (2.7.13)$$

From these equations it is difficult to see whether G_r will be equal to unity.

But at steady state,

for KFP,

$$G_{Fss} = \frac{\sum_{i=1}^{n} L_i}{\sum_{i=0}^{n+d-1} C_i}$$

and for the modified KFP, $G_{Fss} = 1$

Thus the modified KFP reduces to a SP under steady state operation. This result also confirms the ability of the modified KFP to handle deterministic disturbances as shown in section 2.6.

2.7.3 Implementation of the KFP

The dimension of the specific state space formulation used in the KFP increases with the number of time delays. If the time varying KF algorithm given in section (2.3.11) is used for the implementation of the KFP, there will be a large number of matrix operations, especially multiplications, which implies a heavy computational effort. As the time delay increases, the problem would become severe.

The sparse nature of the matrix Φ, due to the observer form used in the state space formulation of the KFP can be used to obtain a simplified algorithm as shown in Appendix E, which has significantly fewer multiplications to perform. If $n=1$, $d=3$, the direct implementation of the KFP using the state space matrix calculation needs 208 multiplications, whereas the simplified implementation needs only 12. This relatively small computational effort justifies the employment of the KFP.

However, instead of implementing the KFP in state space form, it can be implemented in transfer function form, using the configuration obtained via the innovation model illustrated in figure 2.4. This can be implemented as an extension to the SP, by introducing a filter G_F given by equation (2.7.13), whose parameters are calculated using only the equations E1, E3 and E5. This does not reduce the computation effort compared to the simplified implementation in Appendix E, but gives a conceptually simple implementation of the KFP.

2.7.4 Predictive Control with SP and KFP

The predictive control law and incremental predictive control law derived for the KFP and modified KFP in section (2.5 and 6) respectively, are also applicable to the SP. The only difference is that the prediction $\hat{y}(t+d|t)$ of the SP is not the minimum variance estimate, as in the KFP.

From (2.5.29) (without (q^{-1}) in each polynomial),

$$y^*(t+d+1) = (1-A)q\,\hat{y}(t+d) + B'u(t)$$

$$y^*(t+d) = (1-A)\,\hat{y}(t+d) + q^{-1}B'u(t) \qquad (2.5.30)$$

From (2.4.21),

$$\hat{y}(t+d) = q^{-1}A^{-1}B'u(t) + C^{-1}K_1\,[y(t) - A^{-1}Bu(t)\,] \qquad (2.5.31)$$

From (2.5.30) and (2.5.31),

$$\begin{aligned} y^*(t+d) = {} & q^{-1}A^{-1}B'u(t) - (1-A)C^{-1}K_1A^{-1}Bu(t) \\ & + C^{-1}K_1(1-A)y(t) \end{aligned} \qquad (2.5.32)$$

If the true ARMA representation of the process is given by,

$$y(t) = A^{-1}{}_x B_x u(t) + \xi(t) \qquad (2.5.33)$$

then the closed loop equation of the predictive control scheme is given by (from (2.5.32 and 33)),

$$y(t) = \frac{1}{N_1(q^{-1})} y^*(t+d) + \frac{N_2(q^{-1})}{N_1(q^{-1})} \xi(t) \qquad (2.5.34)$$

where,

$$N_1(q^{-1}) = [\ A_x A' B_x^{-1}\ B' q^{-1} - (1-A) C^{-1} K_1 (A_x A^{-1} B_x^{-1}\ B-1)\ ] \qquad (2.5.35)$$

$$N_2(q^{-1}) = A_x B_x^{-1}\ {}_x [\ A^{-1} B^{-1} q^{-1} - (1-A)\ C^{-1} K_1 A^{-1} B\ ] \qquad (2.5.36)$$

The closed loop equation for the KFP under perfect modelling can be obtained from equation (2.5.34) as,

$$y(t) = y^*(t) + [1 - (1-A) C^{-1} K_1 q^{-d}] \xi(t) \qquad (2.7.15)$$

The corresponding equation for the SP can be obtained by substituting $C^{-1}K_1 = 1$, hence,

$$y(t) = y^*(t) + [1 - (1-A) q^{-d}] \xi(t) \qquad (2.7.16)$$

1. With $\xi(t) = 0$ and perfect modelling $y(t) = y^*(t)$ for both SP and KFP, i.e. the output tracks the setpoint perfectly.
2. With $\xi(t) \neq 0$ and perfect modelling the noise in the KFP is filtered by the filter G_F. The stability of the predictive controller for the KFP depends on the roots of the polynomial C, which are proven to be stable.
3. With unmodelled dynamics the analysis becomes complicated.

When there is no noise present in the system the steady state gain of the model should be equal to the steady state gain of the process, to provide perfect steady state tracking. That is,

$$\lim_{q^{-1} \to 1} \{A_*^{-1}\ B_*\} = \lim_{q^{-1} \to 1} \{A^{-1}B\}$$

in both KFP and SP for perfect setpoint tracking.

2.7.5 Variable time-delay processes

The formulation of both KFP and SP is based on the assumption that the time delay of the process is accurately known. But in a practical situation this time delay can vary with time. This variation in time delay may deterioriate the performance of the KFP. In such a situation it is desirable to estimate the variation in the time delay, and the KFP algorithm should be modified to accommodate this variation.

If the variation in the time delay of the process is known as a function of u, y and time, then one could employ a simple computation or a 'table lookup' scheme to estimate the time delay exactly. An on-line dead time estimation scheme based on the cross correlation method can be employed to estimate the time varying dead time (Box and Jenkins, 1976). But this may demand a heavy computational effort. Instead of the full cross correlation method a simpler method is to input a sequence of small pulses to the process input and cross correlate the input and output, only near the region of the delay, to obtain the delay present in the process. But when the process is noisy this may not be a successful method. Another method one could employ is to use a recursive dead time estimation scheme as given in Wang and

En (1985).

Once the dead time variation is known, it can be easily incorporated into the Kalman filter algorithm. The variation in dead time will change the dimension of the state space formulation, but it would not affect the parameters. Difficulties may arise when adding or removing the states in the KF, especially when the initial estimat~~es of the erro~~r covariances are introduced into the algorithm. This might cause all the other Kalman gains to fluctuate. However, the asymptotic convergence of the KF to its steady state KF is still guaranteed.

2.7.6 Input Output model vs State Space Model

When the process is a deterministic system, the SP can be derived by using an ARMA model for the process. There is no real need of a KF in this case. When the process is stochastic, the input-output model given by the ARMAX representation,

$$Ay = Bu + C\xi$$

can be employed to implement an optimal predictor or a minimum variance controller (Astrom, 1970). In this case, ξ, which is a random uncorrelated noise sequence, accounts for both process and measurement noise, and they are not distinguishable.

The advantage of using the state space approach is to incorporate the knowledge of process and measurement noise separately.

This also provides a means to tune the KFP, using the ratio R_w/R_v, to satisfy a certain performance criterion, e.g. minimum variance in output error.

But when the order of the process is high it may be difficult to do proper tunning of R_w/R_v. In such a situation the KFP can be implemented using the input output model given by equation (2.4.18). It is important to note the fact that an input-output model for a process can be obtained via the innovation model of the KFP, but the converse is not true. By identifying the polynomials in the ARMAX model given by equation (2.4.18), the KFP can be implemented without having to know the noise covariances R_w and R_v. It is important to note that the degree of the polynomial $C(q^{-1})$ is greater than that of the polynomials A and B, and it depends on the time delay. A detailed discussion and implementation of this scheme is given in chapter 4, section 4.3.

Another interesting result is the input-output model obtained for the modified KFP using the innovation model, which is given by equation (2.6.19),

$$A(q^{-1})y(t) = B(q^{-1})u(t) + D(q^{-1})q^{-d}\,\frac{\omega(t)}{\Delta} + C(q^{-1})\omega(t) \tag{2.7.17}$$

Although it still needs further investigation, it is easy to interpret (2.7.17) as a combination of the input-output model given by equation (2.4.18) and the input-output model given by equation (2.6.22). The 2nd term on the right hand side of (2.7.17) represents the deterministic disturbances and/or the noise present in the system and can be interpreted as integrated white noise i.e. Brownian type noise. The 3rd term on the right hand side, represents stationary noise.

Thus equation (2.7.17)appears to show that KF theory can be used to derive an input output model for a stochastic process with stationary and non-stationary disturbance terms.

2.8 Simulation Results and Discussion

The simulations were carried out for 1st order and 2nd order processes having the following difference equations.

1st order process,

$$y(t) - 0.9321\ y(t-1) = -0.1717\ u(t-4) \qquad (2.8.1)$$

2nd order process,

$$y(t) - 1.8954\ y(t-1) + 0.8981\ y(t-2) = 0.7975\ u(t-4) - 0.7758\ u(t-5) \qquad (2.8.2)$$

The processes were simulated using the state space formulation given in Section 2.2. A white noise sequences were added, to each state to generate the process noise, and to the output to generate measurement noise. The noise levels used for the above two processes are given in Table 2.1.

The same PI controller parameters were used for both SP and KFP. The controller parameters for the 1st order process were obtained from Meyer (1977), and are given below:

Proportional gain $K_p = -1.36$

Integral time constant $\tau_I = 574$

The improved version of the Smith predictor for stochastic processes was implemented by introducing an exponential filter as discussed in section (2.7.2). The exponential filter is given by,

$$y_F(t) = y_F(t-1) + \alpha[y(t) - y_F(t-1)]$$

where,

$y_F(t)$ - filtered value

$y(t)$ - filter input

1. Deterministic Process

1.1 PI Control

When there is no noise, i.e. $\xi(t) = 0$, and under perfect modelling, the PI control schemes using the KFP and SP perform exactly the same way, as shown in figure 2.8a and b. This confirms the theoretical results obtained in section 2.7.2. Under these ideal conditions the KFP and the SP are functionally the same and behave as open loop predictors.

1.2 Predictive Control

Figures 2.9a and b show the performance of the predictive control scheme using the KFP and the SP, under perfect modelling and noise free conditions and as shown in theory in section 2.7.2, show exactly the same performance.

The predictive control scheme tracks the set point perfectly and shows better performance than the PI control. However, this superior performance is obtained at the expense of high control force (see figure 2.9b). It is also important to note that the minimum variance control scheme due to Astrom (1970) would give the same performance under these conditions.

2. Stochastic Process

2.1 PI Control

Figures 2.10a and b show the performance of the PI control scheme using KFP and SP, in the presence of both process and measurement noise, and under perfect modelling. The criterion for comparing the performance was the minimum variance of the output error.

The ratio R_w/R_v in the KFP and the filter coefficient α in the SP were tuned to obtain the minimum variance performance. (Note that $\alpha=1$ implies zero filtering.)

It is clear from figure 2.10c, that the variance of the output error of the SP increases with decreasing α, i.e. more filtering. For the KFP it is lower than the SP and does not show much variation with respect to R_w/R_v.

As seen from figure 2.10a there is no obvious difference in the performance of the KFP and the SP. However, the better performance of the KFP is demonstrated by figure 2.10c.

2.2 Predictive Control

Figures 2.11a and b show the performance of the predictive control schemes using the KFP and SP, in the presence of process and measurement noise, and under perfect modelling. It is clear that the performance of the SP without a filter ($\alpha=1$), is extremely noisy. Its performance is worse than the PI control scheme in figure 2.10a. The predictive control schemes are quite sensitive to noise present in the feedback loop.

The Smith predictor performance is improved by using the exponential filter. The figures do not show much difference in the performance of the KFP and the improved SP. However figure 2.11c shows that the minimum variance in output error can be achieved by the KFP. This confirms the minimum variance property of the predictive control law based on the KFP as proved in section 2.5.2

Although a significant improvement in the SP is obtained by introducing the filter, it cannot give better performance than any minimum variance predictor. Since the actual ratio of R_w/R_v=1.0, that the minimum variance should occur at that ratio, but the minimum occurs at R_w/R_v=0.3. This may be due to non ideal noise characteristics, finite length of the noise sequences and the fact that only one run was done for each point in figure 2.11c.

Comparing figures 2.10c and 2.11c it is clear that the minimum variance achievable by the KFP in the predictive control scheme is less than the PI control scheme.

The performance of the predictive control scheme for the 2nd order process is shown in figure 2.12a, b and c. The minimum variance performance of the KFP is demonstrated in figure 2.12c.

3. Process with deterministic disturbances

The behaviour of the KFP, SP and MKFP in the presence of deterministic disturbances was investigated, by simulating processes with the following difference equations.

1st order

$$y(t) - 0.9321\ y(t-1) = -0.1717\ u(t-4) + 0.9329\xi(t) \tag{2.8.3}$$

2nd order

$$y(t) - 1.8954\ y(t-1) + 0.8981\ y(t-2)$$
$$= 0.7975\ u(t-4) - 0.7758\ u(t-5) + \xi(t) \tag{2.8.4}$$

$\xi(t)$ is a step occurring at the 100th time instant with amplitudes 0.4 and 0.1 for the 1st and 2nd order processes respectively. It is important to note the steady state values of these disturbances.

For the 1st order process,

$\xi_{ss} = 5.49$,

for the 2nd order process

$\xi_{ss} = 37.037$

and it is clear that they are extremely large disturbances.

3.1 PI Control

From figure 2.13a it is clear that the KFP gives biased output control under deterministic disturbances. There is an approximately 100% error in the controlled output. As explained in section 2.6 this is due to the PD estimation nature of the KFP, which results in biased predictions.

The same figure shows the performance of the SP and MKFP. The modified KFP shows better performance than the SP. The transient behaviour of the disturbance rejection in the MKFP can be tuned by R_v/R_v, instead of retuning the PI controller. Tuning R_w/R_v changes the integral gain L_d of the PID estimation.

3.2 Predictive Control

Figures 2.14 and 2.15 show the performance of the incremental predictive control scheme based on SP and MKFP for the 1^{st} and 2^{nd} order processes. Clearly the performance of the MKFP is significantly better than the SP.

Predictive control using the SP or the KFP show better performance than the PI control scheme, but still at the expense of high control force.

The superiority of the MKFP under incremental predictive control can be explained by the capability of the KFP to predict the disturbances once it has detected the occurrence of a disturbance. Although a thorough investigation of the disturbance prediction mechanism in the MKFP is still needed, this is demonstrated in figure 2.16, which shows the open loop disturbance prediction in both the SP and MKFP. It is clear that the MKFP predicts the disturbance three time steps ahead, which is the time delay in the process, whereas the SP is only capable of simply following the current disturbance.

Since the MKFP is capable of predicting the disturbance, when this is employed in incremental predictive control, the control action is taken in such a way that the disturbance is rejected as soon as it occurs.

4. Process-model mismatch

Figures 2.17a and 2.17b show the performance of the KFP, MKFP and SP under an error in the steady state gain of the model. The steady state gain of the model is 20% less than the actual process gain. All the schemes are sensitive to model error under dynamic control. KFP also gives a small bias or an offset in the steady state. The steady state error can be reduced by increasing the ratio R_w/R_v, but this would result in high Kalman gains and consequently would deteriorate the transiant characteristics.

The MKFP eliminates the offset and can be tuned to give better performance than the SP. Since the Kalman filter treats unmodelled dynamics as noise or disturbances it can be tuned to take into account the process-model mismatch. However these results show that a sufficient formulation of the state space representation is necessary, to take into account the unmodelled dynamics. In this simulation the mismatch in the steady state gain can be interpreted as a deterministic disturbance and hence it is necessary to use the modified KFP.

2.9 Conclusions

1. The Kalman filter predictor scheme, based on a special state space formulation and the Kalman filter is developed for processes with time delays plus process and measurement noise. The KFP gives the optimal or minimum variance predictions of the future process outputs.
2. By using the innovation model, it is shown that the Kalman filter predictor has the same configuration as that of the Smith predictor, except for an additional filter , which performs the minimum variance filtering.
3. Under noise free conditions and perfect modelling the KFP and SP are functionally the same.
4. The SP can be improved, for use with stochastic processes by introducing a filter in the same location as in the KFP.
5. The stability of the KFP, and the convergence of the time varying KFP to a stable steady state KFP are established.
6. A predictive control scheme based on the KFP and the SP are presented. The predictive control scheme based on the KFP results in minimum variance control performance.
7. The Kalman filter predictor is modified by augmenting the state space formulation with an additional state corresponding to noise to give unbiased predictions for processes with deterministic disturbances.
8. An incremental predictive control scheme based on the

modified Kalman filter predictor or SP is proposed for processes with deterministic disturbances. The modified KFP shows significantly better disturbance rejection.

9. A computationally efficient implementation of the Klaman filter predictor is presented.

Table 2.1 Process and Measurement Noise Variances used in Simulations

Process	Process noise $\omega(t)$	Measurement noise $v(t)$
1st order	[0,0.001]	[0,0.1]
2nd order	[0,0.1]	[0,0.1]

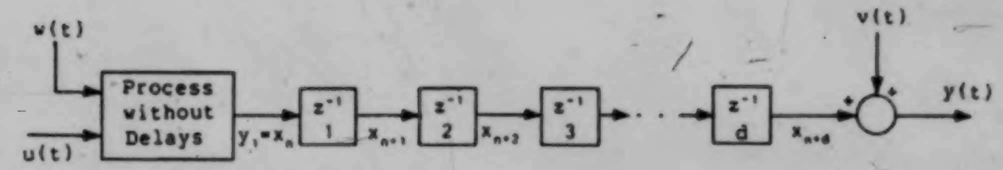

w(t)
u(t)
Process without Delays
$y_1 = x_n$
z^{-1}
1
x_{n+1}
2
x_{n+2}
3
d
x_{n+d}
v(t)
+
+
y(t)

Figure 2.1 Schematic Block Diagram of the State Space Representation for a Process with Time Delays.

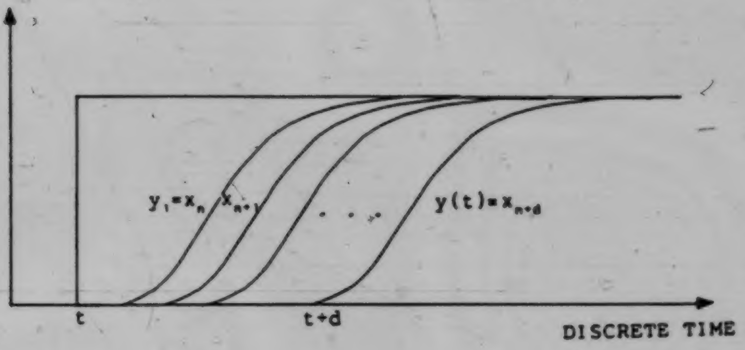


Figure 2.2 Response of Unit Delay Blocks for a Step input to the Process.

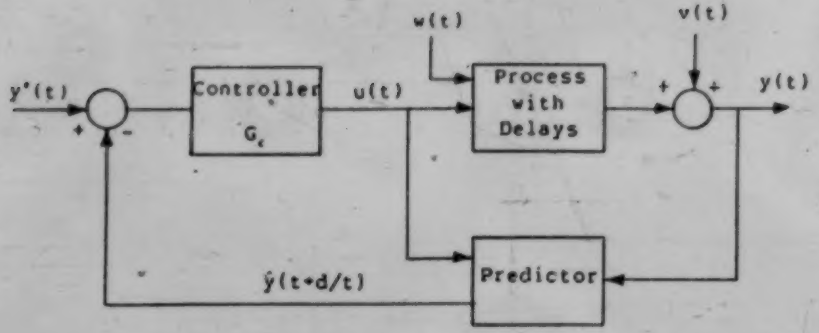

w(t)
v(t)
y*(t)
+
-
Controller
G_c
u(t)
Process
with
Delays
+
+
y(t)
Predictor
ŷ(t+d/t)

Figure 2.3 Feedback Control Scheme Using a Predictor.

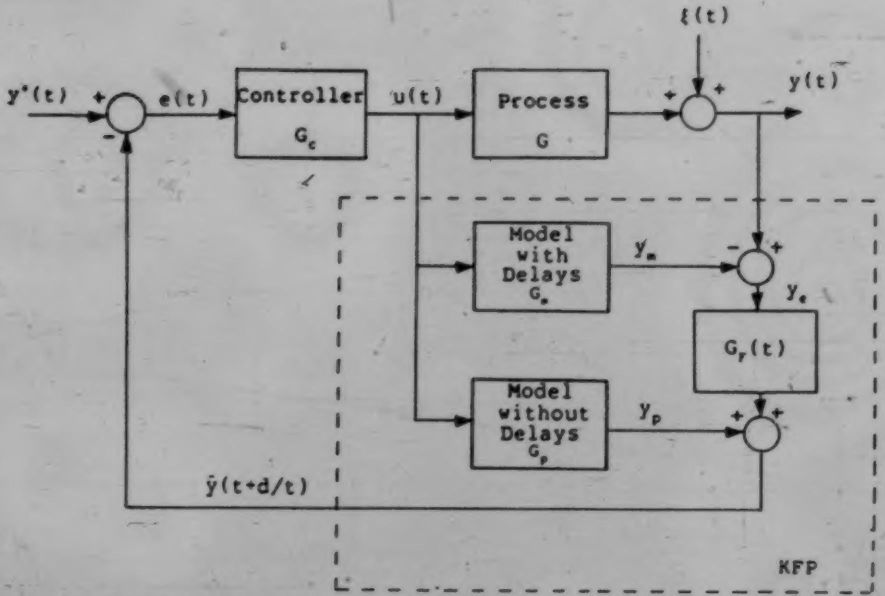

ξ(t)
y*(t)
+
-
e(t)
Controller
G_c
u(t)
Process
G
+
+
y(t)
Model
with
Delays
G_m
y_m
-
+
y_e
$G_f(t)$
Model
without
Delays
G_p
y_p
+
+
ŷ(t+d/t)
KFP

Figure 2.4 Structure of the Kalman Filter Predictor.

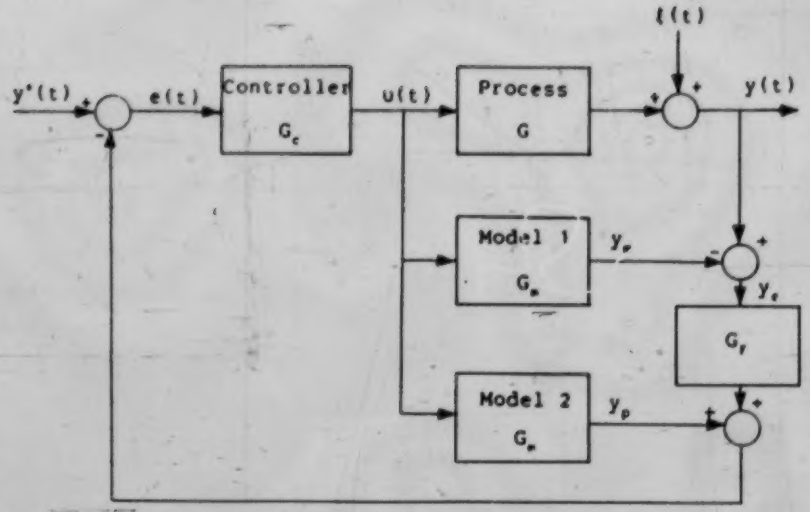

ξ(t)
y*(t)
e(t)
Controller
G_c
u(t)
Process
G
y(t)
Model 1
G_m
y_m
y_e
G_f
Model 2
G_p
y_p

Figure 2.5 Structure of the Internal Model Control System.

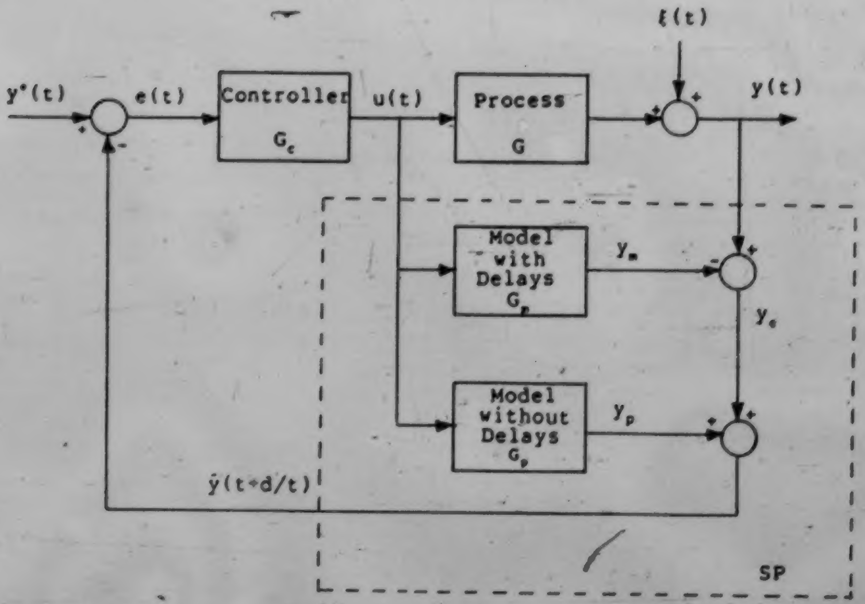

ξ(t)
y*(t)
e(t)
Controller
G_c
u(t)
Process
G
y(t)
Model with Delays
G_m
y_m
y_e
Model without Delays
G_p
y_p
$\hat{y}(t+d/t)$
SP

Figure 2.6 Structure of the Smith Predictor.

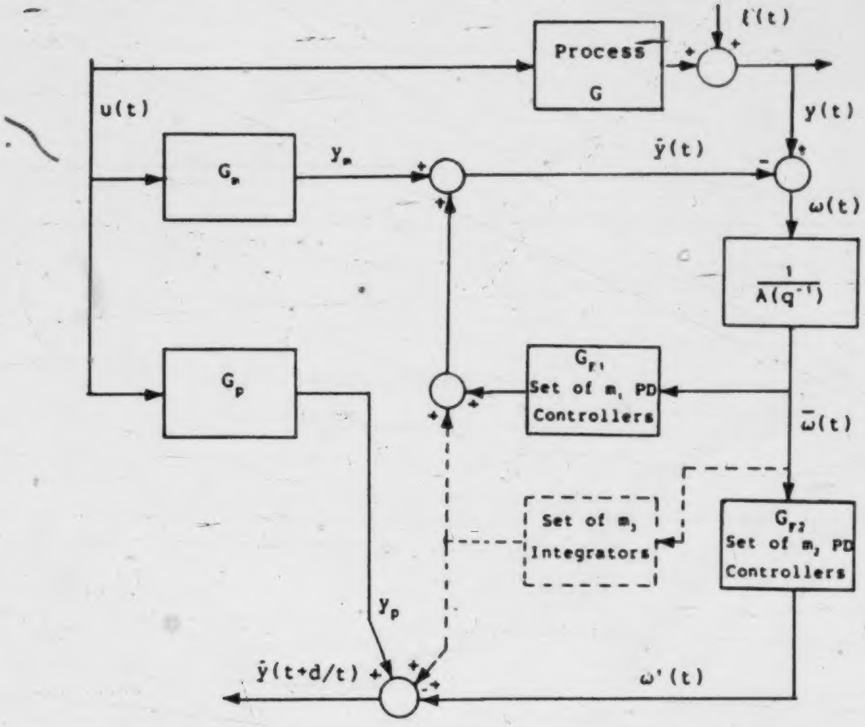
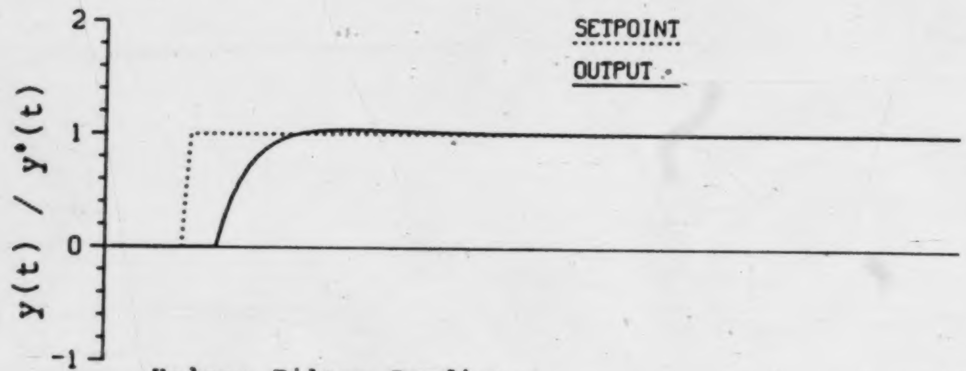
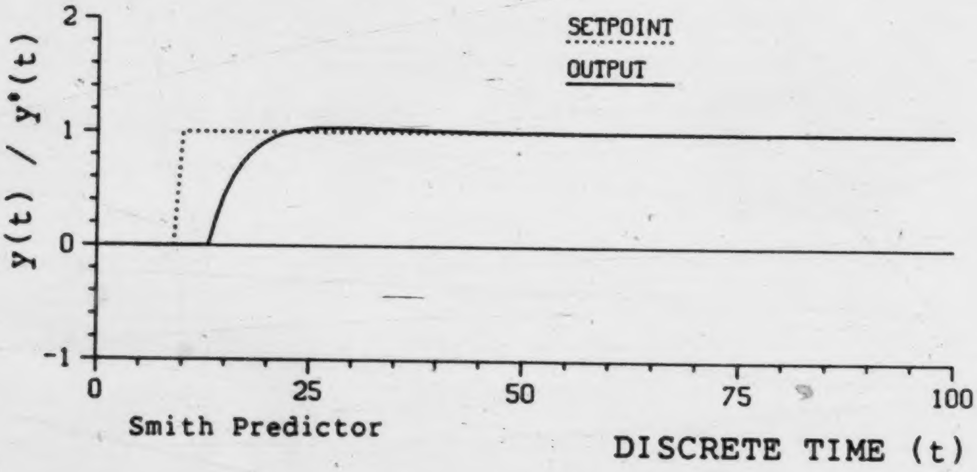

Process
G
ξ(t)
u(t)
y(t)
G_m
y_m
$\hat{y}(t)$
ω(t)
$\frac{1}{A(q^{-1})}$
G_p
G_{f1}
Set of m_1 PD Controllers
$\bar{\omega}(t)$
Set of m_3 Integrators
G_{f2}
Set of m_2 PD Controllers
y_p
$\hat{y}(t+d/t)$
ω'(t)
+
-

Figure 2.7 Elaborated Structure of the Kalman Filter Predictor.
Bank of m_3 Integrators are added to Modify the KFP for Deterministic Disturbances.

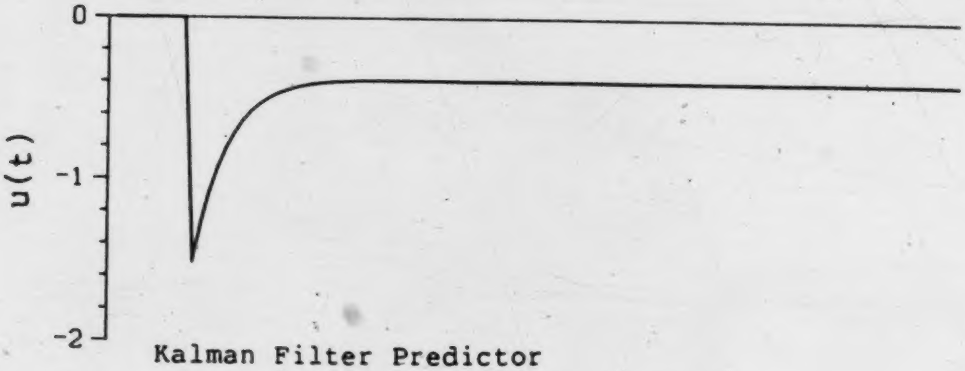


Kalman Filter Predictor

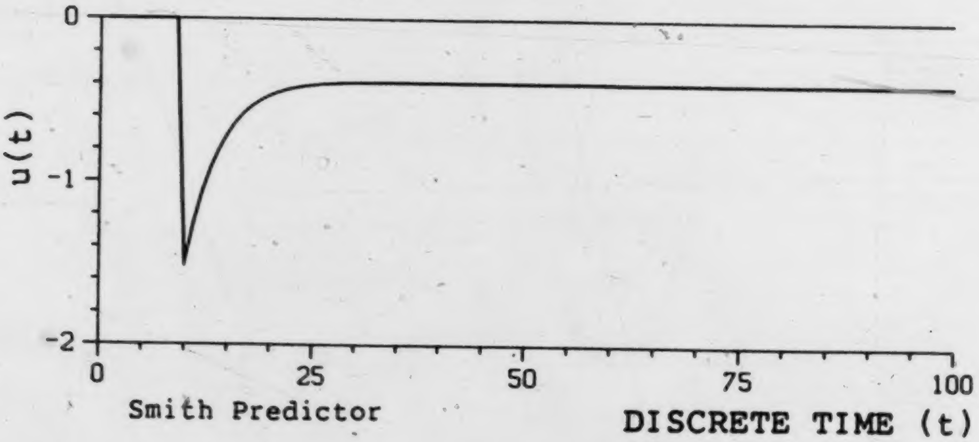


Smith Predictor

Figure 2.8a PI Control of 1^{st} Order Deterministic Process using KFP and SP. $K_p = -1.36$, $\tau = 574$, $R_w/R_v = 0.5$, $M = 10^3$.



Kalman Filter Predictor



Smith Predictor

Figure 2.8b PI Control of 1st Order Deterministic Process using KFP and SP. $K_p = -1.36$, $\tau = 574$, $R_w/R_v = 0.5$, $M = 10^3$.

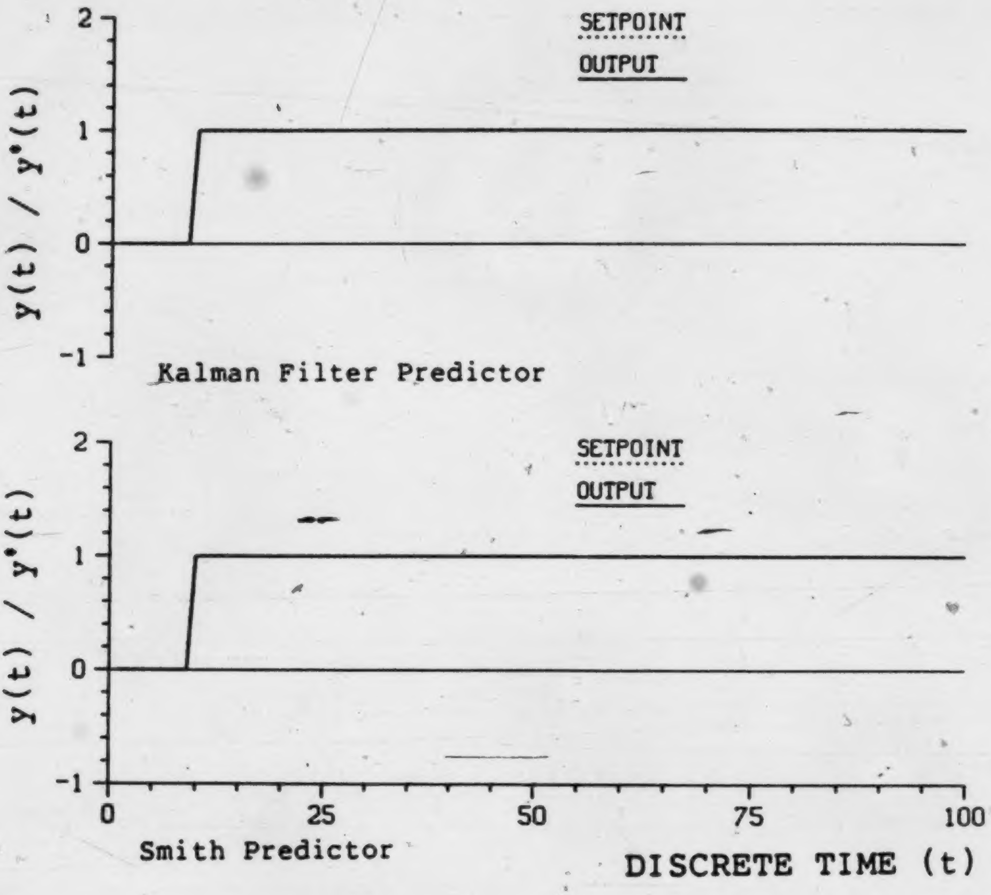
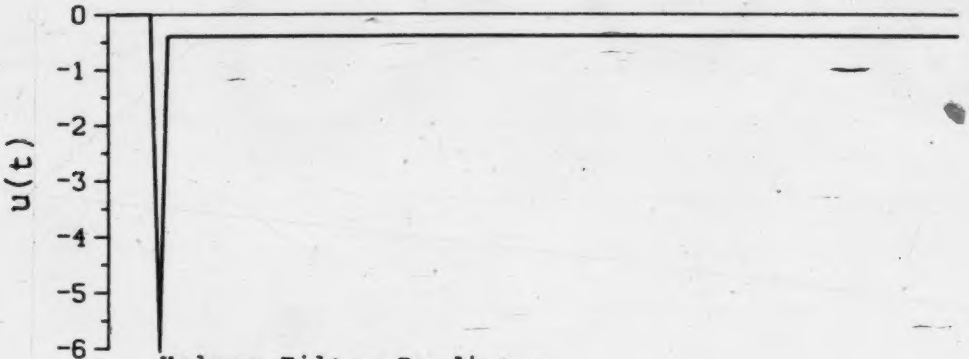
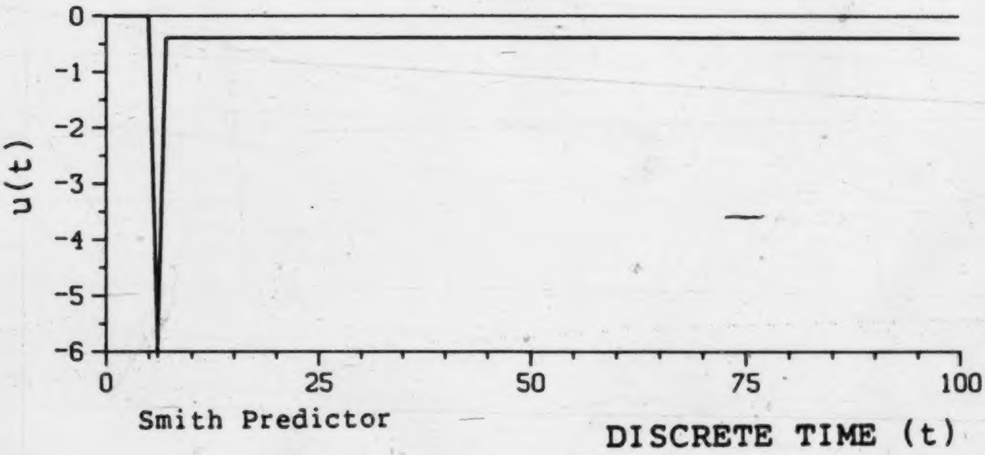


Figure 2.9a Predictive Control of 1st Order Deterministic Process , using KFP and SP. $R_w/R_v = 0.5$, $M = 10^3$.

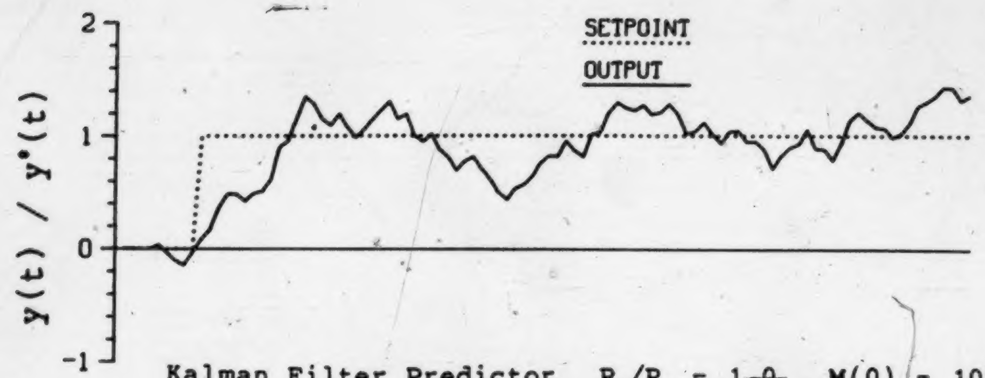


Kalman Filter Predictor

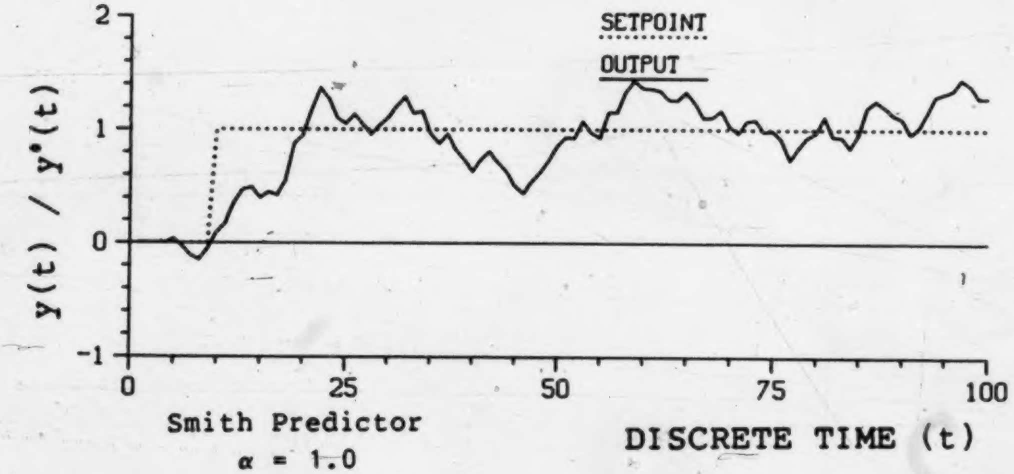


Smith Predictor

Figure 2.9b Predictive Control of 1^{st} Order Deterministic Process , using KFP and SP. $R_w/R_v = 0.5$, $M = 10^3$.



Kalman Filter Predictor R_w/R_v = 1~~.0~~ , M(0) = 10^3



Smith Predictor
$\alpha = 1.0$

Figure 2.10a PI control of a 1^{st} Order Stochastic Process using KFP and SP. v(t) = [0,0.1], w(t) = [0,0.1], $K_p = -1.36$, $\tau_I = 574$.

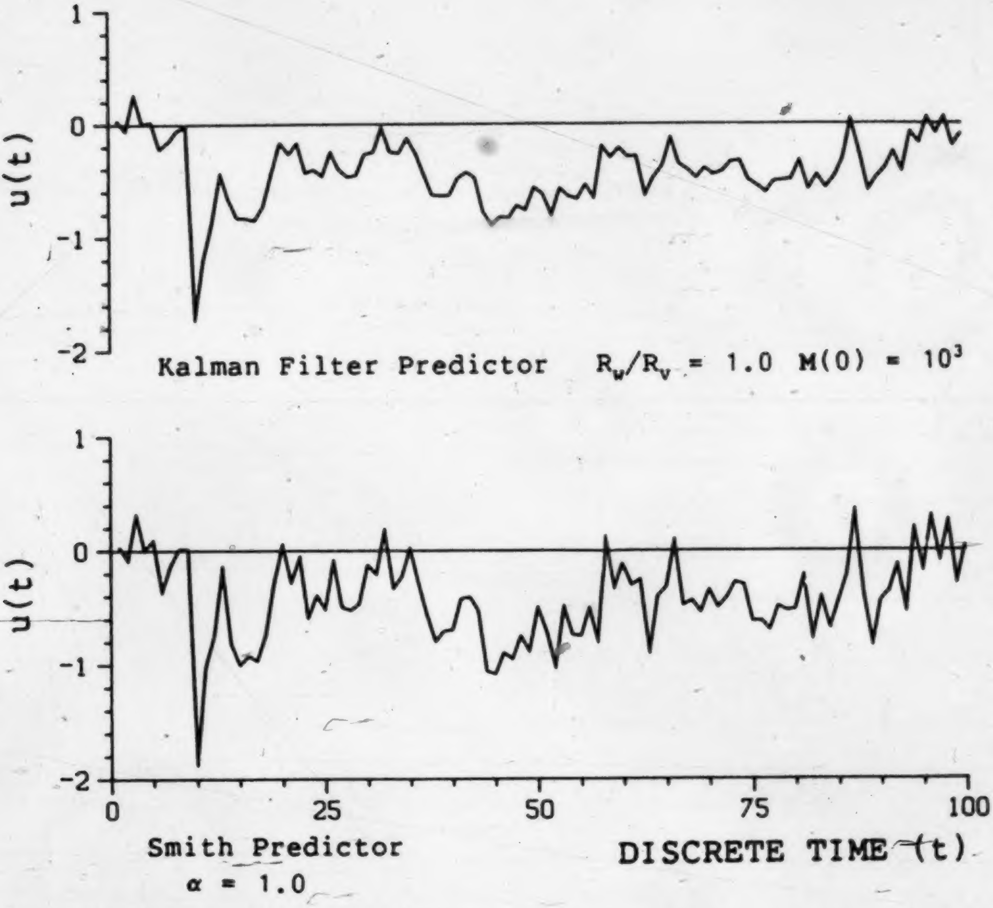


Figure 2.10b PI Control of a 1st Order Stochastic Process using KFP and SP. v(t) = [0,0.1], w(t) = [0,0.1], K_p = -1.36, τ_I = 574.

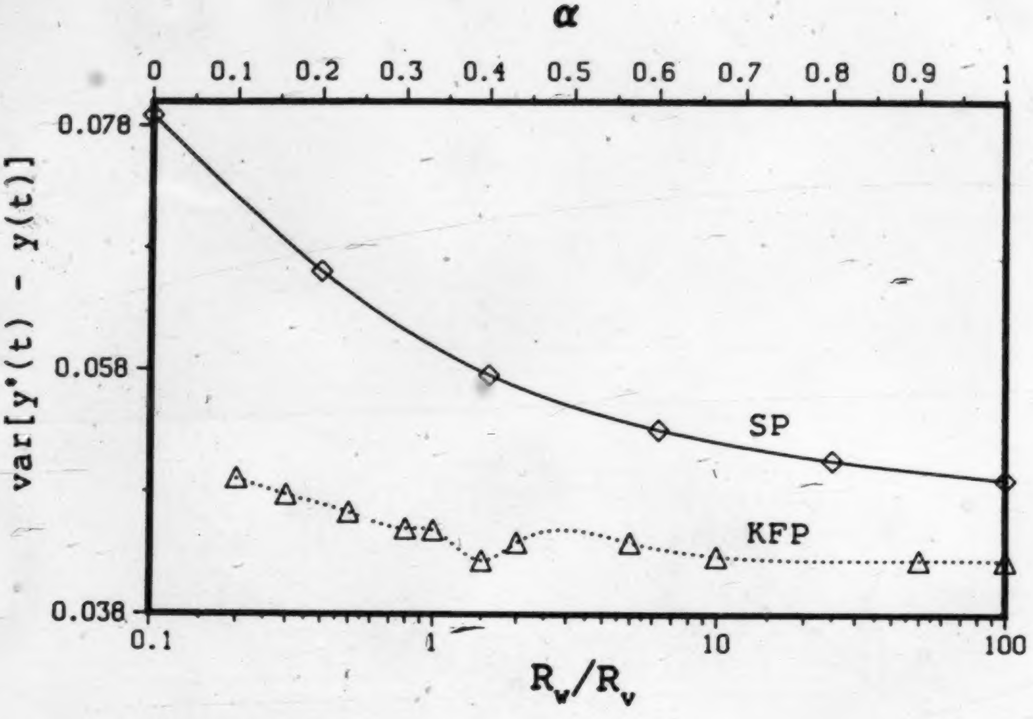
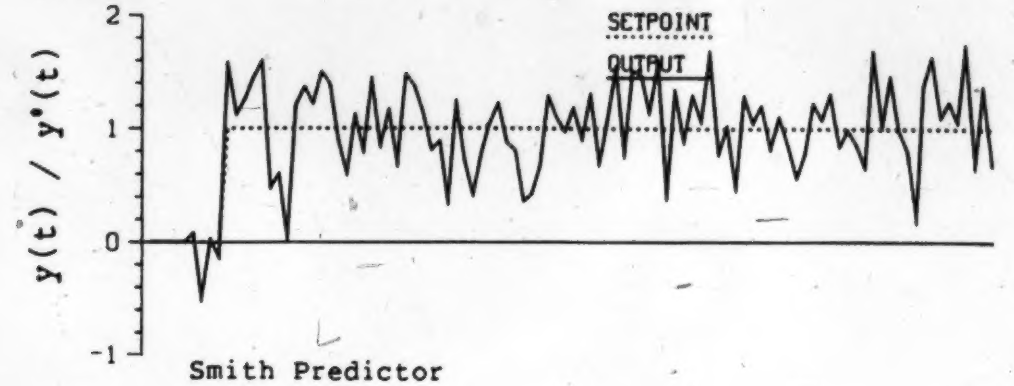
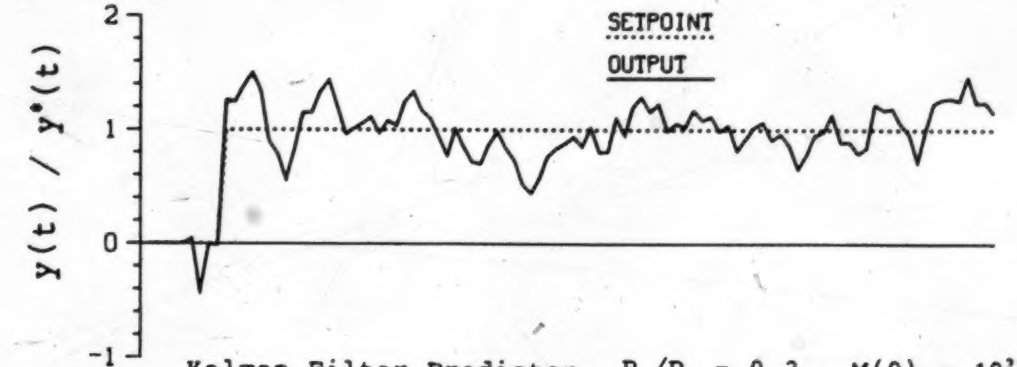


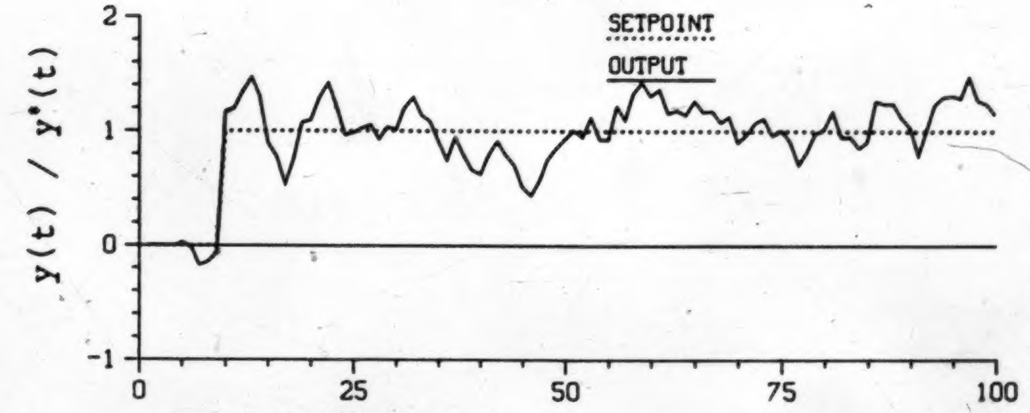
Figure 2.10c Variance of Output Error for PI Control of 1^{st} Order Stochastic Process using KFP and SP. $v(t) = [\ 0,0.1\ ]$, $w(t) = [\ 0,0.1\ ]$, $K_p = -1.36$, $\tau_I = 574$.



Smith Predictor

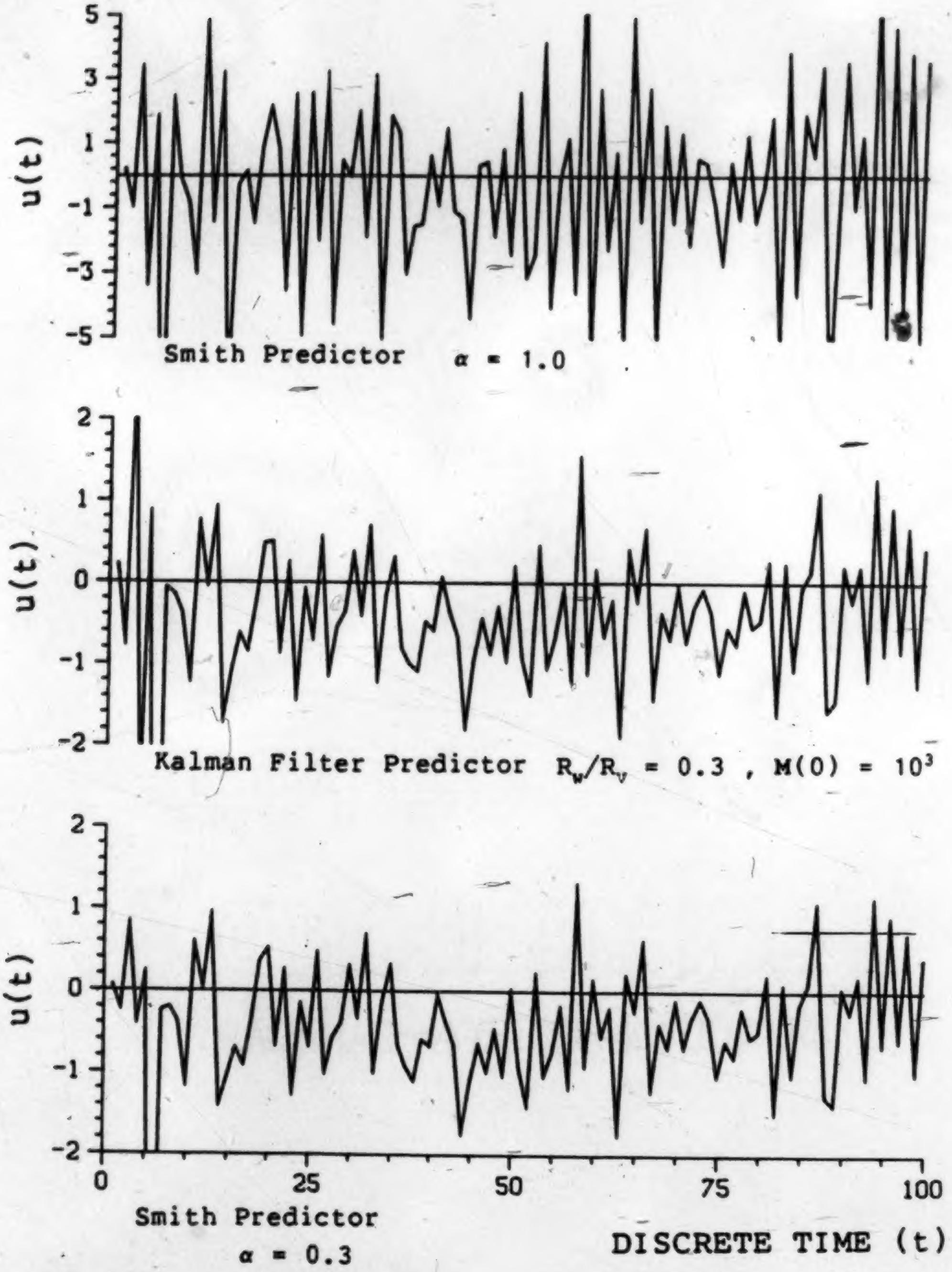


Kalman Filter Predictor $R_w/R_v = 0.3$, $M(0) = 10^2$



Smith Predictor
$\alpha = 0.3$

Figure 2.11a Predictive Control of a 1st Order Stochastic Process using KFP and SP. $v(t) = [\ 0,0.1\ ]$, $w(t) = [\ 0,0.1\ ]$,



Smith Predictor $\alpha = 1.0$

Kalman Filter Predictor $R_w/R_v = 0.3$, $M(0) = 10^3$

Smith Predictor $\alpha = 0.3$

Figure 2.11b Predictive Control of a 1st Order Stochastic Process using KFP and SP. $v(t) = [\ 0, 0.1\ ]$, $w(t) = [\ 0, 0.1\ ]$,

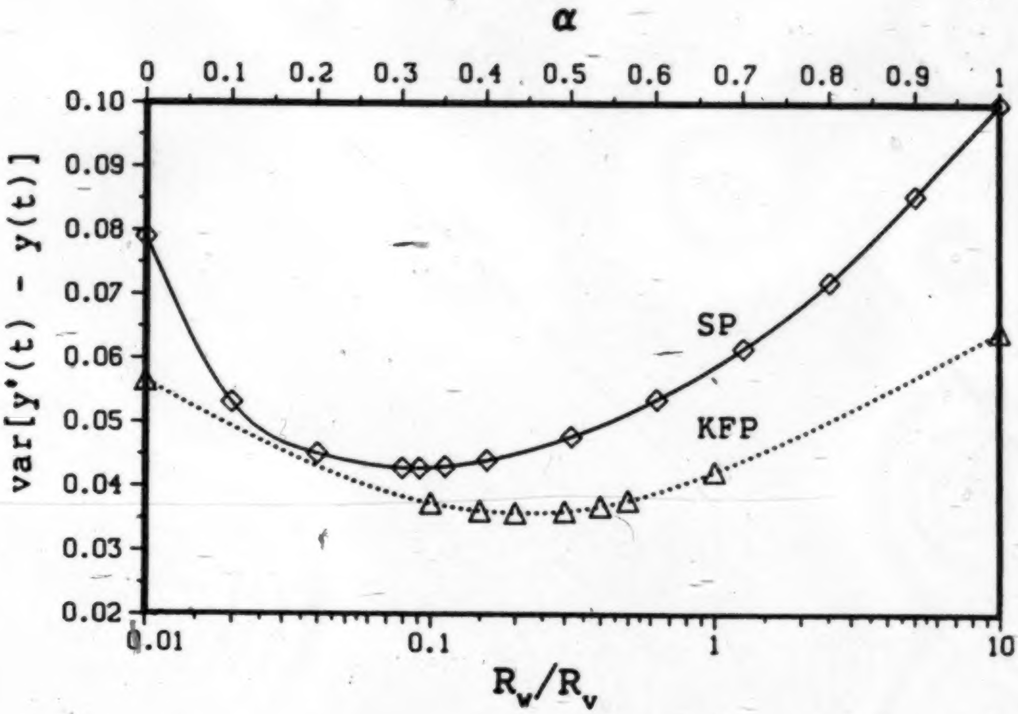
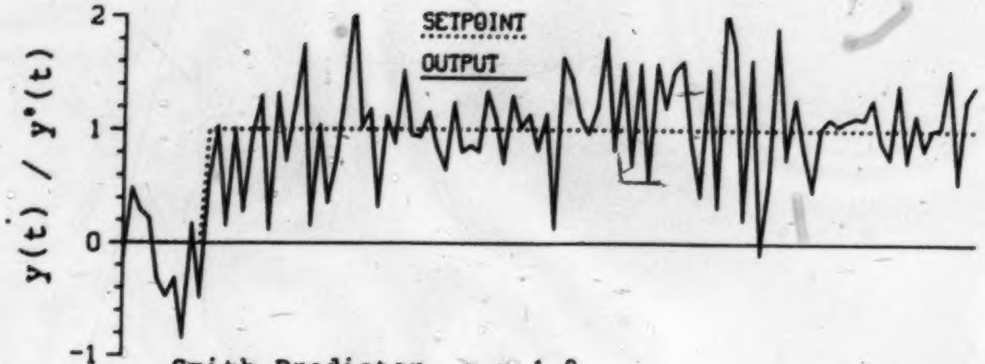
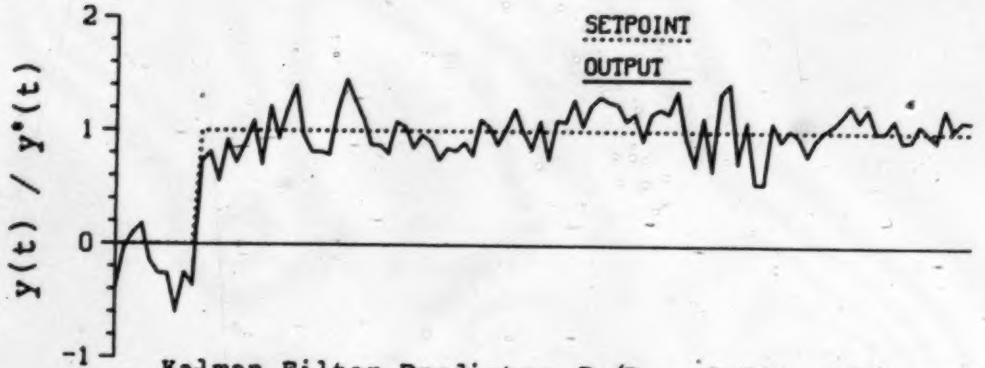


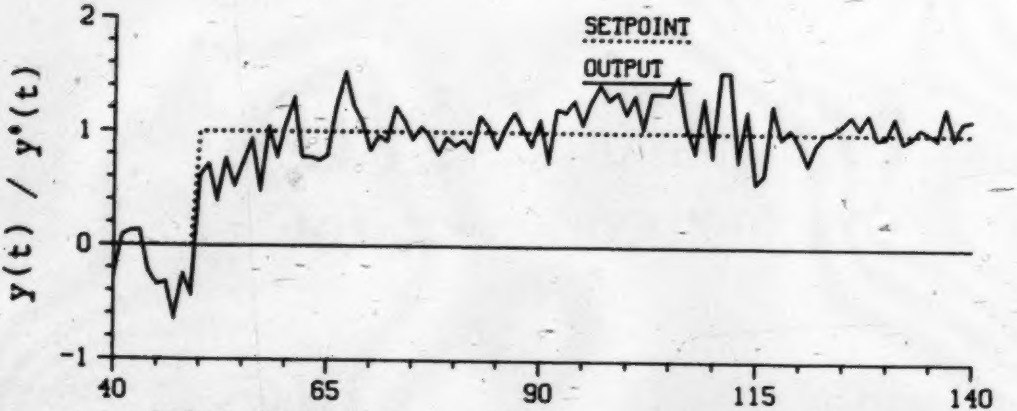
Figure 2.11c Variance of the Output Error for Predictive Control of the 1^{st} Order Stochastic Process using KFP and SP.



Smith Predictor $\alpha = 1.0$

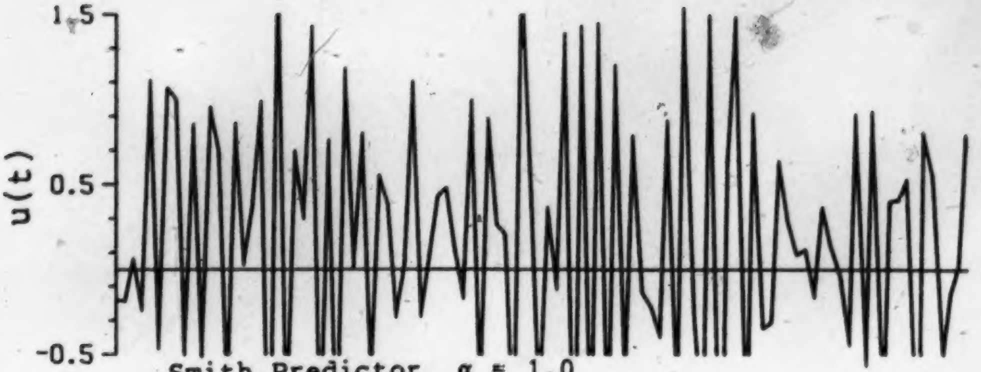


Kalman Filter Predictor $R_w/R_v = 0.001$, $M(0) = 10^3$

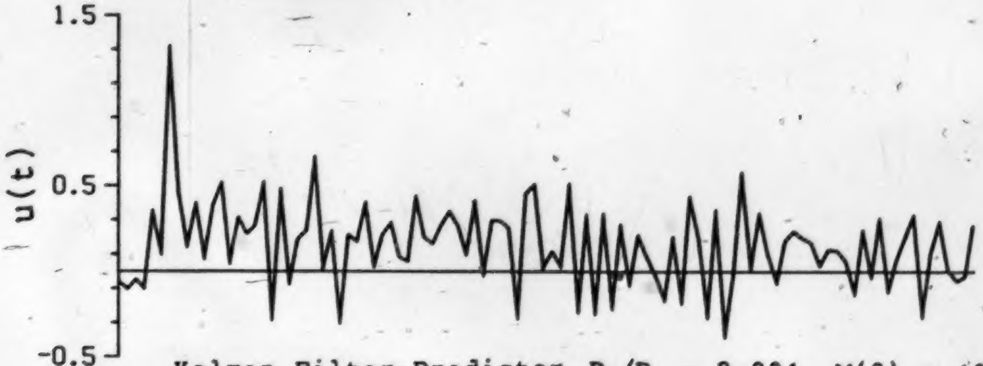


Smith Predictor $\alpha = 0.6$

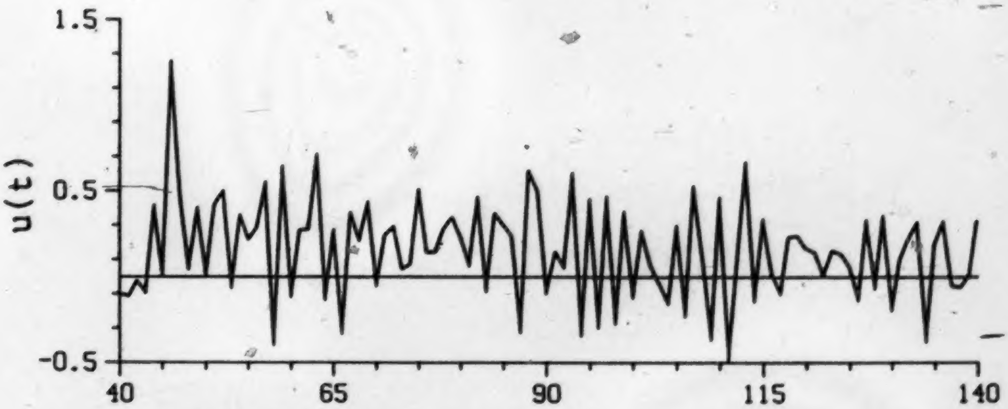
Figure 2.12a Predictive control of a 2nd order Stochastic process using Predictor Schemes. $v(t) = [\ 0,0.1\ ]$, $w(t) = [\ 0,0.01\ ]$,



Smith Predictor $\alpha = 1.0$



Kalman Filter Predictor $R_w/R_v = 0.001$, $M(0) = 10^3$



Smith Predictor $\alpha = 0.6$

Figure 2.12b Predictive Control of a 2nd Order Stochastic Process using Predictor Schemes. $v(t) = [\ 0,0.1\ ]$, $w(t) = [\ 0,0.01\ ]$, $\alpha = 1.0$

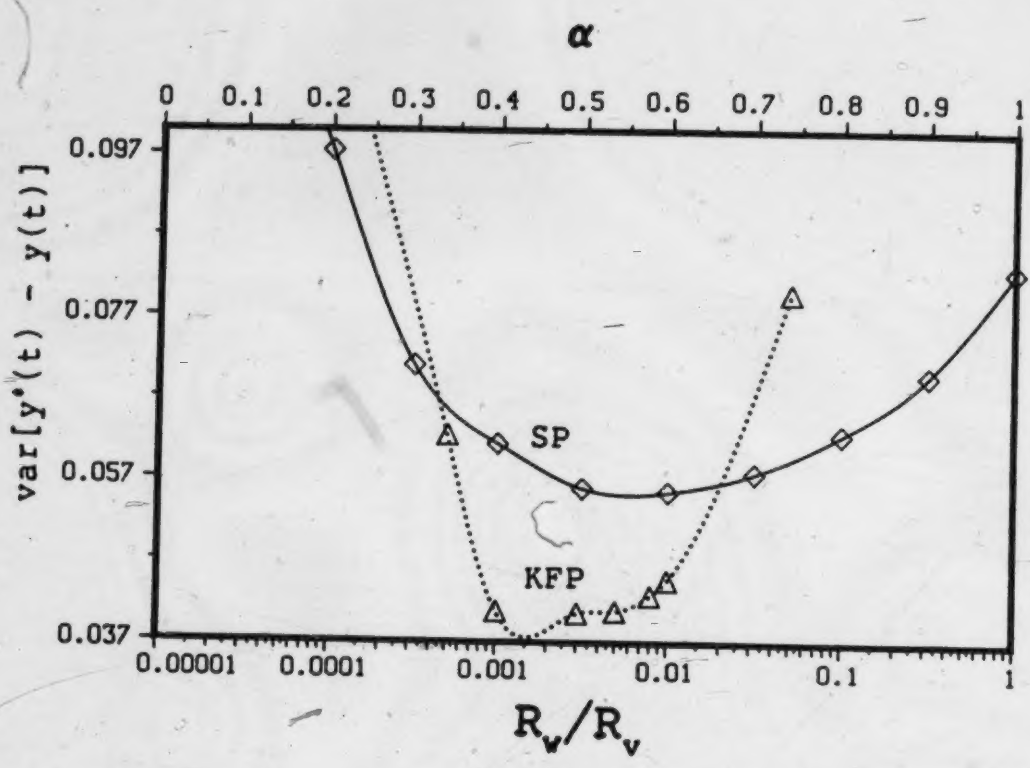


Figure 2.12c Variance of the Output Error for Predictive Control of the 2^{nd} Order Stochastic Process using KFP and SP.

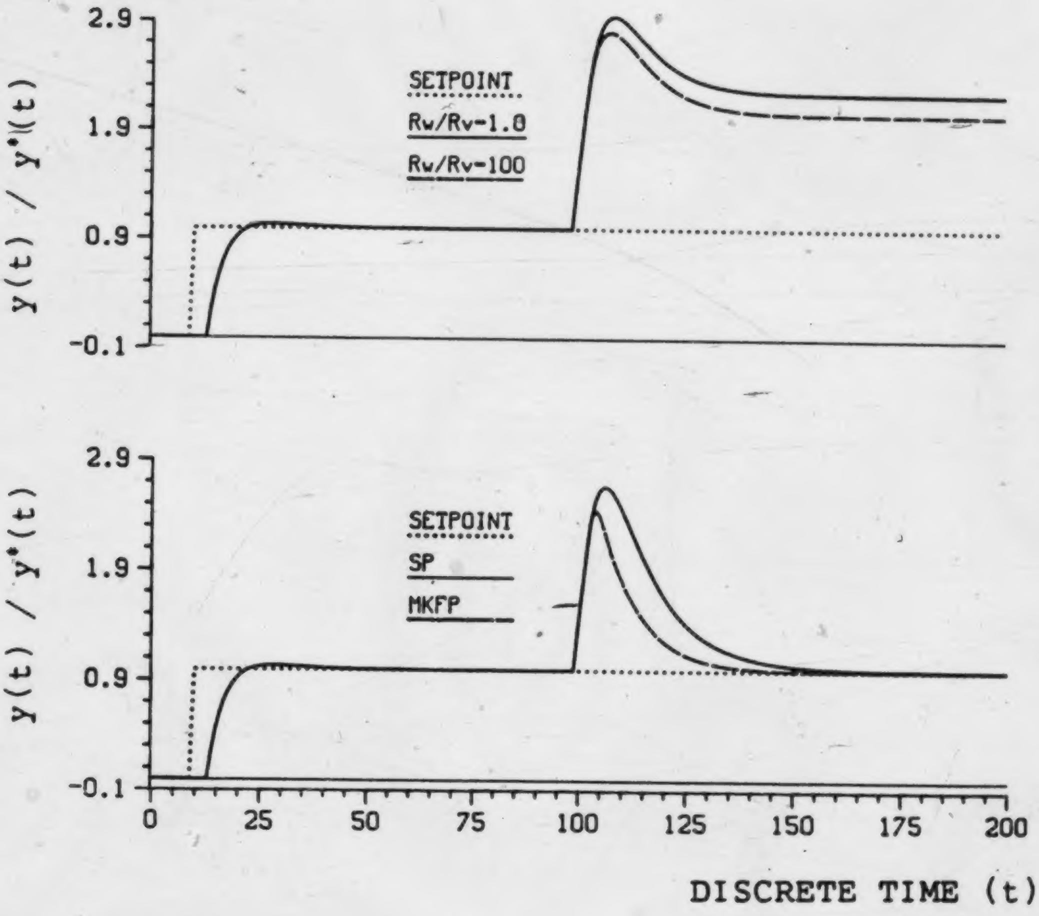


Figure 2.13a PI Control of 1st Order Process in Presence of Deterministic Disturbances using KFP, SP and MKFP.

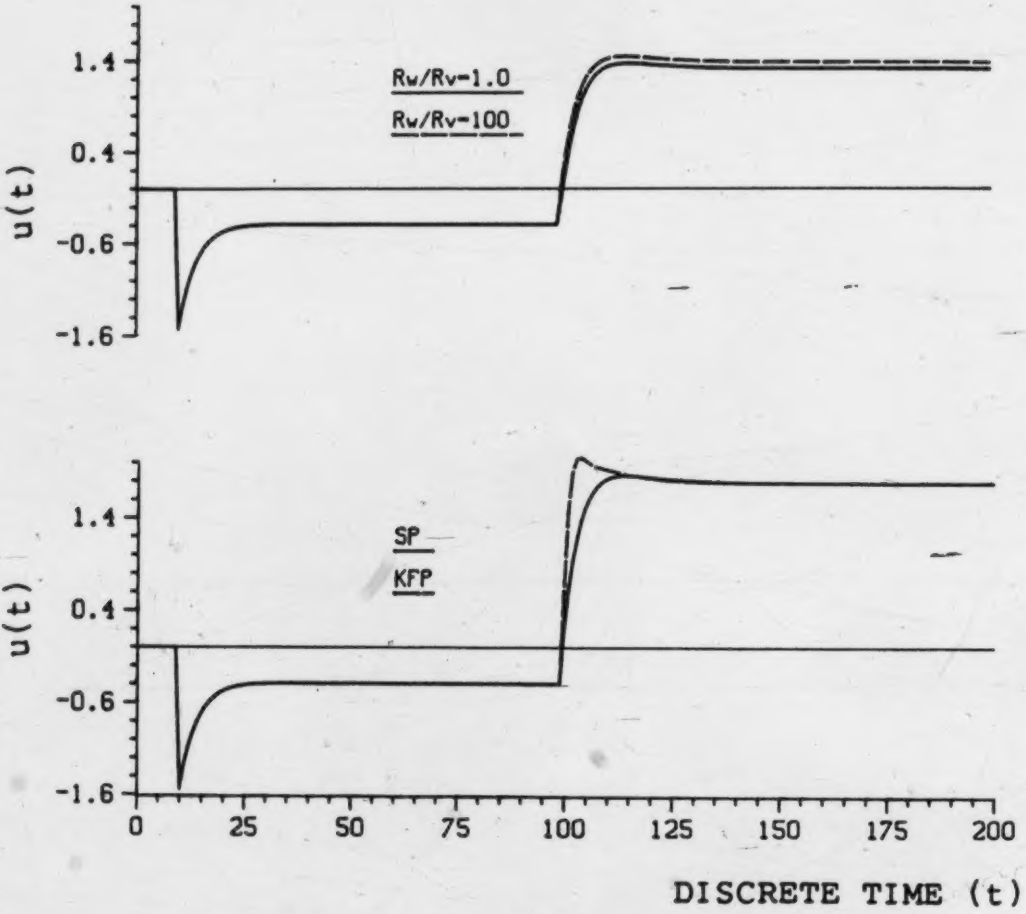


Figure 2.13b PI Control of 1^{st} Order Process in Presence of Deterministic Disturbances using KFP, SP and MKFP.

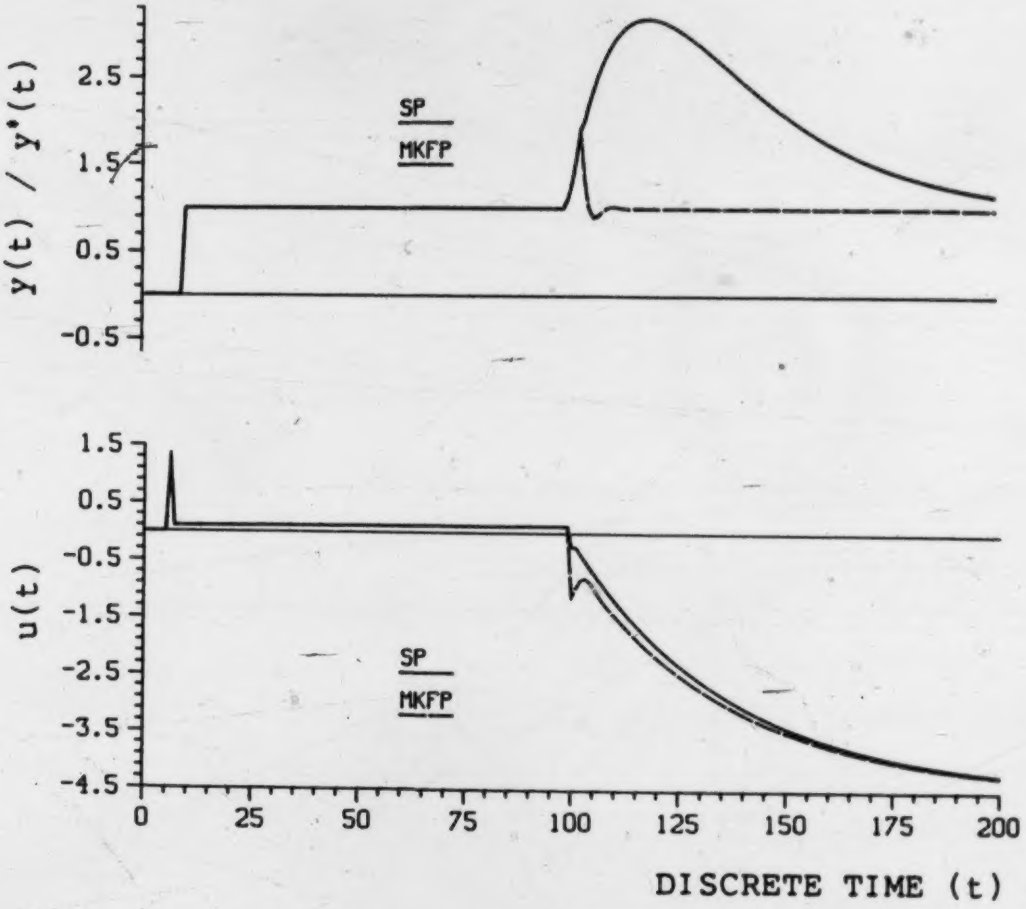


Figure 2.15 Predictive Control of 2nd Order Process in Presence of Deterministic Disturbances using MKFP and SP.

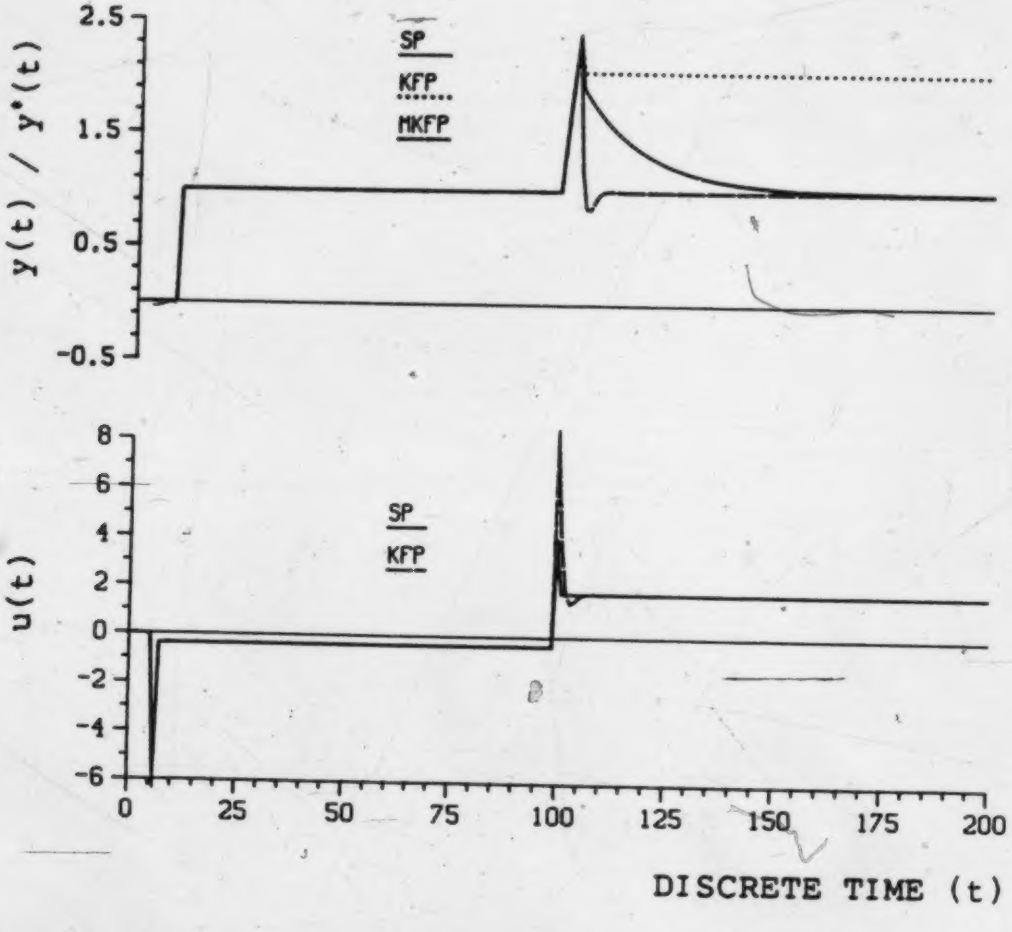
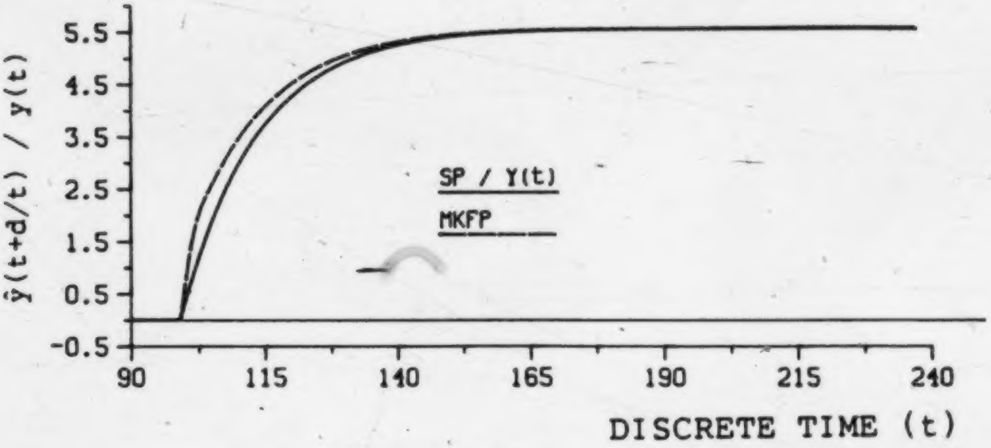


Figure 2.14 Predictive Control of 1st Order Process in Presence of Deterministic Disturbances using KFP, SP and MKFP.



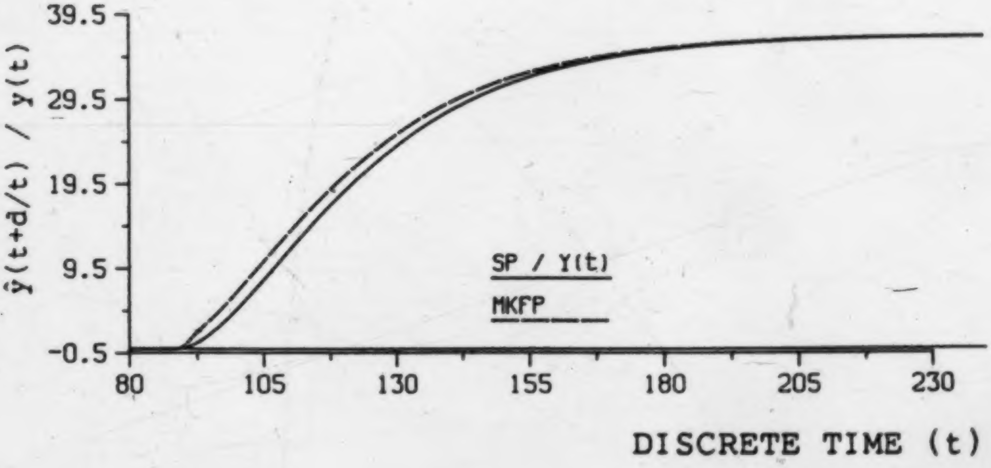
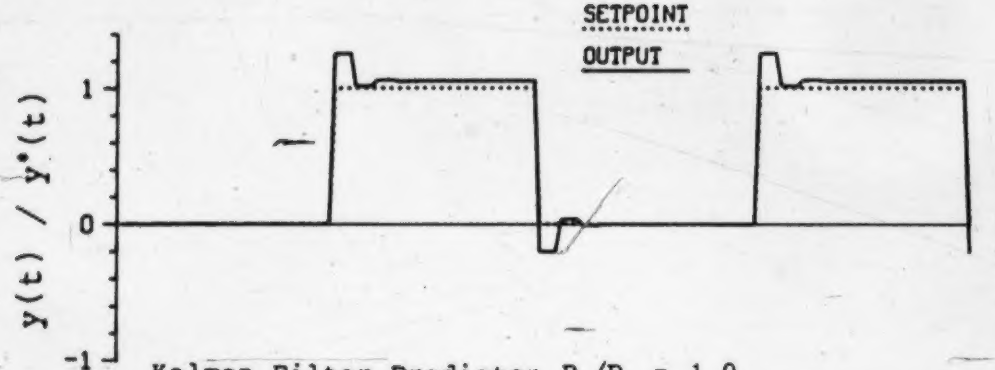
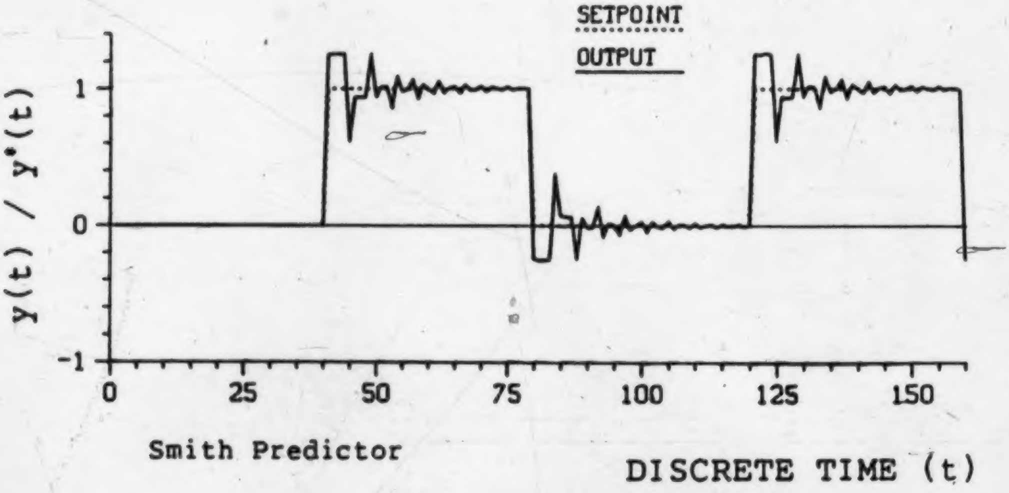


Figure 2.16 Open Loop Prediction in the SP and MKFP under Deterministic Disturbances, for 1st and 2nd Order processes.

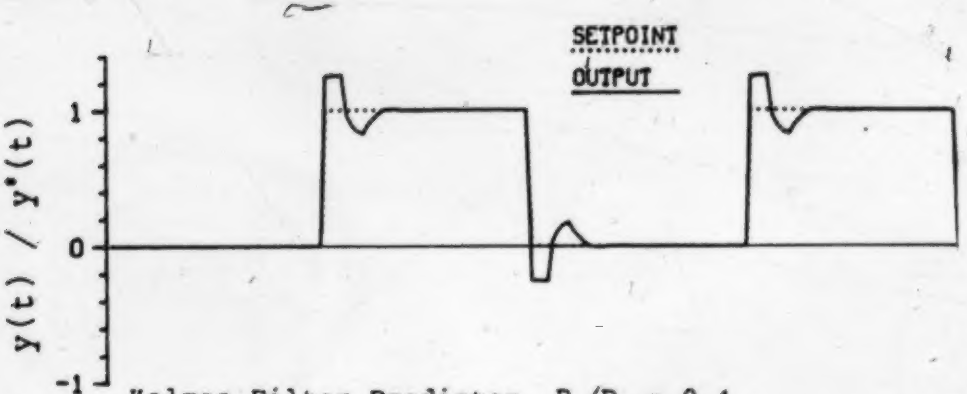


Kalman Filter Predictor $R_w/R_v = 1.0$

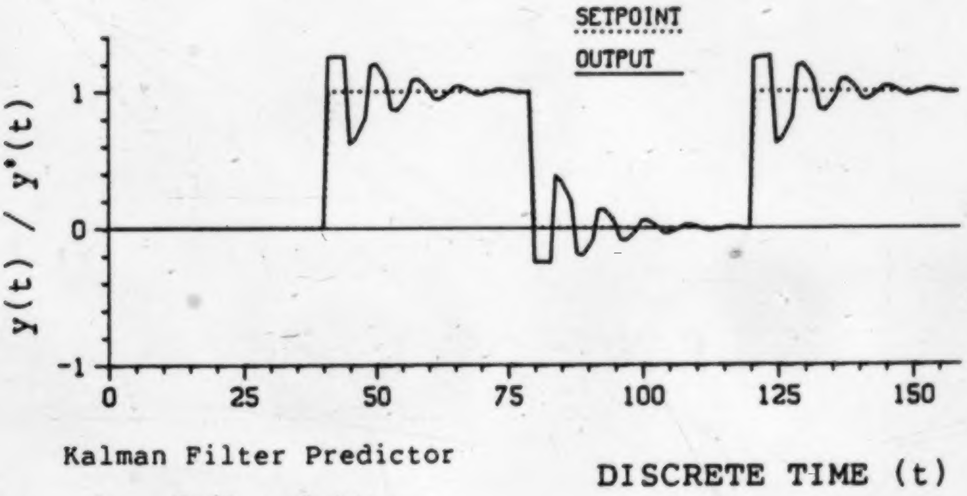


Smith Predictor

Figure 2.17a Predictive Control of SP and KFP under Unmodelled Dynamics.



Kalman Filter Predictor $R_w/R_v = 0.1$



Kalman Filter Predictor

$R_w/R_v = 0.01$

Figure 2.17b Predictive Control of MKFP under Unmodelled Dynamics.

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3. Multivariable Kalman Filter Predictor

3.1 Introduction

Time delays are often encountered in chemical processes, and significantly degrade the performance of conventional control loops. For single input single output (SISO) systems with time delays, the effects of time delays are easy to understand and hence there are a large number of dead time compensation schemes available in literature, e.g. Smith predictor (Smith, 1957, 1959), Kalman filter predictor (Bialkowski, 1978, 1983). However for Multi Input Multi Output (MIMO) systems with time delays, the multivariable controller design techniques, e.g. Multivariable Frequency Domain design techniques, become very difficult.

Moore et al. (1970), and Alevisakis and Seborg (1973, 1974), extended the SISO Smith predictor to MIMO systems having a single delay. Ogunnaike and Ray (1979), presented a multivariable controller design that could handle multiple time delays. A similar scheme has also been presented by Ibrahim and Fisher (1979). This multivariable predictor was obtained by removing all the time delays from the system Transfer Function Matrix (TFM). Although this facilitated the controller design, it did not give a physical meaning to the predictor output. Also, because of the interactions, removing all the time delays did not always produce the best control performance.

Sripada et al. (1985) proposed a dead time compensation scheme for MIMO systems with multiple time delays, based on the interactor factorization due to Wolovich and Falb (1976). The predictor in this scheme is based on the residual TFM obtained by removing the 'natural delay' defined by the interactor, from the system TFM. It is shown that removing only the time delays of the interactor yields an optimal dead time compensation scheme. The simulation results show better closed loop performance of this scheme over the dead time compensation scheme due to Ogunnaike and Ray.

The multivariable dead time compensation schemes discussed above are designed to work with deterministic processes. When process noise and measurement noise are present in the system the performance of these schemes deteriorates due to the noise present in the feedback loop.

In this chapter we present a multivariable stochastic predictor control scheme for processes with time delays. This is a multivariable extension of the Kalman filter predictor presented in chapter 2. This multivariable KFP uses a specific state space formulation to represent the MIMO process with time delay. The MIMO system is represented by p (number of outputs) Multi Input Single Output (MISO) state space formulations, which are serially connected. This state space formulation is based on the interactor factorization due to Wolovich ~~and Falb~~ (1976), and the observable state space formulation due to Kalman (1963). The states of this state space representation are estimated using p serially connected Kalman filters, thus giving serially connected MISO KFPs.

Because of the similarity of each of these MISO KFPs to the SISO KFP presented in Chapter 2, the results obtained for the observability of the state space representation, and the stability, and the convergence of the SISO KFP are directly applicable to each of MISO KFPs.

Using the innovation model concept it is shown that the configuration of the multivariable KFP is equivalent to the Interactor Predictor (IP) due to Sripada et al. (1985), except for a filter which does optimal filtering of noise in the KFP. The IP can be improved to work under a stochastic environment by introducing an adhoc (e.g. exponential) filter TFM in the same location as in the KFP.

The KFP can be used in conjunction with any type of controller design. In this chapter we use PID and predictive control algorithms. It is shown that the KFP plus the predictive controller leads to a minimum variance control system.

The simulation results show the satisfactory performance of both the PID and predictive control scheme based on the KFP.

3.2 MIMO systems with Multiple Time Delays

3.2.1 Introduction

The optimal predictor (Kalman Filter Predictor (KFP)), discussed in this chapter is based on a specific formulation of the state space representation for the MIMO process with time delays. The formulation is relatively straight forward for the SISO system as shown in chapter 2, because the time delay term in the process transfer function can be easily factored out. However, extension of this Kalman filter predictor to a MIMO system with multiple time delays in the transfer function matrix T(q), requires consideration of the following factors.

a. What is the best mechanism to factor out the time delay in a MIMO system having multiple time delays in the transfer function polynomial T(q), as

$$T(q) = D_T(q)R_T(q) \qquad (3.2.1)$$

D(q) - represents the time delay factor

R(q) - residual after factorization.

b. What is a suitable state space realization for R(q) and D(q) that would satisfy the following conditions.

b1. Individual state space models for D(q) and R(q), and the augmented model are observable.

b2. The predicted future outputs defined by the factorization in (3.2.1), are expressed directly as a state, or a linear combination of states.

b3. The elements of the state space matrices which are non zero and non unity are explicitly in the transfer function matrix T(q) or the corresponding ARMA model, as parameters. As discussed later in chapter 5, this structure is necessary to extend this scheme to an adaptive system.

A suitable factorization method and a state space formulation that satisfies the above requirements are presented in the following sections.

3.2.2 Interactor Factorization

The concept of "interactor factorization" which provides a means to factor out the 'natural' delay associated with a MIMO system was first proposed by Wolovich and Falb (1976). The factored matrix that represents the time delays is called the "interactor matrix". Their work has provided a strong mathematical tool for developing MIMO control systems, especially in extending the existing SISO control systems, to MIMO systems with time delays.

The theory of interactor factorization and the factorization procedure are given also in Wolovich and Elliott (1983), and Goodwin and Sin (1984). The interactor matrix has been used to represent the time delays for MIMO systems, when developing adaptive control algorithms for processes with time delays, by Elliott and Wolovich (1984), Dugard, Goodwin and Xianya (1984), Goodwin and Dugard (1983), and Goodwin and Sin (1984). A multivariable dead time compensation scheme based on the interactor factorization was proposed by Sripadaet al. (1985).

For a SISO system with an ARMA model given by,

$$A(q^{-1})\ y(t) = B(q^{-1})\ u(t) \tag{3.2.2}$$

with

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdot\ \cdot\ \cdot + a_nq^{-n} \tag{3.2.3}$$

$$B(q^{-1}) = q^{-(d+1)}(b_1+b_2q^{-1}+\cdot\cdot\cdot+b_{n-1}q^{-(n-1)}) \tag{3.2.4}$$

the time delay (d+1) (actual process delay plus discretization delay) can be formally expressed by defining a scaler function $\xi(q) = q^{d+1}$ such that,

$$\lim_{q \longrightarrow \infty} \xi(q)A^{-1}(q^{-1})B(q^{-1}) = K \tag{3.2.5}$$

where K is a non zero scaler.

The SISO transfer function $T(q^{-1})$ given by,

$$T(q^{-1}) = A^{-1}(q^{-1})B(q^{-1}) \tag{3.2.6}$$

can be written as,

$$T(q^{-1}) = \xi^{-1}(q^{-1})R(q^{-1}) \tag{3.2.7}$$

or as,

$$T(q^{-1}) = q^{-(d+1)}R(q^{-1}) \tag{3.2.8}$$

The same idea has been extended to MIMO systems by Wolovich and Falb (1976), who showed that for every pxm proper, rational transfer function matrix T(q), there is an unique, non-singular, pxp lower-left-triangular matrix $\xi_T(q)$ called the interactor of T(q) such that,

1. $$\det \xi_T(q) = q^g \tag{3.2.9}$$

   where g is an integer.

2. $$\lim_{q \to \infty} \xi_T(q) T(q) = K \qquad (3.2.10)$$

where K is a non-singular matrix of full rank (= min[p,m]).

Since $\xi_T(q)$ is a non-singular matrix, there exists a stable operator matrix called the inverse interactor matrix $\xi_T^{-1}(q)$, which represents a left divisor of T(q). Physically ξ_T^{-1} represents a measure of the natural delay associated with a discretized MIMO system. The T(q) can be factored out as follows:

$$T(q) = \xi_T^{-1}(q)\, R_T(q) \qquad (3.2.11)$$

where,

ξ^{-1} - is a pxp matrix

R_T - is a pxp matrix called the residual.

Note that as in the case of SISO system, $\xi_T^{-1}(q)$ factors out the delay due to discretization also.

When the system transfer function T(q) is causal invertible then the degree of the determinant of the ξ_T^{-1} represents the number of infinite zeros the system. $R_T(q)$ is bicausal, i.e. does not have poles or zeros at infinity. For a discrete time SISO system a zero at infinity implies a delay of one sample period. Thus for discrete time MIMO systems the inverse interactor $\xi_T^{-1}(q)$ factors out the zeros at infinity and provides a measure of the natural delay associated with T(q).

In general a discretized plant $T(q)$ can be obtained using step or pulse response data and put into the following form:

$$T(q) = \begin{bmatrix} q^{-d11}t_{11}(q) & q^{-d12}t_{12}(q) & \cdots & q^{-d1m}t_{1m}(q) \\ q^{-d21}t_{21}(q) & q^{-d22}t_{22}(q) & \cdots & q^{-d2m}t_{2m}(q) \\ \cdot & \cdot & & \cdot \\ q^{-dp1}t_{p1}(q) & q^{-dp2}t_{p2}(q) & \cdots & q^{-dpm}t_{pm}(q) \end{bmatrix} \qquad (3.2.12)$$

where, t_{ij} is a proper rational function in the operator q, and d_{ij} is the delay between i^{th} output and the j^{th} input.

Since discrete systems possess an inherent unit delay due to the zero order hold, their transfer function matrices are always strictly proper and the interactor $\xi_T(q)$ will have the structure,

$$\xi_T(q) = H_T(q)D_T(q) \qquad (3.2.13)$$

where, $H_T(q)$ is a lower triangular, unimodular matrix, with unity values on the diagonal.

$$H_T(q) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ h_{21}(q) & 1 & & \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ h_{p1}(q) & h_{p2}(q) & \cdots & 1 \end{bmatrix} \qquad (3.2.14)$$

$$D_T(q) = \mathrm{Diag}\,(q^{\lambda_1}, q^{\lambda_2}, \cdots, q^{\lambda_p}) \qquad (3.2.15)$$

and $\lambda_i \geq 1$ and $i \leq p$.

Often $H_T(q) = I$ and $\xi_T(q) = \mathrm{diag}\,(q^{\lambda_i})$, in which case λ_i can be interpreted as the minimum delay between any of the inputs and i^{th} output. It also represents the minimum relative degree of any of the transfer functions in the i^{th} row of $T(q)$.

The existence and the uniqueness of the interactor matrix $\xi(t)$, satisfying the above mentioned conditions are proved by Wolovich and Falb (1976), through a constructive procedure, for continuous time systems. The same procedure was used by Goodwin and Sin (1984) for discrete time systems. The factorization procedure along with some numerical examples are given in Appendix F.

Since ξ_T^{-1} extracts the delay due to discretization from some elements (at least one element of each row) of $T(q)$, the corresponding elements in the TFM $R_T(q)$ will not be strictly proper. This would cause problems in the state space realization given in section (3.4). To retain the delay due to ZOH, in the residual define the factorization is defined as follows:

$$T(q) = \xi^{-1}(q)\, R(q) \tag{3.2.16}$$

where

$$R(q) = q^{-1}\, R_T(q) \tag{3.2.17}$$

$$\xi^{-1}(q) = q\, \xi_T^{-1}\ (q) \tag{3.2.18}$$

Now the residual $R(q)$ is strictly proper, but this might remove the strictly proper property of the inverse interactor $\xi^{-1}(q)$. However, this will not cause any problems in the design of the KFP.

Example:

If,

$$\xi_T^{-1}(q) = \begin{bmatrix} q^{-1} & 0 \\ 0 & q^{-2} \end{bmatrix}$$

then

$$\xi^{-1}(q) = \begin{bmatrix} 1 & 0 \\ 0 & q^{-1} \end{bmatrix}$$

In this example $\xi_T^{-1}(q)$ is strictly proper but $\xi^{-1}(q)$ is not.

3.3 Predictors for MIMO systems with Time delays

3.3.1 Introduction

A number of schemes to predict the undelayed output of a SISO process with time delays are available in literature, e.g. the Smith predictor. The formulation of the predictor is easy for the SISO case, because for a linear system the time delay can be factored from the rest of the dynamics. The undelayed output of the process is clearly defined for a SISO system as the output of the process when the time delays are not present.

The main problem in extending this predictor concept to the multivariable case is to define an "undelayed" output vector of the process, which can be predicted.

Moore et al (1970), Alevisakis and Seborg (1973 ,1974), extended the SISO Smith predictor (1957 ,1959) to MIMO system having a single delay. Ogunnaike and Ray (1979), presented a multivariable controller design that could handle multiple delays. A similar scheme was also presented by Ibrahim and Fisher (1979). In this scheme the predictor was obtained by removing all the delays from the system transfer function matrix. Note that this cannot be represented by a factorization as in equation (3.2.1), except for the special case when each element of a row in the transfer function matrix has the same delay as shown in Appendix G. Removing all the time delays from the system TFM, facilitates the controller design. Any conventional controller design technique for processes without time delays , e.g. multivariable frequency domain controller design, can be employed. But the main drawback in this scheme is that it does not give much physical meaning to the 'undelayed' output. Also the relationship of the output of the real plant to the output of the delay free model is not a simple set of time delays. Thus "optimal" tuning of the feedback controller based on the model output may not be optimal for the real plant.

Sripada et al. (1985) proposed a predictor for MIMO systems with time delays, based on the interactor factorization due to Wolovich and Falb (1976). This predictor was based on the residual $\underline{R}_T(q)$, which was obtained after removing the 'natural delay' defined by the interactor, from the system TFM as demonstrated in Appendix H. It was also shown that using the interactor matrix to represent the time delay, is optimal under certain conditions.

The Interactor Predictor (IP) due to Sripada, Fisher and Shah (1985) is designed for deterministic system. The MIMO Kalman Filter Predictor (KFP) presented in this chapter is based on the interactor factorization approach, and designed to handle a stochastic system.

3.3.2 Predictor Concept for MIMO systems

For a single input single output system defined by the ARMA model (3.2.2),

$$A(q^{-1})\ y(t) = q^{-(d+1)}\ B(q^{-1})\ u(t) \qquad (3.3.1)$$

the future value of the output at time t+d+1 is given by y(t+d+1).

But for a MIMO system the concept of future output values is not so straightforward.

Consider,

$$y(t) = \xi_T^{-1}(q)\ R_T(q)\ u(t) \qquad (3.3.2)$$

and define a filtered output:

$$\text{where } \overline{y}(t) = \xi_T(q)\ y(t) = q\ \xi(q)\ y(t) \qquad (3.3.3)$$

$\overline{y}(t)$ is defined as the prediction in a MIMO system.

A schematic block diagram of the MIMO system, based on the interactor factorization is given in figure 3.1. The first block in figure 3.1 represents the MIMO system dynamics after appropriate time delays have been factored as in equation (3.2.16). The vector $y^0(t)$ of dimension p, which is the output of the first block is given by,

$$\begin{aligned} y^0(t) &= q^{-1} R_T(q)\ u(t) = R(q)\ u(t) \\ &= q^{-1}\ \bar{y}(t) \end{aligned} \tag{3.3.4}$$

Also from figure 3.1, the output of the second block is,

$$\begin{aligned} y(t) &= q\ \xi_T^{-1}(q)\ y^0(t) \\ &= \xi^{-1}(q)\ y^0(t) \end{aligned} \tag{3.3.5}$$

The quantity we are interested in for the design of a predictor is $\bar{y}(t)$. Since the discretization delay is included in R(q), the quantity we are interested is $\bar{y}(t-1)$. From equation (3.3.4) it is clear that the output of the first block $y^0(t)$ gives the prediction $\bar{y}(t-1)$.

The Interactor predictor due to Sripada et al (1985) is a deterministic predictor that provides the estimates of $y^0(t)$, based on equation (3.3.4).

The objective of this present work is to estimate this output $y^0(t)$, in a stochastic environment. Since we are interested in using a Kalman filter to estimate the vector $y^0(t)$, it is necessary to obtain a suitable state space formulation for the system given by equations (3.3.4 and 5).

3.3.3 Relationship between ARMA and TFM models for the MIMO system with Interactor Factorization

Let the $(i,j)^{th}$ element of the transfer function matrix R(q) be expressed as,

$$R_{ij}(q) = \frac{b^{ij}(q)}{a^{i}(q)}\, q^{-(d_{ij}+1)} \tag{3.3.6}$$

where

$$a^{i}(q) = q^{ni} + a^{i}_{1}\, q^{ni-1} + \cdot \cdot \cdot \cdot + a^{i}_{ni} \tag{3.3.7}$$

$$b^{ij}(q) = b^{ij}_{1}\, q^{ni} + \cdot \cdot \cdot + b^{ij}_{ni-1}\, q + b^{ij}_{ni} \tag{3.3.8}$$

d_{ij} is the time delay, with at least one, $d_{ij} = 0$, and, $j = 1, \cdot \cdot, m$ for each $i = 1, \cdot \cdot, p$.

Now consider,

$$\xi^{-1}(q) = \begin{bmatrix} q^{-\lambda_1} & & & \\ g^{21}(q) & q^{-\lambda_2} & \cdot\ \cdot\ \cdot & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & & \cdot \\ \cdot & & \cdot & \cdot \\ g^{p1}(q) & \cdot\ \cdot & g^{p,p-1}(q) & q^{-\lambda_p} \end{bmatrix} \tag{3.3.9}$$

where,

$$g^{ij}(q) = \frac{g^{ij}_{1}\, q^{\lambda_i - 1} + g^{ij}_{2}\, q^{\lambda_i - 2} + \cdot \cdot \cdot \cdot + g^{ij}_{\lambda_i}}{q^{\lambda_i}} \tag{3.3.10}$$

$1 < i \leq p, \qquad j \leq i-1$

or,

$$g^{ij}(q^{-1}) = g^{ij}_{1}\, q^{-1} + \cdot \cdot \cdot + g^{ij}_{\lambda_i}\, q^{-\lambda_i} \tag{3.3.11}$$

It is important to note that the degree of the polynomial $g^{ij}(q^{-1})$ is limited as follows for simplicity:

minimum degree of $g^{ij}(q^{-1}) \geq 1$

maximum degree of $g^{ij}(q^{-1}) \leq \lambda_i$

That is, it is assumed in the following development that the maximum delay term in the ith row of $\xi^{-1}(q^{-1})$ is given by the delay in the diagonal element of that row. This is not proved for the general case. However the formulation is not restricted to these conditions.

The TFM R(q) can be interpreted as consisting of p MISO systems. Based on the equation (3.3.6) the ARMA model for the j^th MISO system can be written as,

$$\alpha_j(q^{-1})y_j^0(t) = q^{-1}\beta^j(q^{-1})\ u(t) \qquad (3.3.12)$$

where

$$\alpha_j(q^{-1}) = 1 + a_1^j\ q^{-1} \cdots + a_{nj}^j\ q^{-nj} \qquad (3.3.13)$$

$$\begin{aligned}\beta^j\ (q^{-1})\ u(t) &= [\beta^{j1}(q^{-1})\ \beta^{j2}(q^{-1}) \cdots \beta^{jm}\ (q^{-1})\ ]\ u(t)\\ &= \beta_1^j + \beta_2^j\ q^{-1} \cdots + \beta_{nj}\ q^{-n'j}\end{aligned} \qquad (3.3.14)$$

with

$$\beta^{jk}\ (q^{-1}) = q^{-djk}(b_1^{jk} + b_2^{jk}\ q^{-1} + \cdots + b_{nj}^{jk}\ q^{-nj}) \qquad (3.3.15)$$

and

$$n_j = n_j + \max\ [d_{jk}] \text{ for } k = 1,m.$$

Note that coefficients of α_j and β^{jk} are same as that of $a^j(q)$ and $b^{jk}(q)$ respectively.

The ARMA model for the TFM R(q) can be written as,

$$A(q^{-1})\ y^0(t) = q^{-1}\ B(q^{-1})\ u(t) \qquad (3.3.16)$$

where

$$\begin{aligned}A(q^{-1})\ y^0(t) &= [\alpha^1(q^{-1})\ \alpha^2(q^{-1}) \cdots \alpha^p(q^{-1})]y^0(t)\\ &= (I + A_1q^{-1} + A_2q^{-2} + \cdots + A_nq^{-n})y^0(t)\end{aligned} \qquad (3.3.17)$$

with

$$A_j = \text{diag}\ [a_j^1\ \ a_j^2 \cdots a_j^p\ ] \qquad (3.3.18)$$

and $n = \max[n_j]$ for $j=1,p$.

$$B(q^{-1})u(t) = \begin{bmatrix} \beta^{11}(q^{-1}) & \cdot & \cdot & \cdot & \beta^{1p}(q^{-1}) \\ \cdot & & & & \\ \cdot & & & & \\ \beta^{p1}(q^{-1}) & \cdot & \cdot & \cdot & \beta^{pm}(q^{-1}) \end{bmatrix} u(t) \tag{3.3.19}$$

$$= B_1 + B_2 q^{-1} + \cdot \cdot \cdot + B_{n'} q^{-n'}$$

where $n' = \max\,[n_j^i]$

As shown in appendix I, B_1 is non-singular.

The inverse interactor given by equation (3.3.9) can be rewritten as,

$$\xi^{-1}(q) = D(q) + G_H(q) \tag{3.3.20}$$

where

$$D(q) = \text{Diag}\,(q^{\lambda_1}, q^{\lambda_2}, \cdot \cdot \cdot, q^{\lambda_p}) \tag{3.3.20}$$

$$G_H(q) = \begin{bmatrix} 0 & & & & \\ g^{21}(q) & 0 & & & 0 \\ \cdot & & \cdot & & \\ \cdot & & & \cdot & \\ g^{p1}(q) & \cdot & \cdot & g^{p,p-1}(q) & 0 \end{bmatrix} \tag{3.3.22}$$

The ARMA model for the process given by the TFM $T(q)$ is,

$$y(t) = A_T^{-1}(q^{-1}) B_T(q^{-1})\, u(t) \tag{3.3.23}$$

Since $T(q) = \xi^{-1}(q) R(q)$,

$$A_T^{-1} B_T(q^{-1}) = \xi_T^{-1} A^{-1}(q^{-1}) B(q^{-1}) \tag{3.3.24}$$

3.4 State Space Formulation

3.4.1 Observable State Space Realization for Discrete Time MIMO Systems

There are a number of observable state space realizations for discrete MIMO systems available in literature. Most of them are concerned with obtaining an observable state space formulation from the TFM. Wolovich and Elliott (1983) present an observable state space realization for discrete systems, which is based on the state space realization for continuous systems by Wolovich (1974). A review of MIMO observable state space realizations for continuous systems which can also be used for discrete systems is given in Kailath (1980). The difficulty in using those schemes is that the respective ARMA parameters of the TFM do not appear explicitly in the state space model. They are obtained by a linear transformation on the existing state space realizations, e.g. Guidorzi (1978), El-Sherief (1981), Sinha and Kwong (1979), and El-Sherief and Sinha (1979).

The state space formulation used in this work is based on the observable canonical form due to Kalman (1963), which was developed for continuous systems. We have developed the discrete version of the Kalman's canonical form, which was developed from the equivalent continuous time system given in Mayne (1968).

Let T(q) be the transfer function matrix for which we are interested in obtaining a state space representation.

$$T(q) = \begin{bmatrix} t_{11}(q) & t_{12}(q) & \cdot\cdot & t_{1m}(q) \\ t_{21}(q) & t_{22}(q) & \cdot\cdot & t_{2m}(q) \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ t_{p1}(q) & t_{p2}(q) & \cdot\cdot\cdot & t_{pm}(q) \end{bmatrix} \qquad (3.4.1)$$

This is a pxm proper, rational transfer function matrix in q. For physical realization T(q) should be causal.

A discrete, linear system described by a rational transfer function matrix is causal if and only if T(q) is proper, i.e.

$$\lim_{q \longrightarrow \infty} T(q) = K$$

where K is a finite pxm matrix.

T(q) is strictly proper if K=0.

If each element $t_{ij}(q)$ of T(q) is obtained through discretization using a ZOH then,

$$\lim_{q \longrightarrow \infty} t_{ij}(q) \text{-->} 0 \;\forall\; i,j \qquad (3.4.2)$$

and thus T(q) is strictly proper.

In the system equation

$$y(t) = T(q)\ u(t) \qquad (3.4.3)$$

where

y(t) - px1 output vector

u(t) - mx1 input vector

the i^{th} output $y_i(t)$ is given by,

$$y_i(t) = t_{i1}(q)u_1(t) + t_{i2}u_2(t) \cdot \cdot \cdot + t_{im}(q)u_m(t) \quad (3.4.4)$$

or

$$y_i(t) = q^{-d_{i1}} \frac{b^{i1}(q)}{a^i(q)} u_1(t) + q^{-d_{i2}} \frac{b^{i2}(q)}{a^i(q)} u_2(t) +$$

$$\cdot \cdot \cdot + q^{-d_{im}} \frac{b^{im}(q)}{a^i(q)} u_m(t) \quad (3.4.5)$$

where $a^i(q)$ is the least common multiplier of the denominator of the elements of the i^{th} row of $T(q)$, and given by,

$$a^i(q) = q^{ni} + a^i_{ni} q^{ni-1} + \cdot \cdot \cdot + a^i_2 q + a^i_1 \quad (3.4.6)$$

$$b^{ij}(q) = b^{ij}_{ni} q^{ni-1} + \cdot \cdot \cdot + b^{ij}_2 q + b^{ij}_1 \quad (3.4.7)$$

and d_{ij} is the time delay associated with each transfer function element, excluding the ZOH delay.

Now considering that the system given by the equation (3.4.3), consists of p Multi Input Single Output (MISO) systems given by equation (3.4.5), the observable canonical representation for the system given by equation (3.4.5) is as follows:

$$\mathbf{x}_i(t+1) = \Phi_i \mathbf{x}_i(t) + \Lambda_i \ u(t-d_i) \quad (3.4.8)$$

$$y_i(t) = \Theta_i \ \mathbf{x}_i(t) \quad (3.4.9)$$

where

$u(t-d_i) = [\ u_1(t-d_{i1}),\ u_2(t-d_{i2}) \cdot \cdot u_m(t-d_{im})\ ]^T$

$\mathbf{x}_i(t)$ - n_ix1 state vector.

$$\Phi_i = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 & -a^i_1 \\ 1 & 0 & \cdot & \cdot & \cdot & 0 & -a^i_2 \\ 0 & 1 & \cdot & \cdot & \cdot & 0 & -a^i_3 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & 0 & 1 & -a^i_{ni} \end{bmatrix}_{ni \times ni} \quad (3.4.10)$$

or

$$\Phi_i = [\ e_2\ e_3 \cdot \cdot\ e_{ni},\ -\mu_i\ ] \qquad (3.4.11)$$

the unit vector e_i of dimension n_i given by,

$$e_i^T = [0 \cdot \cdot \cdot 0\ 1\ 0 \cdot \cdot \cdot 0\ ] \qquad (3.4.12)$$

has the i^{th} element equal to unity, and,

$$\mu^i = [\ a_1^i \cdot \cdot \cdot a_{ni}^i\ ] \qquad (3.4.13)$$

$$\Lambda_i = [\ \beta^{i1}\ \beta^{i2} \cdot \cdot \cdot \beta^{im}\ ]_{nixm} \qquad (3.4.14)$$

with

$$\beta^{ij} = [\ b_1^{ij} \cdot \cdot \cdot b_{ni}^{ij}\ ]^T \qquad (3.4.15)$$

and,

$$\Theta_i = e_{ni}^T = [0 \cdot \cdot \cdot 0\ 1]_{1xni}^T \qquad (3.4.16)$$

Since the systems given by equations (3.4.8 - 16) are observable, the observability matrix W_i is given by,

$$W_i = \begin{bmatrix} \Theta_i \\ \Theta_i\Phi_i \\ \Theta_i\Phi_i^2 \\ \cdot \\ \cdot \\ \cdot \\ \Theta_i\Phi_i^{ni-1} \end{bmatrix} \qquad (3.4.17)$$

and has a rank of n_i.

A similar set of state space representations which are in observable form can be written for each of the p outputs. By augmenting the states of all these p realizations into one state space realization, the following equations are obtained:

$$x(t+1) = \Phi x(t) + \Lambda u(t-d) \tag{3.4.18}$$

$$y(t) = \Theta x(t) \tag{3.4.19}$$

where

$$\underline{\Phi} = \text{diag}\,[\Phi_1\ \Phi_2\ \cdot\ \cdot\ \Phi_p]_{N \times N} \tag{3.4.20}$$

$$N = \sum_{i=1}^{p} n_i \tag{3.4.21}$$

$$\Theta = \text{diag}\,[\Theta_1\ \Theta_2\ \cdot\ \cdot\ \Theta_p]_{(P \times N)} \tag{3.4.22}$$

or

$$\Theta = \begin{bmatrix} e_{n1}^T \\ e_{(n1+n2)}^T \\ \cdot \\ \cdot \\ e_{(n1+n2\quad + np)}^T \end{bmatrix} \tag{3.4.23}$$

$$\Lambda = \text{diag}\,[\Lambda_1\ \Lambda_2\ \cdot\ \cdot\ \Lambda_p]_{(N \times (m \times p))} \tag{3.4.24}$$

and

$$u(t) = [\ u(t-d_1)^T,\ u(t-d_2)^T,\ \cdot\ \cdot,\ u(t-d_p)^t]^T$$

For the system given by equations (3.4.18 and 19), the observability matrix is given by,

$$\bar{W} = \begin{bmatrix} \Theta \\ \Theta\Phi \\ \Theta\Phi^2 \\ \cdot \\ \cdot \\ \cdot \\ \Theta\Phi^{N-1} \end{bmatrix} \tag{3.4.25}$$

$$W = \begin{bmatrix} \text{diag}\,[\Theta_1\ \Theta_2\ \cdot\ \cdot\ \Theta_p] \\ \text{diag}\,[\Theta_1\Phi_1\ \Theta_2\Phi_2\ \cdot\ \cdot\ \Theta_p\Phi_p] \\ \cdot \\ \cdot \\ \cdot \\ \text{diag}\,[\Theta_1\Phi_1^{N-1}\quad \Theta_2\ \Phi_2^{N-1}\quad \cdot\ \cdot\ \Theta_p\Phi_p^{N-1}\ ] \end{bmatrix} \tag{3.4.26}$$

rearranging the rows of (3.4.26),

$$W = \begin{bmatrix} W_1 & & 0 \\ & W_2 & \\ 0 & & \\ & & W_p \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot & \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot & \end{bmatrix} \tag{3.4.27}$$

From equation (3.4.27) it is obvious that W has a rank of $N=\sum_{i=1}^{p} n_i$. Hence, the state space representation of the MIMO system given by equation (3.4.18 and 19) is observable.

3.4.2 State Space formulation for the Kalman Filter Predictor

The observable state space realization given by the equations (3.4.18 and 19) can be used to obtain the state space formulations for the systems given by equations (3.3.4 and 5).

For equations (3.3.4) the state space equation is given by,

$$x^1(t+1) = \Phi_1 x^1(t) + \Lambda_1\, u(t-d) \tag{3.4.28}$$

$$y^0(t) = \theta_1 x^1(t) \tag{3.4.29}$$

where

$$u(t-d) = [u(t-d_1)^T,\ u(t-d_2)^T, \cdot\ \cdot\ \cdot, u(t-d_p)^T]^T$$

with

$$u(t-d_i)^T = [u_1(t-d_{i1}),\ u_2(t-d_{i2}) \cdot\ \cdot\ \cdot,\ u_m(t-d_{im})]^T$$

$$x^1(t) = [x_1^1\ ,\ x_2^1\ \cdot\ \cdot\ \cdot\ x_{N1}^1\ ]$$

$$\Phi_1 = \mathrm{diag}\ [\ \Phi_{11}\ \Phi_{12}\ \cdot\ \cdot\ \cdot\ \Phi_{1p}]_{N1xN1} \tag{3.4.30}$$

with,

$$N_1 = \sum_{i=1}^{p} n_i \tag{3.4.31}$$

and

$$\Phi_{1j} = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & -a^{j1} \\ 1 & 0 & \cdot & \cdot & \cdot & -a^{j2} \\ 0 & 1 & \cdot & \cdot & \cdot & -a^{j3} \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ 0 & 0 & \cdot & \cdot & 1 & -a^{j,nj} \end{bmatrix}_{njxnj} \tag{3.4.32}$$

$$\Lambda_1 = \mathrm{diag}\ [\Lambda_{11}\ \Lambda_{12}\ \cdot\ \cdot\ \cdot\ \Lambda_{1p}]_{N1x(mxp)} \tag{3.4.33}$$

with

$$\Lambda_{ij} = [\Lambda_1^{j1}\quad \Lambda_1^{j2}\quad \cdot\ \cdot\ \cdot\ \Lambda_1^{jm}\ ]_{njxm} \tag{3.4.34}$$

and

$$\Lambda^{ij} = [b_1^{ij}\quad \cdot\ \cdot\ \cdot\ b_{ni}^{ij}\ ]^T \tag{3.4.35}$$

$$\Theta_1 = \begin{bmatrix} e_{n1}^T \\ e_{(n1+n2)}^T \\ \cdot \\ \cdot \\ \cdot \\ e_{(n1+n2\ \ +np)}^T \end{bmatrix}_{(pxN1)} \qquad (3.4.36)$$

Consider equation (3.3.5),

$$y(t) = \xi^{-1}(q)\ y^0(t) \qquad (3.3.5)$$

Using the TFM $\xi^{-1}(q)$ as described by equations (3.3.9, 10 and 11), the observable state space formulation for the system given by equation (3.3.5) is,

$$z(t+1) = \Phi_2 z(t) + \Lambda_2 y_0(t) \qquad (3.4.37)$$

$$y(t) = \Theta_2 z(t) \qquad (3.4.38)$$

where

$$\Phi_2 = \text{diag}\ [\Phi_{21}\ \Phi_{22}\ \cdot\ \cdot\ \cdot\ \Phi_{2p}]_{N2xN2}$$

with

$$N_2 = \sum_{i=1}^{p} \lambda_i$$

and

$$\Phi_{2j} = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \vdots & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot \\ \cdot \\ 0 & 0 & \cdot & \cdot & 1 & 0 \end{bmatrix}_{\lambda_j x \lambda_j} = \begin{bmatrix} 0 \\ e_1^T \\ e_2^T \\ \cdot \\ \cdot \\ e_{\lambda_j-1}^T \end{bmatrix} \qquad (3.4.39)$$

$$\Lambda_2 = [\Lambda_{21}\ \Lambda_{22}\ \cdot\ \cdot\ \cdot\ \Lambda_2]_{pxN2}^T \qquad (3.4.40)$$

with

$$\Lambda_{2j} = [\Lambda_2^{j1}\quad \Lambda_2^{j2}\quad \cdot\ \cdot\ \cdot\ \Lambda_2^{j,j-1}\qquad e_1\ 0\ \cdot\ \cdot\ 0]_{\lambda_j xp} \qquad (3.4.41)$$

and

$$\Lambda_2^{jk} = [g_{\lambda_j}^{jk}\quad g_{\lambda_j-1}^{jk}\ \cdot\ \cdot\ \cdot\ g_1^{jk}\ ]^T \qquad (3.4.42)$$

$$z(t) = [z_1,\ z_2,\ \cdot\ \cdot\ \cdot\ z_{N2}] \qquad (3.4.43)$$

From equations (3.4.28 and 37),

$$z(t+1) = \Phi_2 Z(t) + \Lambda_2 \theta_1 x'(t) \qquad (3.4.44)$$

Now formulate an augmented state vector,

$$x(t) = [x'(t):z(t)]^T \qquad (3.4.45)$$

and rename the states of vector x(t) as follows for convenience.

$$x(t) = [x_1,\ x_2,\ \cdot\ \cdot,\ x_{N1},\ x_{N1+1},\ \cdot\ \cdot\ x_{N1+N2}] \qquad (3.4.46)$$

The corresponding state space equation for the augmented system is given by,

$$x(t+1) = \Phi x(t) + \Lambda u(t-d_1) + \Gamma w(t) \qquad (3.4.47)$$

$$y(t) = \theta x(t) + v(t) \qquad (3.4.48)$$

where

$$\Phi = \begin{bmatrix} \Phi_1 & 0 \\ \Lambda_2\theta_1 & \Phi_2 \end{bmatrix}_{(N1+N2)x(N1+N2)} \qquad (3.4.49)$$

$$\Lambda = \begin{bmatrix} \Lambda_1 \\ 0 \end{bmatrix}_{(N1+N2)x(mxp)} \qquad (3.4.50)$$

$$\Gamma = \begin{bmatrix} \Gamma_1 \\ 0 \end{bmatrix}_{(N1+N2)x1v} \qquad \Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{1,1v} \\ \gamma_{n1,1} & \gamma_{n1,1v} \end{bmatrix}$$

$$\theta = [0\ :\ \theta_2]_{px(N1+N2)} \qquad (3.4.51)$$

Also note that the process noise w(t) and measurement noise v(t) are included in the state space model.

w(t) - process disturbance vector of dimension l_v, and it is assumed to be zero mean uncorrelated noise with a covariance matrix $R_w = [w(t)\ w(t)^T]$ having a dimension of $l_w x l_w$. It is also important to note that the process disturbances are considered to affect only the states x_1 to x_{N1}, or the states corresponding to the residual R(q), i.e. no noise is added to the delayed states since they are interpreted as 'pure' delays.

v(t) - measurement noise vector of dimension p, and is assumed to be zero mean uncorrelated noise with a covariance matrix $R_v = [v(t)v(t)^T]$ with a dimension of pxp.

The predictor equation to estimate $y^0(t)$ is given by,

$$y^0(t) = [\theta_1 : 0]\ x(t) \tag{3.4.52}$$

Any state estimator can be used to estimate the states of the augmented system given by equations (3.4.47 and 48). Thus the vector $y^0(t)$ can be predicted. However before proceeding further with the estimator, it is important to consider the following factors.

1. The state transition matrix Φ of equation (3.4.47) will have a large dimension if the time delays present in the system are large. This would cause a heavy burden on the computer, because the state estimation algorithm has a large number of matrix operations. It is also clear that Φ is highly sparse. This means a large number of unnecessary multiplications by zero in the standard matrix operations.
2. The observability of the augmented state space model is important when an estimator is used for state estimation. Especially when the Kalman filter is used,

it is important for the stability and the convergence of the KF. The observability of the augmented system when the interactor is diagonal, can be easily established. But, because of the complexity involved, and because it is not required for the implementation discussed later, the observability of the augmented system for the triangular interactor is not proved. Taking these two factors into account, a different approach is used for the KFP. Instead of using the single state space formulation with a single state estimator, the system is decomposed into p subsystems.

The decomposition of the total system, in schematic block diagram form is clearly shown in figure 3.2. The transfer function matrices R(q) and $\xi^{-1}(q)$ are decomposed separately into p MISO subsystems in each. Interconnections of these MISO systems for a general triangular interactor are shown in figure 3.2. The particular nature of the inputs to the subsystems in $\xi^{-1}(q)$ is due to the triangular nature of the interactor. An observable state space formulation given by (3.4.18 and 19) can be written for each subsystem in R(q) and $\xi^{-1}(q)$ by treating each of them as a MISO system. Finally, subsystem j of the total system is obtained by augmenting the j^{th} subsystem in R(q) with the j^{th} subsystem in $\xi^{-1}(q)$. In this augmentation the j^{th} element of $y^0(t)$ is considered as a state, while all the other j-1 elements are treated as inputs to the j^{th} subsystem of $\xi^{-1}(q)$. This is the key feature of this formulation, and it was possible only because of the triangular nature of the interactor. The important point is that the p subsystems of the total system are not in general independent, but are serially connected as shown in figure 3.2.

Consider the j^{th} subsystem of the total system. The input to the j^{th} subsystem in R is vector u. Input to the j^{th} subsystem in ξ^{-1} are y_k^0 $(k \le j-1)$ of vector $y_0(t)$.

Treating the j^{th} subsystem as an augmentation of two MISO systems, the state space formulation of the j^{th} subsystem can be written using equations (3.4.47 and 48) as follows:

$$x^j(t+1) = \Phi_j x^j(t) + \Lambda_j u^j(t) + \Gamma^j w_j(t) \tag{3.4.53}$$

$$y_j(t) = \Theta_j x_j(t) + v_j(t) \tag{3.4.54}$$

and the predictor equation is given by,

$$\begin{aligned} y_j^0(t) &= [e_{nj}^T]_{nj+\lambda_j} x^j(t) \\ &= x_{nj}^j(t) \end{aligned} \tag{3.4.55}$$

where

$$\Phi_j = \left[\begin{array}{c|c} \Phi_{1j} & 0 \\ \hline e_{nj}^T & \\ 0 & \Phi_{2j} \\ . & \\ . & \\ 0 & \end{array}\right]_{(nj+\lambda j)x(nj+\lambda j)} \tag{3.4.56}$$

$$\Lambda_j = \begin{bmatrix} \Lambda_{1j} & 0 \\ 0 & \Lambda_{2j} \end{bmatrix}_{(nj+\lambda j)(m+j-1)} \tag{3.4.57}$$

with

$$\Lambda_{2j} = [\Lambda_2^{j1} \quad \Lambda_2^{j2} \quad \cdot \cdot \cdot \quad \Lambda_2^{j,j-1} \quad] \qquad (3.4.58)$$

and

$$\Lambda_2^{jk} = [g^{jk}_{\lambda_j} \quad g^{jk}_{\lambda_j-1} \quad \cdot \cdot \cdot \quad g_1^{jk} \quad]^T \qquad (3.4.59)$$

the state vector

$$x^j = [x_1^j \quad x_2^j \quad \cdot \cdot \quad x_{nj}^j \; x_{nj+1}^j \quad \cdot \quad x_{nj+\lambda j}^j \quad]^T$$

(3.4.60) and the input vector

$$u^j(t) = [u(t-d_1^j)^T : [y_1^0 , \cdot \cdot , y_{j-1}^0]^T]^T \qquad (3.4.61)$$

$$\Theta_j = [e^T_{nj+\lambda_j}] \qquad (3.4.62)$$

Φ_{1j}, Φ_{2j}, and Λ_j are as defined by equations (3.4.32, 33 and 39).

$$\Gamma_j = [\gamma_1^j \quad \gamma_2^j \quad \cdot \cdot \quad \gamma_{nj}^j \; 0 \; \cdot \; \cdot \; 0]^T \; (n_j+\lambda_j) \qquad (3.4.63)$$

and w^j and v^j are the process noise and measurement noise of the j^{th} subsystem, and are zero mean uncorrelated random noise with covariances given by,

$$R_{wj} = [w_j w_j^T] \text{ and } R_{vj} = [v_j v_j^T]$$

respectively.

As far as the state transition matrix Φ_j and the output equation (3.4.54) are concerned, the state space formulation of the subsystem is the same as the SISO case given in chapter 2, the observability of which has already been proven. Thus the system given by the TFM T(q) has been decomposed into p observable MISO subsystems.

3.5 Multivariable Kalman Filter Predictor

The single state space formulation given by equation (3.4.47 and 48), to represent a MIMO system with multiple time delays can be used to obtain an estimate of $y^0(t)$. Since we are concerned here with a stochastic system, a suitable state estimator to estimate the states of the above formulation, thus to predict $y^0(t)$, is the Kalman filter. A single multivariable Kalman filter could be employed to obtain the minimum variance estimates of $y^0(t)$. The schematic diagram of the multivariable KFP is given in figure 3.4.

However, implementation of this multivariable KFP might cause problems with respect to the computational effort, because of the large dimension (NxN of equation 3.4.47), of the state space formulation. In addition to the number of matrix multiplications present in the Kalman filter algorithm, there is a matrix inversion in the MIMO Kalman filter, which would not only increase the computational load, but also might cause numerical problems.

An alternative approach to the design of the multivariable KFP, which is more convenient to implement and more convenient to analyze theoretically, is to use the decomposed state space formulation as given by equations

(3.4.53 and 54). This formulation consists of p MISO subsystems as shown in figure 3.2. To estimate the vector $y^0(t)$, which is now given by the predictions $y_1^0(t)$, $y_2^0(t)$ $\cdot \cdot \cdot$ $y_p^0(t)$, of each subsystem, p MISO Kalman filters can be used as shown in figure 3.4. Although the number of states estimated is the same as before, the computational effort is drastically reduced. In both figures 3.2 and 3.4 the subsystems are structurally parallel but must be executed sequentially. Another important feature in implementing a MISO KF instead of a MIMO KF is that the former does not require a matrix inversion.

A further reduction in computational time can be achieved by taking advantage of the sparse nature of Φ_j, Λ_j and Θ_j. More specifically, the KF algorithm given in Appendix A can be significantly reduced by using the algorithm given in Appendix E, which is derived for the KFP in particular. This also justifies the use of the observable canonical form given in section (3.4.1).

3.5.1 Stability and convergence of the Multivariable Kalman Filter Predictor

The stability of the multivariable KFP which consists of p MISO KFP, depends on the stability of the individual KFP. Because of the serial nature of these KFPs, instability in any one of the KFP would introduce an unbonded input signal to the succeeding KFPs, thus causing instability in the succeeding predictions.

The stability of the each KFP depends on the stability of the Kalman filter, with respect to the solution of the ARE given in Appendix B.

Since each of the KFP is a MISO system the stability and convergence analysis presented in Chapter 2 can still be used because they are concerned only with the pairs (Φ,E) and (Θ,Φ). The only difference between the SISO and MISO system is the matrix Λ_j. Thus the results obtained in Chapter 2 for the SISO case is directly applicable to each MISO Kalman filter predictor.

The final results are presented here for convenience.

Result 1: The state space representation given by equations (3.4.53 and 54) is observable, i.e. the pair (Θ,Φ) is observable (see Lemma 2.3.1 in Chapter 2).

Result 2: The eigenvalues of the system given by (3.4.53 and 54) lie within the unit circle, if the original system is stable, i.e. the pair (Φ,E) is stabilizable (see Lemma 2.3.2 in Chapter 2). Results 1 and 2 satisfy the conditions for the steady state transition matrix $\bar{\Phi}$ of the KF (see Appendix B) to be stable.

Thus, each KFP in the multivariable KFP gives a stable steady state KFP, i.e. $\bar{\Phi}_j$ - the steady state transition matrix of the KFP is stable.

Results 1 and 2 also satisfy the conditions for theorem B1, which guarantees the time varying KF will converge to the steady state Kalman filter. According to theorem B1 the error covariance $M(t)$, Kalman gains $L(t)$ and $\bar{\Phi}(t)$ in the

Kalman filter predictor converge exponentially fast to their corresponding steady state values **M**, **L** and Φ, if M_0 (initial covariance) ≥ 0.

3.6 Innovation Model approach for the Multivariable Kalman Filter Predictor

By using the innovation model approach for a SISO KFP, its configuration and certain properties were more easily investigated, (see chapter 2 section 2.4). The same approach can be used for the multivariable KFP, to get a better insight into its configuration and behaviour.

The innovation model concept given in Appendix C is applied to each Kalman filter predictor of the Multivariable Kalman filter predictor.

The innovation model for the jth KFP is given by,

$$\hat{x}^j(t+1) = \Phi_j\hat{x}^j(t+1) + \Lambda_j u^j(t) + L^j(t+1)\omega_j(t+1) \qquad (3.6.1)$$

where

$$\Theta_j\Phi_j = [0\ 0\ \cdot\ \cdot\ 1\ 0] \qquad (3.6.2)$$

$$\Theta_j\Lambda_j = [0\ \cdot\ \cdot\ 0,\ g_1^{j1}\quad g_1^{j2}\quad \cdot\ \cdot\ g_1^{j,j-1}\quad]_{(m+j-1)} = N^j \qquad (3.6.3)$$

Equations (3.6.2 and 3) are true for $\lambda_j \geq 1$.

From (C.2) the estimated current output $\hat{y}_j(t)$ is given by,

$$\hat{y}_j(t+1) = \hat{y}_j(t/t-1) = \Theta_j\Phi_j\hat{x}^j(t-1) + \Theta_j\Lambda_j u^j(t-1) \qquad (3.6.4)$$

Hence,

$$\hat{y}_j(t) = \hat{x}^j_{nj+\lambda_j-1} + N^j u^j(t-1) \qquad (3.6.5)$$

The innovation sequence $\omega_j(t)$ is given by,

$$\omega_j(t) = y_j(t) - \hat{y}_j(t) \tag{3.6.6}$$

Rewrite the equation (3.6.1) as,

$$\begin{aligned}\hat{x}^j(t+1) = {} & I'\hat{x}^j(t) + \Phi'_j\hat{x}_{jn1}(t) + \Lambda_j u^j(t) + \\ & L_j(t+1)\omega_j(t+1)\end{aligned} \tag{3.6.7}$$

where

$$I' = \underset{(nj+\lambda j)x(nj+\lambda j)}{\begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ 0 & 0 & \cdot & \cdot & 1 & 0 \end{bmatrix}} \qquad \Phi'_j = \underset{(nj+\lambda j)x1}{\begin{bmatrix} -a_{nj} \\ -a_{nj-1} \\ \cdot \\ -a_1 \\ 0 \\ \cdot \\ 0 \end{bmatrix}}$$

$$\Lambda_j = \left[\begin{array}{c|c} \text{diag}[\Lambda_1^{j1} \quad \Lambda_1^{j2} \quad \cdot \quad \cdot \quad \Lambda_1^{jm} \quad] & 0 \\ \hdashline 0 & \Lambda_2^{j1} \quad \Lambda_2^{j2} \quad \cdot \quad \cdot \quad \Lambda_2^{j,j-1} \end{array}\right]$$

$$\begin{aligned}u^j(t) = [& u_1(t-d_{j1}),\ u_2(t-d_{j2}),\ \cdot\ \cdot\ u_m(t-d_{jm}), \\ & \hat{y}_1^0\ ,\ \hat{y}_2^0\ ,\ \cdot\ \cdot\ \hat{y}_{j-1}^0]\end{aligned}$$

Using Lemma D.1 to equation (3.6.7) to solve for $\hat{x}^j_{nj}(t)$ we get,

$$\hat{x}^j_{nj}(t) = [I - \alpha_j]\hat{x}_{nj}(t) + q^{-1}\beta^j u(t) + K^j_1(t)\omega_j(t) \tag{3.6.8}$$

where α_j and β^j are given by equations (3.3.13 and 14) respectively, and

$$K^j_1(t)(q^{-1}) = L^j_{nj}(t) + L^j_{nj-1}(t-1)q^{-1} + \cdot \cdot \cdot L^j_1(t-n_j)q^{-nj+1} \tag{3.6.9}$$

Since the predicted value of $y^0_j(t)$ is given by,

$$\hat{y}^0_j = \hat{x}_{nj}(t) \tag{3.6.10}$$

the predictor equation (3.6.8) for the j^th^ KFP can now be written as,

$$\hat{y}^0_j(t) = q^{-1}\alpha^{-1}_j \beta_j u(t) + \alpha^{-1}_j K^j_1(t)\omega_j(t) \tag{3.6.10}$$

Using Lemma D.1 in equation (3.6.7), to solve for $\hat{x}^j_{nj+\lambda j-1}$,

$$\hat{x}^j_{nj+\lambda j-1}(t) = [1-\alpha_j]\,\hat{x}^j_{nj}(t) + q^{-\lambda_j}\beta_j u(t) + g^1_j\,\hat{y}^0(t) + K^j(t)\omega_j(t) \tag{3.6.11}$$

where

$$g^1_j(q^{-1}) = [g^1_{j1}(q^{-1})\ g^1_{j2}(q^{-2}) \cdot \cdot g^1_{j,j-1}(q^{-1}),\ 0 \cdot \cdot 0]_{pxp} \tag{3.6.12}$$

with

$$g^1_{jk}(q^{-1}) = g^{jk}_2\ q^{-1} + g^{jk}_3\ q^{-2} + \cdot \cdot + g^{ij}_{\lambda_j}\ q^{-\lambda_j+1} \tag{3.6.13}$$

$$K^j(t)(q^{-1}) = K_2^j(t)(q^{-1}) + q^{-\lambda_j+1}K_1^j(t)(q^{-1}) \qquad (3.6.14)$$

with

$$K_2^j(t)(q^{-1}) = L_{nj+\lambda j-1}(t) + L_{nj+\lambda j-2}(t)q^{-1} + \cdots$$

$$+ L_{nj+1}(t-\lambda j+2)q^{-\lambda j+2} \qquad (3.6.15)$$

Since,

$$\begin{aligned} y_j(t) &= \hat{y}_j(t) + \omega_j(t) \\ &= \hat{x}^j_{nj+\lambda j-1}(t-1) + N^j u^j(t-1) + \omega_j(t) \end{aligned} \qquad (3.6.16)$$

Substituting (3.6.16) in (3.6.11), we get

$$\alpha_j(q^{-1})y_j(t) = q^{-(\lambda j+1)}\beta_j(q^{-1})u(t)$$

$$+ \alpha_j[1+q^{-1}K_2^j(t-1) + q^{-\lambda j}A_j^{-1}\ K_1^j(t-\lambda j)]\omega_j(t)$$

$$+ \alpha_j g_j(q^{-1})\hat{y}^0(t-1) \qquad (3.6.17)$$

where

$$g_j(q^{-1}) = [g^{j1}(q^{-1})\ g^{j2}(q^{-1}) \cdot \cdot g^{j,j-1}(q^{-1}),\ 0 \cdot \cdot 0] \qquad (3.6.18)$$

with $g^{jk}(q^{-1})$ as defined by equation (3.3.11).

Define,

$$F_j(q^{-1})(t) = \alpha_j[1+q^{-1}K_2^j(t-1) + q^{-\lambda j}\alpha_j^{-1} K_1^j(t-\lambda j)]$$

$$= 1+f_1^j(t)q^{-1} + \cdot \cdot \cdot + f^j_{nj+\lambda j-1}(t)q^{-\lambda j-nj+1} \tag{3.6.19}$$

The equation (3.6.17) can now be written as,

$$\alpha_j y_j(t) = q^{-(\lambda j+1)}\beta_j u(t) + F_j(t)w_j(t) + \alpha_j g_j \hat{y}^0(t) \tag{3.6.20}$$

The equation (3.6.11) can be written by substituting for $\hat{x}^j_{nj+\lambda j-1}$ from (3.6.16) as,

$$\hat{y}_j(t) = q^{-(\lambda j+1)}\alpha_j^{-1} \beta_j u(t) + q^{-1}g_j\hat{y}^0(t) + K^j(t)\omega_j(t) \tag{3.6.21}$$

From equations (3.6.10) and (3.6.20),

$$\hat{y}_j^0(t) = q^{-1}\alpha_j^{-1}(q)\beta_j(q^{-1})$$

$$+ \frac{K_1^j(t)}{F_j(t)}[y_j(t) - q^{-(\lambda j+1)}\alpha_j^{-1}\beta_j u(t) + g_j\hat{y}^0(t)] \tag{3.6.22}$$

Writing corresponding equations (3.6.22), for all the KFPs, and augmenting them, the following equation is obtained:

$$\hat{y}^0(t) = q^{-1}A^{-1}Bu(t) + G_F[y(t) - Dq^{-1}A^{-1}Bu(t) + G_H\hat{y}^0(t)] \tag{3.6.23}$$

where A, B, D and G_H are as defined by equations (3.3.17, 19, 21 and 22), and,

$$G_F(q^{-1}) = \mathrm{diag}[g_{F1}(q^{-1}),\ g_{F2}(q^{-1})\ \cdot\ \cdot\ g_{Fp}(q^{-1})] \tag{3.6.24}$$

with

$$g_{Fj}(q^{-1}) = \frac{K_1^j\,(t)(q^{-1})}{F_j(t)(q^{-1})} \tag{3.6.25}$$

$$\omega(t) = [\omega_1(t),\ \omega_2(t),\ \cdot\ \cdot,\ \omega_p(t)]^T$$

The configuration of the multivariable KFP based on equation (3.6.23) is shown in figure 3.5.

The block ξ_s^{-1} defined by,

$$\xi_s^{-1} = \lim_{q \longrightarrow 1} \xi^{-1}(q)$$

is the steady state compensater as explained in section (3.8.1).

It is clear from figure 3.5 and figure H.2, that the multivariable KFP and the multivariable interactor predictor have a similar configuration. These two schemes are discussed in the next section.

From equation (3.6.10), the composite equation for the total system can be written as,

$$\hat{y}^0(t) = q^{-1}A^{-1}Bu(t) + A^{-1}K_1(t)\omega(t) \tag{3.6.26}$$

where

$$K_1(t) = \mathrm{diag}[K_1^1(t), K_1^2(t), \cdot\ \cdot, K_1^p(t)] \qquad (3.6.27)$$

Similarly for equation (3.6.20) we can write,

$$y(t) = Dq^{-1}A^{-1}Bu(t) + A^{-1}F(t)\omega(t) + G_H\hat{y}^0(t) \qquad (3.6.28)$$

where

$$F(t) = \mathrm{diag}[F_1(t), F_2(t), \cdot\ \cdot, F_p(t)]$$

From equations (3.6.26 and 28)

$$y(t) = Dq^{-1}A^{-1}Bu(t) + [A^{-1}F(t) + G_HA^{-1}K_1(t)]\ \omega(t)$$
$$+ G_Hq^{-1}A^{-1}Bu(t) \qquad (3.6.29)$$

From equations (3.3.20 and 5),

$$\xi_T^{-1}\ (q) = q^{-1}(D+G_H) \qquad (3.6.30)$$

From equations (3.6.29 and 30), we get,

$$y(t) = \xi_T^{-1}\ A^{-1}Bu(t) + [A^{-1}F(t) + G_HA^{-1}K_1(t)]\omega(t) \qquad (3.6.31)$$

Equation (3.6.31) gives an input output model for the MIMO process obtained via the innovation model.

Consider the time invariant Kalman filter where $F(t)(q^{-1}) \text{--->} F(q^{-1})$, $K_1(t)(q^{-1}) \text{--->} K_1(q^{-1})$ and $K_2(t)(q^{-1}) \text{--->} K_2(q^{-1})$ as $t \text{--->} \infty$.

From equation (3.6.31), we get,

$$A\ \xi_T\ y(t) = B\ u(t) + C\ \omega(t) \qquad (3.6.32)$$

where

$$C(q^{-1}) = A\ \xi^T\ [A^{-1}F + G_HA^{-1}K_1\ ]$$
$$= A\ \xi_T\ [I+q^{-1}K_2 + \xi^{-1}A^{-1}K_1\ ] \qquad (3.6.33)$$

The transfer function matrix $\xi_T(q)$ can be expressed in the polynomial matrix form as,

$$\xi'_T(q) = \xi_1q + \xi_2q^2 + \cdot\ \cdot + \xi_{n\lambda}q^{n\lambda} \qquad (3.6.34)$$

where $n\lambda$ is the maximum degree of the elements of the TFM $\xi_T(q)$.

Then,

$$C(q^{-1}) = c_1 q^{n\lambda} + c_2 q^{n\lambda - 1} + \cdot \cdot + c_N q^{-n' - \lambda' + 1} \qquad (3.6.35)$$

where

$$N = n\lambda + n' + \lambda'$$

and

$$n' = \max^m[nj,\ j=1,p]$$

and

$$\lambda' = \max^m[\lambda_j,\ j=1,p]$$

3.7 Comparison of MIMO KFP with MIMO Interactor Predictor

The results obtained in the previous section suggest a similarity between the interactor predictor due to Sripada et al (1985) and the KFP. (The key steps in the formulation of the interactor predictor are summerized in Appendix H) A comparative study of the behaviour of these two schemes is presented below:

a) The difference between the KFP and the IP as seen from their configurations in figure 3.5 and figure H.2, are,

1. The Kalman filter predictor has a filter G_F, which is based on the Kalman gains of each KF in the system.
2. The steady state compensation that is needed in PID control as explained in section (3.7.1) is performed at different locations for these two schemes.
3. In the KFP the signal $G_H\hat{y}^0(t)$ is dependent on the process measurement, because $\hat{y}^0(t)$ for the j^{th} KF comes

from the previous j-1 predictions, whereas in the IP $y_1(t)$ depends only on the process input, u(t).

b) In the IP $y_e(t)$, the difference between the model output y_m and the measured output y(t), which gives a signal corresponding to any noise, disturbance or model mismatches is directly added to the predicted output, whereas in the KFP it is filtered by the filter G_F.

This also suggests a possible improvement to the IP for noisy systems by introducing an exponential filter as shown by the dotted lines in figure H.2. This filter G_A is given by the transfer function matrix as,

$$G_A(q^{-1}) = \mathrm{diag}[G_A^1(q^{-1}) \cdot \cdot \cdot G_A^p(q^{-1})] \qquad (3.7.1)$$

where G_A^i can be any filter, e.g. an exponential filter.

However the interesting point to note is that the filter G_F based on the Kalman gains, i.e. based on the knowledge of the process and the noise statistics, gives the minimum variance filter that can be used to filter the error vector $y_e(t)$.

c) When the interactor is diagonal $G_H(q^{-1})=0$ and $\xi_s^{-1} = 1$. This makes the IP and KFP exactly same, in configuration, and functional blocks except for the filter G_F in the KFP. In this case KFP, has p independent MISO KFPs.

d) When there is no noise present in the system, and under perfect modelling, the prediction for both IP and KFP are given by,

$$\hat{y}^0(t) = \xi_s^{-1} q^{-1}A^{-1}Bu(t) \qquad (3.7.2)$$

Equation (3.7.2) is the open loop predictor equation. Thus under ideal conditions both the KFP and the IP behave like an open loop predictor.

e) The stability of the KFP depends on the stability of the filter G_F. The diagonal filter transfer function matrix consists of individual filters for each MISO Kalman filter.

The stability of each filter $g_{Fi}(q^{-1})$ depends on the roots of the polynomial $F_j(q^{-1})$. It is shown on chapter 2 that the roots of $F_j(q^{-1})$ are the same as that of the steady state filter transition matrix $\bar{\Phi}_j$ of the KF, which is stable if the conditions given in section (3.5.1) are satisfied.

3.8 Feedback Control System using the Kalman Filter Predictor

The predicted vector $\hat{y}^0(t)$ can be used in any control systems design in place of the actual measured output y(t). In the Interactor predictor due to Sripada et al (1985), the predicted output was used for dead time compensation. In this section, a multivariable dead time compensation scheme based on the multivariable KFP is presented. A predictive control strategy based on the predictions from the Interactor predictor and KFP are also presented.

3.8.1 PID dead time compensator

It was shown by Sripada et al (1985) that in some cases the PID dead time compensator based on the predictor of $\hat{y}^0(t)$ performed better than the predictor due to Ogunnaike and Ray (1979), in which all the time delays are removed in the predictor.

The PID dead time compensator based on the interactor predictor for deterministic systems can be extended to stochastic systems by using the KFP.

The PID dead time compensator based on the KFP is schematically shown in figure 3.5. The reason to include the steady state compensator ξ_s^1 as shown in figure 3.5 is explained below.

In an ideal situation when there is no noise and under perfect modelling the prediction $\hat{y}^0(t)$ is given by,

$$\begin{aligned}\hat{y}^0(t) &= R(q^{-1})\ u(t) = q^{-1}A^{-1}B\ u(t)\\ &= q^{-1}\xi(q)\ y(t) \qquad (3.8.1)\end{aligned}$$

The process output y(t) is given by,

$$y(t) = \xi^{-1}(q^{-1})\ A^{-1}B\ u(t) \qquad (3.8.2)$$

Consider the steady state gains,

$$G_{s1} = \lim_{q \longrightarrow 1} A^{-1}B \qquad (3.8.3)$$

$$G_{s2} = \lim_{q \longrightarrow 1} \xi^{-1}\ A^{-1}B \qquad (3.8.4)$$

If the steady state value of y(t) is y_{ss} and the steady state value of $\hat{y}^0(t)$ is $\hat{y}^0_{ss}$ the prediction $\hat{y}^0_{ss}=y_{ss}$ only if $G_{s1}=G_{s2}$. This condition is satisfied only if $\xi(q)$ is diagonal.

In the general case,

$$G_{s2} = \xi_s^{-1}\ G_{s1} \qquad (3.8.5)$$

where $$\xi_s^{-1} = \lim_{q \longrightarrow 1} \xi^{-1}(q^{-1}) \qquad (3.8.6)$$

If the PID controller is designed for the open loop predictor G_m given in figure H.1, then the prediction $\hat{y}^0(t)$ will track the desired output at steady state, but it does not guarantee that the actual output will track the desired output or set point, because of the steady state error between the prediction $\hat{y}^0_{ss}$ and output y_{ss}. This will require retunning of the PID controller. This is overcome by introducing a steady state compensator ξ^1_s as shown. This is necessary for any error driven controller. The controller is now designed to control the steady state compensated prediction $\hat{y}_c$.

3.8.2 Predictive Control

Both the Interactor predictor and the Kalman filter predictor are based on apriori knowledge of the process model. If the process model is known, it is convenient to use a predictive controller based on the known parameters, rather than tune a PID controller. In this section a stochastic predictive controller design for the KFP is presented.

The predictive control law for the SISO KFP is obtained by minimizing the cost function given by,

$$J(t+d+1) = E\{\, [y(t+d+1) - y^*(t+d+1)\,]^2 \,\} \tag{3.8.7}$$

For the multivariable KFP the corresponding cost function is obtained by defining new variables instead of the actual outputs and the desired set point.

Define following two variables:

the filtered output $\bar{y}(t)$

$$\bar{y}(t) = \xi_T(\bar{q})\ y(t) \tag{3.8.8}$$

and the filtered desired set point $\bar{y}^*(t)$

$$\bar{y}^*(t) = \xi_T(q)\ y^*(t) \tag{3.8.9}$$

The predictive control law for the KFP is calculated such that the cost function,

$$J(t) = E\ \{\ [\bar{y}(t) - \bar{y}^*(t)]\ [\bar{y}(t) - \bar{y}^*(t)]^T\ \} \tag{3.8.10}$$

is minimized. J(t) is a pxp matrix.

Equation (3.8.10) can also be written as,

$$J(t) = E\ \{\ E\ \{\ [\bar{y}(t) - \bar{y}^*(t)]\ [\bar{y}(t) - \bar{y}^*(t)]^T/Y_t\ \} \tag{3.8.11}$$

If $J^*(t)$ is the optimal value of J(t) then,

$$J^*(t) = E\ \{\ \min E\ \{\ [\bar{y}(t) - \bar{y}^*(t)][\bar{y}(t) - y^*(t)]^T/Y_t\ \}\} \tag{3.8.12}$$

Consider the model of the process obtained via the innovation model, and which is given by equation (3.6.32),

$$A\ \xi_T\ y(t) = B\ u(t) + C\ \omega(t) \tag{3.8.13}$$

$[\omega(t)]$ is the innovation sequence vector, satisfying,

$$E\ [\ \omega(t)/Y_t\ ] = 0 \tag{3.8.14}$$

$$E\ [\ \omega(t)\ \omega(t)^T/Y_t\ ] = \sigma^2 \tag{3.8.15}$$

From equation (3.8.13),

$$\xi_T\ y(t) = [I-A]\ \xi_T\ y(t) + B\ u(t) + C(q)\ \omega(t) \tag{3.8.16}$$

Taking the expectation of equation (3.8.16),

$$E\ [\ \xi_T\ y(t)/Y_t\ ] = E\ \{\ [I-A]\ \xi_T\ y(t)/Y_t\ \} + B\ u(t) \tag{3.8.17}$$

Since,

$$y^0(t) = \xi\ y(t) = q\ \xi_T\ y(t) \qquad (3.8.18),$$

the term

$$E\ \{\ [I-A]\xi_T\ y(t)/Y_t\ \} = -A_1E\ \{y^0(t)/Y_t\} -$$

$$A_2E\{y^0(t-1)/Y_{t-1}\} \cdot \cdot \cdot A_{n'}.E\{y^0(t-n'+1)/Y_{t-n'+1}\} \qquad (3.8.19)$$

The Kalman filter predictor gives the minimum variance estimates of the vector $y^0(t)$, i.e.

$$\hat{y}^0(t/t) = E\ \{\hat{y}^0(t)/Y_t\} \qquad (3.8.20)$$

with an error covariance given by,

$$\sigma_y^2 = E\ \{\ [\bar{y}(t) - \hat{y}^0(t)]\ [\bar{y}(t) - \hat{y}^0(t)]^T/Y_t\ \} \qquad (3.8.21)$$

$$= E\ \{\ [y_0(t) - \hat{y}^0(t) + \bar{v}(t)]\ [y_0(t) - \hat{y}^0(t) + \bar{v}(t)]^T/Y_t\} \qquad (3.8.22)$$

where

$$\bar{v}(t) = \xi\ v(t)$$

Since vector $[y_0(t) - \hat{y}^0(t)]$ and $\bar{v}(t)$ are independent,

$$\sigma_y^2 = E\ \{\ [y_0(t) - \tilde{y}_0(t)]\ [y_0(t) - \tilde{y}_0(t)]^T\ \} + E\ [\ \bar{v}(t)\ \bar{v}(t)^T\ ] \qquad (3.8.23)$$

Let us define the prediction,

$$E\ \{\xi_T\ y(t)/Y_t\} = E\ \{\bar{y}(t)/Y_t = \bar{y}(t/t) \qquad (3.8.24)$$

then the predictor equation for $\bar{y}(t)$, based on the predictions $\tilde{y}_0(t)$ from the KFP is given by,

$$\bar{y}(t/t) = [I-A]\ \hat{y}^0(t/t) + B\ u(t) \qquad (3.8.25)$$

From the cost function (3.8.12),

$$J^{*}(t) = E\,\{\min\{\,[(\bar{y}(t) - \bar{y}(t/t)) + (\bar{y}(t/t) - y^{*}(t))]\; [(\bar{y}(t) - \bar{y}(t/t)) + (y^{*}(t))]^{T}\,\}\,\} \tag{3.8.26}$$

$$J^{*}(t) = E\,\{\min E\{\,[\bar{y}(t) - \bar{y}(t/t)]\,[\bar{y}(t) - \bar{y}(t/t)]^{T} + [\bar{y}(t/t) - \bar{y}^{*}(t)]\,[\bar{y}(t/t) - y^{*}(t)]^{T}\,\}\,\} \tag{3.8.27}$$

Consider the quantity,

$\bar{y}(t) - \bar{y}(t/t)$,

from (3.8.25) and (3.8.16) we get,

$$\bar{y}(t) - \bar{y}(t/t) = [I-A]\,[\bar{y}(t) - \hat{y}^{0}(t)] + C(q)w(t) \tag{3.8.28}$$

From (3.8.25), (3.8.22) and (3.8.15),

$$E\,\{\,[\bar{y}(t) - \bar{y}(t/t)]\,[\bar{y}(t) - \tilde{y}(t/t)]^{T}\,\} =$$
$$E\,\{\,[(I-A)\,(\bar{y}(t) - \tilde{y}_{0}(t))]\,[(I-A)\,(\bar{y}(t) - \bar{y}_{0}(t))]^{T}\,\} +$$
$$E\,\{\,[F\,w(t)]\,[F\,w(t)]^{T}\,\} = \sigma_{e}^{2} \tag{3.8.29}$$

σ_{e}^{2} can be evaluated using (3.8.15 and 22), and it is an independent quantity as far as the minimization of the cost function is concerned.

Hence equation (3.8.27) can be written as,

$$J^{*}(t) = E\{\min E\{\,[\bar{y}(t/t) - y^{*}(t)]\,[\bar{y}(t) - y^{*}(t)^{T}]^{T}\} + \sigma_{e}^{2}\,\} \tag{3.8.30}$$

Clearly the minimum $J^{*}(t)$ is obtained when,

$$\bar{y}(t/t) - y^{*}(t) = 0 \tag{3.8.31}$$

Thus the minimum variance control law using the KFP is obtained by substituting (3.8.3) into (3.8.25).

The predictive control law is given by,

$$u(t) = B_1^{-1} \{\bar{y}^*(t) - [I-A]\,\hat{y}^0(t) - [B-B_1]\,u(t)\} \tag{3.8.32}$$

where, B_1 is non-singular as shown in Appendix I.

From the definition of the cost function it should be noted that the one step ahead control law (3.8.32) does not now lead to perfect dynamic tracking of the designed output $y^*(t)$. Instead the filtered output $\bar{y}(t)$ tracks the filtered desired output $\bar{y}^*(t)$.

From equations (3.8.32) and (3.8.16),

$$\bar{y}(t) - \bar{y}^*(t) = [I-A]\,[y^0(t) - \hat{y}^0(t)] + C(q)\omega(t) \tag{3.8.33}$$

when there is no noise present and under perfect modelling the equation (3.8.33) becomes,

$$\bar{y}(t) - \bar{y}^*(t) = 0$$

$$\lim_{t \to \infty} \bar{y}(t) - \bar{y}^*(t) = \lim_{t \to \infty} \xi_T\,[y(t) - y^*(t)] = 0$$

If ξ_T has all its roots at the origin then,

$$\lim_{t \to \infty} [y(t) - y^*(t)] = 0$$

Thus $y(t)$ converges to $y^*(t)$ after the transition produced by the dynamics of the interactor $\xi(q)$ have decayed.

3.9 Discussion

1) In a predictor design for a multivariable process with multiple time delays it is important to define a suitable factorization method to factor out the time delays from the rest of the dynamics of the process TFM model.

In the Ogunnaike and Ray Predictor (ORP), instead of factoring out the time delay to obtain the predictor G_p, all the time delays in the process TFM model are removed. The main disadvantages of this predictor are as follows:

1. Removing all the time delays cannot be represented as a mathematical operation or a matrix factorization. This limits the theoretical investigation of the ORP.
2. Removing all the time delays from the TFM is not always the best thing to do. In certain cases this might cause an adverse effect on the dynamic control performance.
3. The prediction $\hat{y}^0(t)$ in the ORP (see figure G.1) does not have any direct relationship to the actual variables of the process.

In the Kalman filter predictor and the interactor predictor the time delay is represented by the inverse interactor matrix ξ^{-1}.

It is important to note the following properties of the Interactor.

1. Interactor factors out only the natural delay of the process.
2. In a general case when the interactor is triangular it may also remove some dynamics of the process along with

the time delays.

3. Under certain conditions, i.e. diagonal interactor, the interactor removes the minimal time delays associated with each output.
4. For a particular TFM of the process model there is a unique interactor.
5. The factorization can be mathematically represented and provides a powerful tool for theoretical investigations.

2) The interactor predictor provides good predictions of $\hat{y}^0(t)$ when the process is deterministic. When the process is disturbed by process noise and measurement noise the Kalman filter predictor provides the minimum variance estimates of $\hat{y}^0(t)$, based on the process model and the noise statistics. When the noise statistics are not known the ratio of the process noise covariance to measurement noise covariance, i.e. R_{wj}/R_{vj}, can be used as a tunning parameter of the KFP.

3) The state space formulation of a MIMO system is usually of high dimension and this demands high computational effort if a time varying KF is to be employed for state estimation.

Therefore, in the multivariable KFP, instead of a single MIMO state space representation and a single MIMO KF, p MISO state space representations are used with p MISO Kalman filters.

This reduces the computation effort drastically. A further reduction in computation effort is achieved by using the KFP algorithm given in Appendix E. Another advantage of using the MISO KF instead of the MIMO KF is, that it avoids a matrix inversion, which would not only demand heavy computational effort but also may cause numerical problems.

4) Since the time varying KF converges exponentially to the steady state KF, if the conditions given in Appendix B are satisfied, one could use a steady state KF instead of the time varying KF. The optimal steady state gains of the KF can be evaluated off-line, using the algorithm due to Laub (1979) and Pappas et al (1980), or the recursive algorithm presented in Appendix A or E. Using the steady state KF would reduce the computation effort further.

5) It is shown in section 3.7 that the KFP and the IP have similar configurations. When the interactor is diagonal, the KFP and the IP have the same functional blocks except for the filter in the KFP.

It is important to present here a result obtained by Sripada et al (1985), regarding the optimality of the IP under deadbeat control. This is shown for the case when the interactor is diagonal and the system is free of noise. The same results holds true for the KFP, under these conditions.

When there is no noise present in the system ($y_\eta=0$), and under perfect modelling ($G_m=G$),
for ORP we have,

$$y(t) = GG_p^{-1} \; y_p(t) \tag{3.9.1}$$

and for both the KFP and the IP having a diagonal interactor we have,

$$y(t) = \xi^{-1} \, y_p(t) \tag{3.9.2}$$

If the controller G_c is an output deadbeat, i.e. the set point is reached in one sample period, then,

$$y_p(t+1) = y^*(t) \tag{3.9.3}$$

and this is the best possible for discrete systems.

From equations (3.9.1, 2 and 3) it follows that for ORP

$$y(t) = GG_p^{-1} \quad y^*(t-1) \tag{3.9.4}$$

and for IP and KFP,

$$y(t) = \xi^{-1} \; y^*(t-1)$$

or

$$y_i(t+\lambda_i+1) = y_i^* \, (t) \tag{3.9.5}$$

It is clear from equation (3.9.4) that the ORP does not guarantee that the output will track the set point subject only to a delay as in the IP scheme. Thus the ORP does not guarantee the same optimal performance for deadbeat control as the interactor predictor. It is important to note that when the interactor is diagonal it represents the minimal time delay associated with an output.

However, when the interactor is triangular a general result regarding the optimality is difficult to establish.

6) The predictive control algorithm for the Interactor predictor is given in Appendix J. It is clear from equations (J.10) and (3.7.32) that the IP and the KFP use the same predictive control law. The only difference in these two schemes is that the prediction $\hat{y}^0(t)$ in the KFP is the minimum variance prediction under stochastic conditions and this guarantees a minimum variance control. Obviously, if the IP is used in a stochastic situation, the minimum

variance of the prediction $\hat{y}^0(t)$ is not guaranteed.

When there is no noise present in the system ($y_n=0$) and under perfect modelling, the closed loop equation for the IP and the KFP is given by,

$$\bar{y}(t) = \bar{y}^*(t) \tag{3.9.6}$$

When the interactor is diagonal,

$$y_j(t+\lambda_j+1) = y_j^*(t+\lambda_j+1) \tag{3.9.10}$$

Thus the IP and the KF give perfect set point tracking when the interactor is diagonal.

When the interactor is triangular, we have the following set of equations:

$$y_1(t+\lambda_1+1) = y_1^*(t+\lambda_1+1)$$

$$h_{21}(q^{-1})\, y_1(t) + y_2(t+\lambda_2+1) = h_{21}(q^{-1})\, y_1^*(t) + y_2^*(t+\lambda_2+1)$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$h_{p1}(q^{-1})\, y_1(t) + h_{p2}(q^{-1})\, y_2(t) \cdot \cdot + y_p(t+\lambda_p+1)$$

$$= h_{p1}(q^{-1})\, y_1^*(t) + h_{p2}(q^{-1}\, y_2^*(t) \cdot \cdot + y_p^*(t+\lambda_p+1)$$

From the set of above equations we get,

$$y_1(t) = y_1^*(t)$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$y_p(t) = y_p^*(t)$$

Thus the IP and the KFP give perfect set point tracking for the triangular interactor.

It is also important to note that in the formulation of the KFP and the IP for the predictive control scheme the steady state compensator ξ_s^{-1} not present.

7) The steady state gain of the filter G_F in the KFP is given by,

$$G_{Fss} = \text{diag} [\; g_{F1ss} \; g_{F2ss} \; \cdot \; \cdot \; \cdot \; g_{Fpss} \;]$$

where

$$g_{Fjss} = \frac{\sum_{k=1}^{nj} L_k^j}{\sum_{k=0}^{j+nj-1} f_k^j} \qquad \text{with } f_0^j = 1 \qquad (3.9.11)$$

If,

$$\sum_{k=1}^{nj} L_k^j \neq \sum_{k=0}^{j+nj-1} f_k^j$$

then the KFP will cause problems if there are deterministic disturbances present in the process. The prediction $\hat{y}^0(t)$ would not give the correct information to the controller and consequently cause bias in the output control.

This problem can be overcome by augmenting the state space formulation with additional states corresponding to the stochastic model of the deterministic disturbances as suggested by Balcan et al (1970, 1973). Further discussion of this approach is given in chapter 5.

3.10 Simulation Results and Discussion

Two MIMO processes, one with a diagonal interactor and the other with a triangular interactor were used for simulations.

Case 1. Diagonal Interactor

The discretized version of the continuous time model of the binary distilltion column obtained by Wood and Berry(1973), and used by Ogunnaike and Ray (1979) and Sripada et al (1985), is used for simulation and is given by the following discrete transfer function matrix.

$$T(q) = \begin{bmatrix} \dfrac{0.8975q^{-2}}{1-0.9419q^{-1}} & \dfrac{-0.8719q^{-4}}{1-0.9535q^{-1}} \\[2ex] \dfrac{0.5786q^{-8}}{1-0.912q^{-1}} & \dfrac{-1.301q^{-4}}{1-0.933q^{-1}} \end{bmatrix} \qquad (3.10.1)$$

with

$$y(t) = T(q)u(t)$$

where

y_1 = top composition

y_2 = bottom composition

u_1 = reflux flow rate

u_2 = steam flow rate

The interactor factorization of T(q) is given by,

$$\xi^{-1}(q) = \begin{bmatrix} q^{-1} & 0 \\ 0 & q^{-3} \end{bmatrix}$$

$$R(q) = \begin{bmatrix} \dfrac{0.8975q^{-1}}{1-0.9419q^{-1}} & \dfrac{-0.8719q^{-3}}{1-0.9535q^{-1}} \\[2ex] \dfrac{0.5786q^{-5}}{1-0.912q^{-1}} & \dfrac{-1.301q^{-1}}{1-0.933q^{-1}} \end{bmatrix}$$

The state space formulation for the MISO systems of the above process are given by (from equations 3.3.6 ,3.4.53 and 3.4.54),

MISO system 1.

$$x^1(t+1) = \begin{bmatrix} 0 & -0.8981 & 0 \\ 1 & 1.8954 & 0 \\ 1 & 1 & 0 \end{bmatrix} x^1(t)$$

$$+ \begin{bmatrix} -0.8558 & 0.8212 \\ 0.8975 & -0.8719 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t-1) \end{bmatrix} + \begin{bmatrix} \gamma_{11} \\ \gamma_{12} \\ 0 \end{bmatrix} w_1(t)$$

$$y_1(t) = [0\ 0\ 1]x^1(t) + v_1(t)$$

MISO System 2

$$x^2(t+1) = \begin{bmatrix} 0 & -0.8509 & 0 & 0 & 0 \\ 1 & 1.845 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} x^2(t)$$

$$+ \begin{bmatrix} -0.5398 & 1.1865 \\ 0.5786 & -.301 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1(t-4) \\ u_2(t) \end{bmatrix} + \begin{bmatrix} \gamma_{21} \\ \gamma_{22} \\ 0 \\ 0 \\ 0 \end{bmatrix} w_2(t)$$

$$y_2(t) = [0\ 0\ 0\ 0\ 1]x^2(t) + v_2(t)$$

The process is simulated using the above state space formulation, and the noise added to the process is shown in Table 3.1.

The controller parameters for the PI controller, to be used with the ORP and IP are obtained from Sripada et al

(1985). The PI parameters for the KFP are same as that used for IP.

The PI controller is implemented using following equation:

$$G_c(q^{-1}) = \frac{l_0 + l_1 q^{-1}}{1 - q^{-1}}$$

The controller parameters were tuned to obtain the best performance of the predictor output y_p.The l_0 and l_1 parameters for the different predictor schemes are given Table 3.2.

Both the IP and the ORP were modified to handle noise by introducing exponential filter TFM in the same location as in the KFP.

The exponential filter has the following form:

$$y_r(t) = y_r(t-1) + \alpha[y(t) - y_r(t-1)]$$

The predictive control laws for u_1 and u_2 are given by,

$$u_1(t) = [y_1^*(t+2) - 1.8954\ \hat{y}_1^0(t) + 0.8981\ \hat{y}_1^0(t-1)$$
$$+ 0.8558\ u_1(t-1) + 0.8719\ u_2(t-2)$$
$$- 0.8212\ u^2(t-3)]/0.8975$$

$$u_2(t) = [y_2^*(t+4) - 1.845\ \hat{y}_2^0(t) + 0.8509\ \hat{y}_2^0(t-1)$$
$$- 1.1865\ u_2(t-1) - 0.5786\ u_1(t-4)$$
$$+ 0.5398\ u_1(t-5)]/(-1.301)$$

It is clear from the state space equations and the control law equations, that the two MISO KFPs and the predictive controllers are independent.

a. PI Control

1. Deterministic Case

When there is no noise present, and under perfect modelling, as shown in figure 3.6a and b, the KFP and IP perform exactly the same. The set points for the top and bottom composition of the distillation column are changed at the 10th and 100th time instants respectively. If the controllers are tuned to control the output y_p of the model G_p, then the KFP and IP show better performance than the ORP. The controllers used with the ORP could be retuned to improve the performance, but then the whole purpose of tuning to y_p would be lost. Figures 3.6c and d show the manipulated variables of each of these predictor schemes.

Since a decoupling technique has not been used to remove the interactions, a change in set point for one output influences the other.

2. Stochastic Case

Figures 3.7a and b show the performance of the 3 predictor schemes under noisy conditions. As shown in figure 3.7e introducing an exponential filter in the IP and ORP give higher tracking error variance. The results shown in figures 3.7a and b, for IP and ORP are without filtering. The lowest tracking error variance is achieved by using the KFP.

The manipulated variables u_1 and u_2 of the KFP in figures 3.7c and d show higher fluctuations than in the

ORP or IP. This is the result of the minimum variance estimation of the KFP.

b. Predictive Control

1. Deterministic Case

Figure 3.8a and b show the top and bottom composition of the distillation column under the predictive control using IP and KFP. The setpoints for the top and bottom compositions were changed at the 10th and 100th time instants respectively. Both IP and KFP show exactly the same performance under ideal conditions, and this confirms the results in section 3.7.

It is also important to note that the outputs are not affected by interactions as in the case of PI control, i.e. y_1 is not affected by the set point change in y_2 and visa versa. This is a property of the predictive controller based on the interactor matrix. It provides the natural decoupling of the process. However, as shown in figures 3.8c and d, the maximum deviations in the manipulated variable from the predictive control schemes are higher than in the PI control scheme.

Since all the MISO KFPs are independent in this case the prediction $\hat{y}^0(t)$ is a future value of y(t).

It is also assumed in predictive control that the future values of the set points are known at least up to the number of time instants given by the delays in the diagonal elements of the inverse interactor ξ^{-1}.

2. Stochastic Case

Figures 3.9a and b show the performance of the KFP and IP in the presence of both process and measurement noise. In the presence of noise the performance of the interactor predictor is very noisy. The Kalman filter shows satisfactory performance under this condition. The ratio of the covariances R_w/R_v, in the KFP is tuned to obtain the minimum variance control performance.

The IP was improved by introducing exponential filters. As shown in figure 3.9a and b, the IP plus exponential filter shows improved performance (filtering). Figure 3.9e shows the variance of tracking error for both the outputs, with respect to the ratio R_w/R_v and the filter coefficient α. The KFP exhibits the minimum variance. For the top composition, the minimum variance, achieved by the IP is very close to that of the KFP.

Case 2. Triangular Interactor

The discrete transfer function matrix of the MIMO process, that has a triangular interactor is given by,

$$T(q) = \begin{bmatrix} \dfrac{0.2658q^{-4} - 0.061q^{-5}}{1-1.6q^{-1} + 0.63q^{-2}} & \dfrac{0.012q^{-5} + 0.0044q^{-6}}{1-1.6q^{-1} + 0.63q^{-2}} \\[2ex] \dfrac{0.083q^{-5} + 0.03q^{-6}}{1-1.6q^{-1} + 0.63q^{-2}} & \dfrac{0.1063q^{-6} - 0.0244q^{-7}}{1-1.6q^{-1} + 0.63q^{-2}} \end{bmatrix}$$

$$\xi^{-1}(q) = \begin{bmatrix} q^{-3} & 0 \\ 0.3198q^{-4} & q^{-5} \end{bmatrix}$$

$$R(q) = \begin{bmatrix} \dfrac{0.2558q^{-1} - 0.061q^{-2}}{1-1.6q^{-1} + 0.63q^{-2}} & \dfrac{0.012q^{-2} + 0.0044q^{-3}}{1-1.6q^{-1} + 0.63q^{-2}} \\ \dfrac{0.0495q^{-1}}{1-1.6q^{-1} + 0.63q^{-2}} & \dfrac{0.1025q^{-1} - 0.0258q^{-2}}{1-1.6q^{-1} + 0.63q^{-2}} \end{bmatrix}$$

The state space formulation for each MISO system is given by,

$$x^1(t+1) = \begin{bmatrix} 0 & -0.63 & 0 & 0 & 0 \\ 1 & 1.6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} x^1(t)$$

$$+ \begin{bmatrix} -0.061 & 0.0044 \\ 0.2558 & 0.012 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u^1(t) \\ u_2(t-1) \end{bmatrix} + \begin{bmatrix} \gamma_{11} \\ \gamma_{12} \\ 0 \\ 0 \\ 0 \end{bmatrix} w_1(t)$$

$$y_1(t) = [0\ 0\ 0\ 0\ 1]\ x^1(t) + v_1(t)$$

$$x^2(t+1) = \begin{bmatrix} 0 & -0.63 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} x^2(t)$$

$$+ \begin{bmatrix} 0.0 & -0.0258 & 0.0 \\ 0.0495 & 0.1025 & 0.0 \\ 0.0 & 0.0 & 0.3198 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ y_1^0(t) \end{bmatrix} + \begin{bmatrix} \gamma_{21} \\ \gamma_{22} \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix} w_2(t)$$

$$y_2(t) = [0\ 0\ 0\ 0\ 0\ 0\ 1]\ x^2(t) + v_2(t)$$

The predictive control law for the MISO systems are given by,

$$\begin{aligned} u_1(t) = {} & [y^*(t+4) - 1.6\ \hat{y}_1^0(t) + 0.63\ \hat{y}_1^0(t-1) \\ & + 0.061u_1(t-1) - 0.012u_2(t-1) \\ & - 0.0044\ u_2(t-2)]/0.2658 \end{aligned}$$

$$\begin{aligned} u_2(t) = {} & [-0.0495\ u_1(t) - 0.3197\ y_1^*(t+5) \\ & + y_2^*(t+6) - 1.6\ \hat{y}_2^0(t) + 0.63\ \hat{y}_2^0(t-1) \\ & + 0.0258\ u_2(t-1)]/0.10246 \end{aligned}$$

It is important to note that the control law $u_2(t)$ is calculated not only for the set point $y_2^*(t)$ but also for $y_1^*(t)$, and it also depends on the current manipulated variable $u_1(t)$. In Case 1 when the interactor was diagonal, this predictive control system is composed of independent MISO schemes. But in Case 2 they are serially coupled.

a. Predictive Control

1. Deterministic Case

As in the diagonal interactor case both KFP and IP show the same performance under ideal conditions and are shown in figures 3.10a and b. In the diagonal ξ^{-1} case the prediction $\hat{y}^0(t)$ is a future value of the actual output y(t). But for the triangular ξ^{-1}, only $\hat{y}_0^1$ is a future value of $y_1(t)$. As shown in figure 3.10b (the top plot) the prediction $\hat{y}_0^2$ is not a future value of the output, but is related to the future output through a certain T.F. Figures 3.10c and d show the manipulated variable u_1 and u_2. As in the diagonal ξ^{-1} case the interactions are decoupled.

2. Stochastic Case

Figures 3.11a and b show the performance of the KFP and IP under noisy conditions.The IP is quite noisy if no filter is used. The KFP shows better performance than the IP. Figure 3.11e shows the variance of the output tracking error of y_1 and y_2, for the KFP and the IP plus filter. As proved theoretically the KFP shows minimum variance control performance.Figures 3.11c and d show the manipulated variable u_1 and u_2.

The simulation results confirm the theoretical results obtained in the previous sections, and can be summarized as follows:

1. Under noise free condition and perfect modelling the KFP is the same as the IP.

2. Under noisy conditions the Kalman filter predictor gives minimum variance predictions.
3. The predictive control law with KFP gives minimum variance control performance.
4. Although the IP can be improved to handle noise by introducing (exponential) filters, it does not guarantee minimum variance control performance.

3.11 Conclusions

1. The multivariable Kalman filter predictor based on the interactor factorization gives a minimum variance predictor for MIMO processes with time delays, in the presence of stochastic noise.
2. The multivariable KFP and the Interactor predictor have similar structures (cf. figures 3.5 and H.2).
3. Under ideal conditions, i.e. noise free and perfect modelling, the KFP and the IP are functionally the same.
4. The predictive control scheme based on the multivariable KFP and the interactor factorization gives minimum variance control performance, and naturally decouples the interactions.
5. A practical multivariable KFP is obtained by implementing p MISO KFPs, instead of a single KFP. The computational effort is reduced further by incorporating the simplified KFP algorithm.
6. The state space formulation of each MISO KFP guarantees the stability of each MISO KFP and the convergence of the time varying KFP to the steady state KFP.
7. The ratio of the noise covariances for each MISO KFP can be used as a tuning parameter for the KFPs.
8. The interactor predictor can be extended to handle noise by introducing exponential filters, and tuning them in an adhoc manner.

Table 3.1 Process and Measurement Noise Variances Used in Simulations

MISO System	Process Noise	Measurement Noise
1	[0.0,0.005]	[0.0,0.1]
2	[0.0,0.005]	[0.0,0.1]

Table 3.2 Controller Parameters for Different Predictor Schemes

System	Loop 1		Loop 2	
	l_0	l_1	l_0	l_1
ORP	0.45	-0.30	-0.25	0.15
IP	0.35	-0.30	-0.18	0.15
KFP	0.35	-0.30	-0.18	0.15

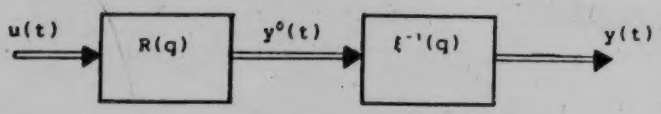

u(t)
R(q)
y°(t)
ξ⁻¹(q)
y(t)

Figure 3.1 Schematic block diagram of a MIMO process with time delays, based on the interactor factorization .

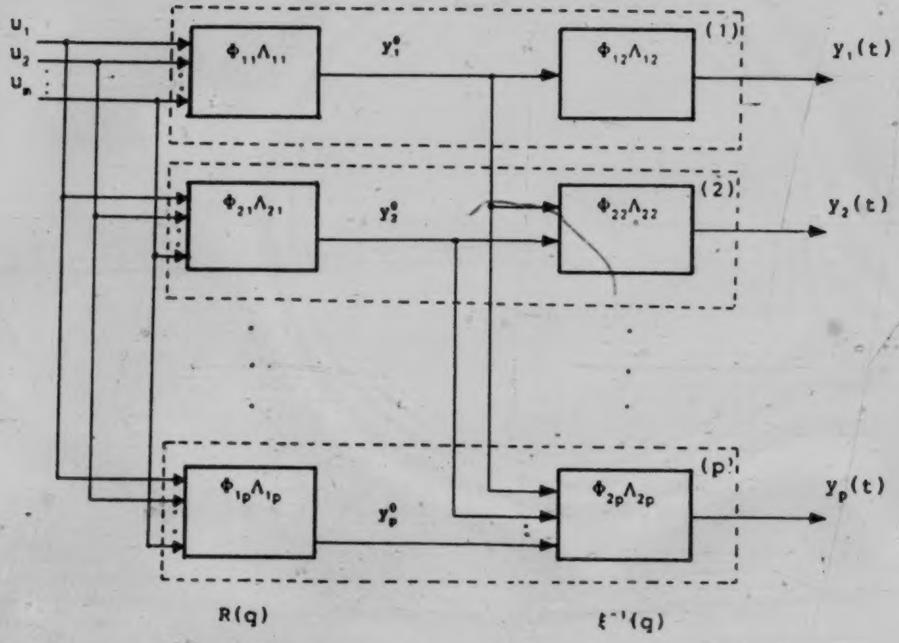

u₁
u₂
uₚ
Φ₁₁Λ₁₁
y°₁
Φ₁₂Λ₁₂
(1)
y₁(t)
Φ₂₁Λ₂₁
y°₂
Φ₂₂Λ₂₂
(2)
y₂(t)
Φ₁ₚΛ₁ₚ
y°ₚ
Φ₂ₚΛ₂ₚ
(p)
yₚ(t)
R(q)
ξ⁻¹(q)

Figure 3.2 Schematic block diagram of a MIMO process, decomposed into a p MIMO processes, based on interactor factorization.

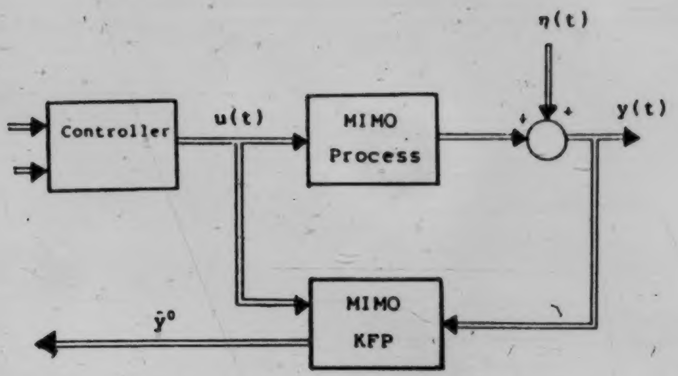

η(t)
Controller
u(t)
MIMO
Process
+
+
y(t)
MIMO
KFP
ŷ°

Figure 3.3 Schematic block diagram of the multivariable Kalman Filter Predictor.

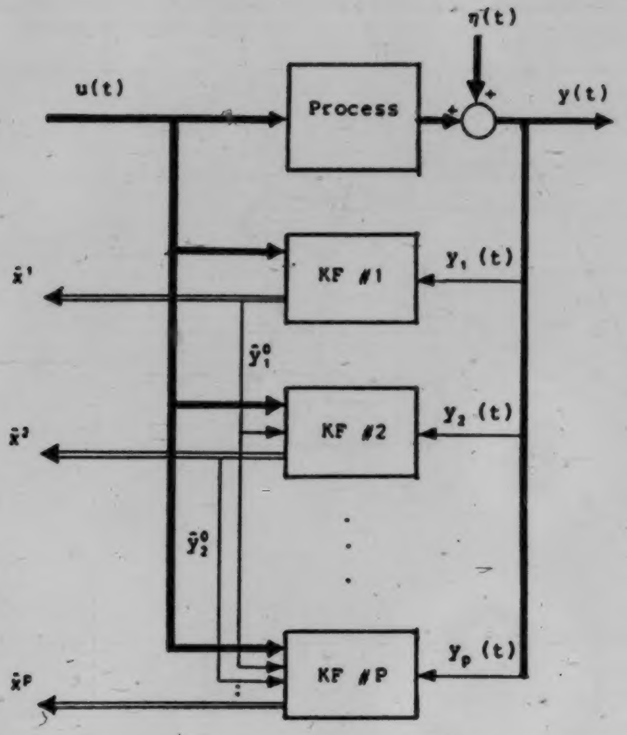

$\eta(t)$
u(t)
Process
y(t)
+
+
KF #1
KF #2
KF #P
$y_1(t)$
$y_2(t)$
$y_p(t)$
$\hat{x}^1$
$\hat{x}^2$
$\hat{x}^p$
$\hat{y}_1^0$
$\hat{y}_2^0$

Figure 3.4 Serially connected p MISO Kalman Filter Predictors.

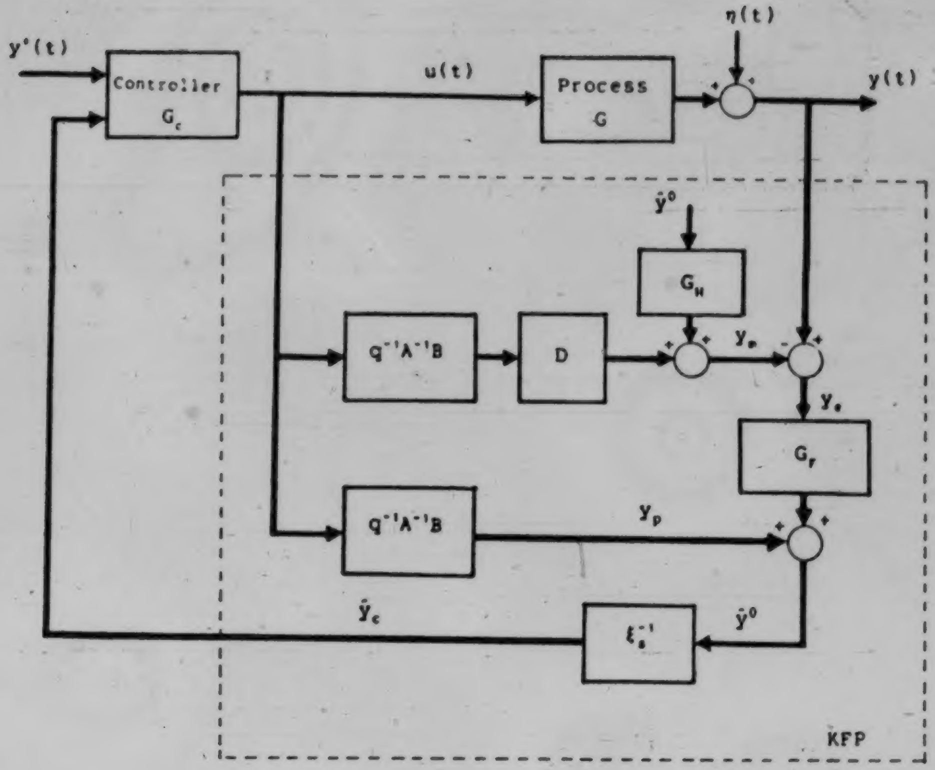
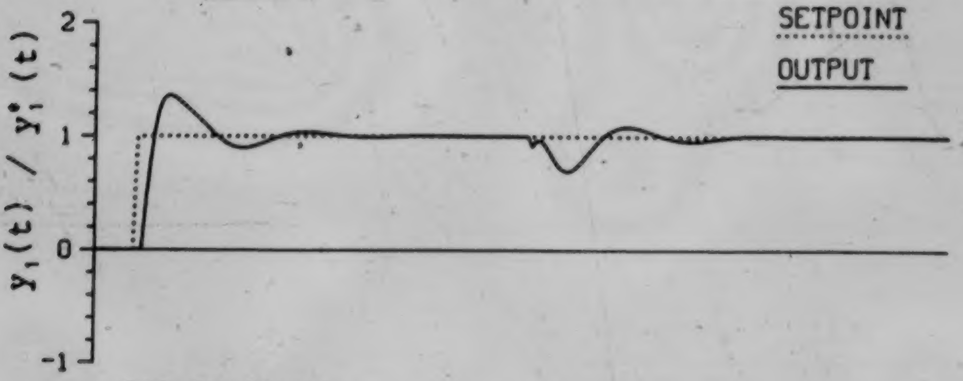
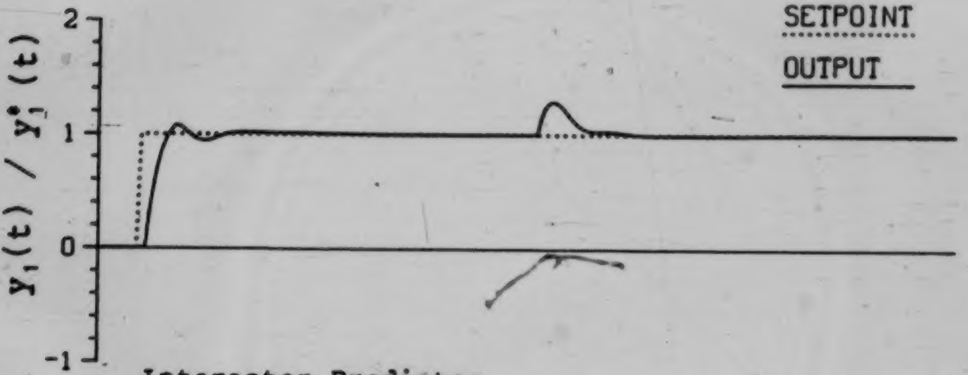

$y^r(t)$
Controller
G_c
$u(t)$
Process
G
$\eta(t)$
$y(t)$
$\hat{y}^0$
G_H
$q^{-1}A^{-1}B$
D
y_m
y_e
G_f
$q^{-1}A^{-1}B$
y_p
$\hat{y}_c$
ξ_s^{-1}
$\hat{y}^0$
KFP

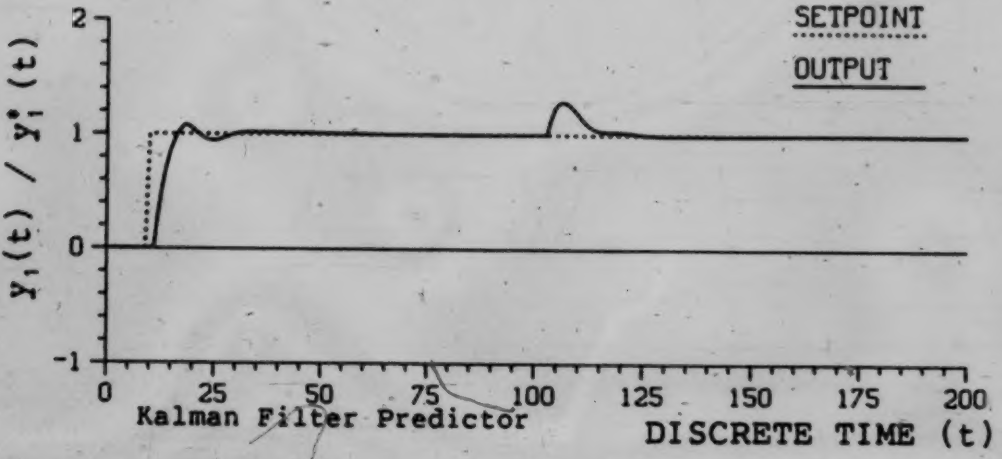
Figure 3.5 Structure of the Multivariable Kalman Filter Predictor.



O&R Predictor



Interactor Predictor



Kalman Filter Predictor

Figure 3.6a PI control of Top Composition using KFP, IP and ORP (Deterministic Case).

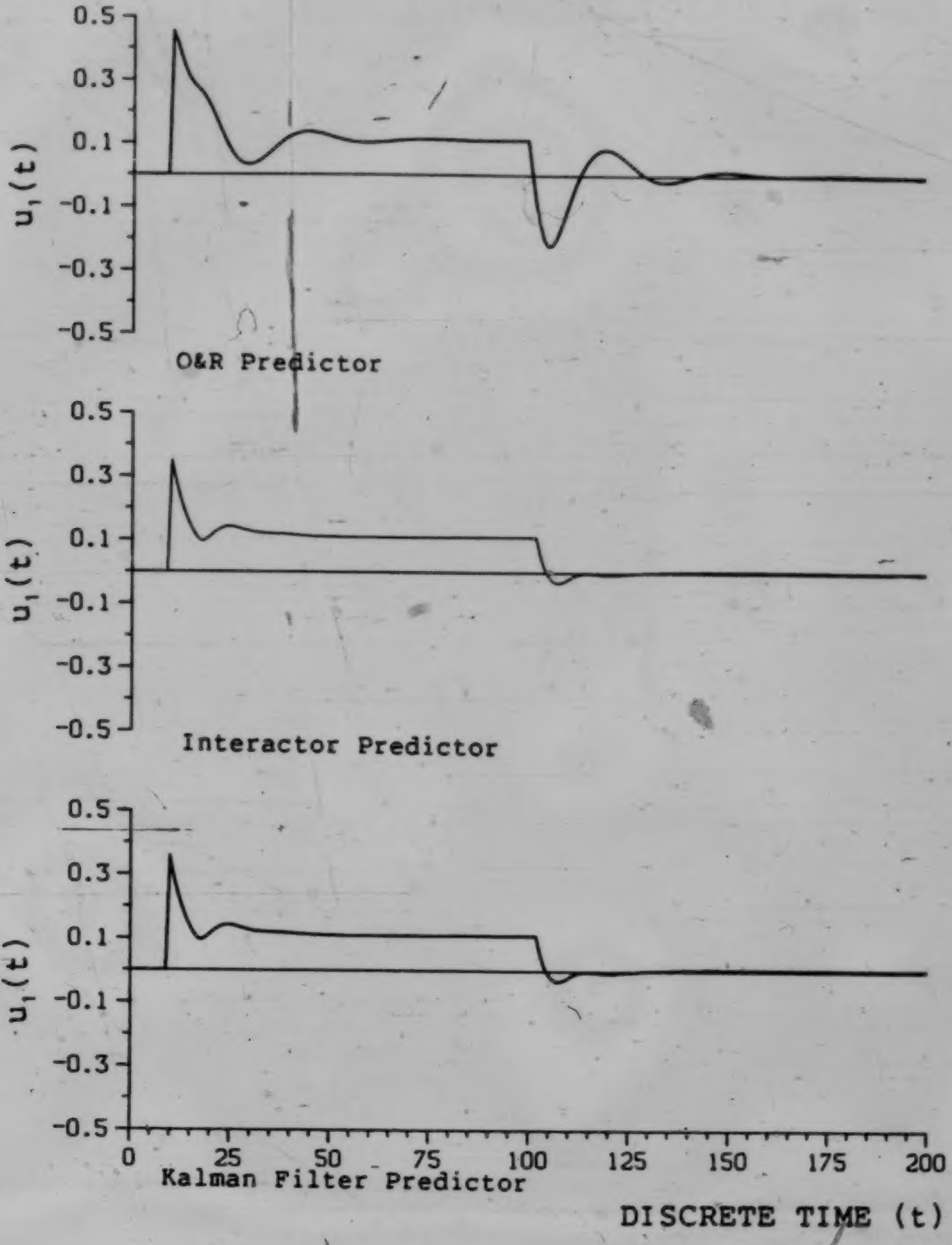
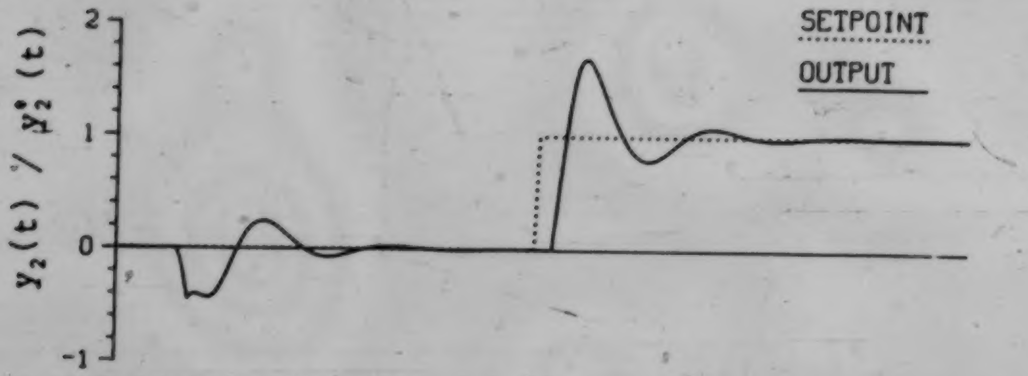
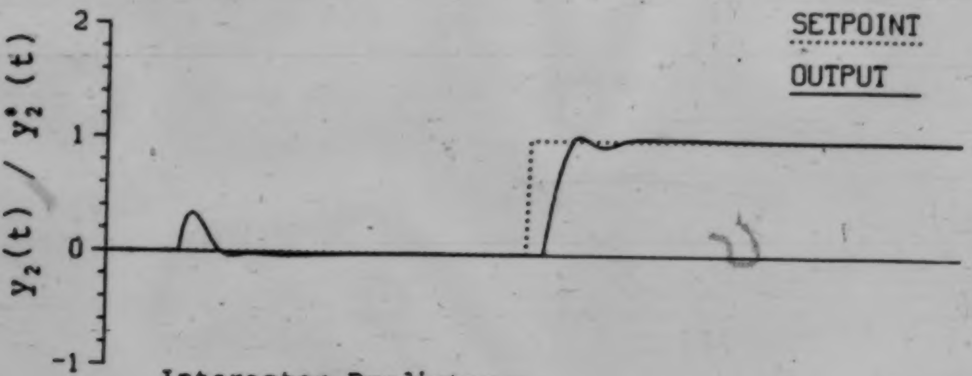


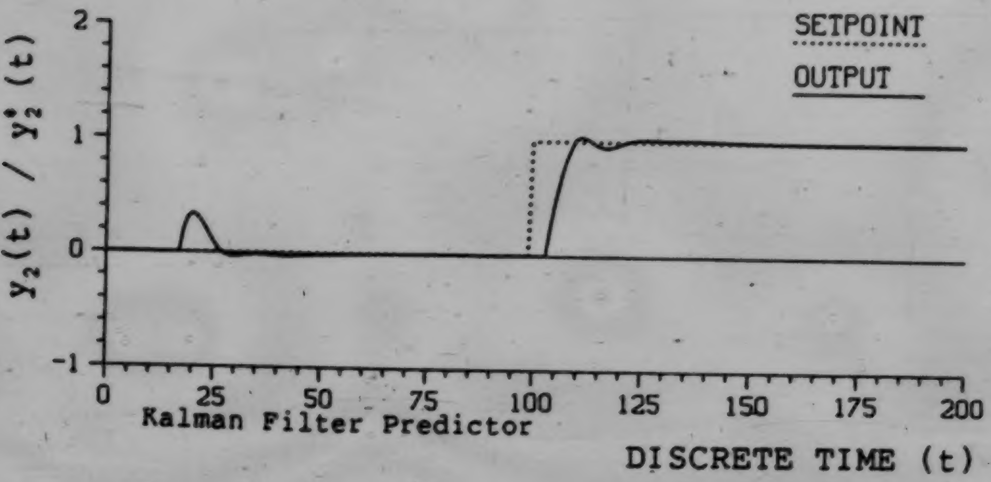
Figure 3.6b PI Control of Bottom Composition using KFP, IP and ORP (Deteministic Case).



O&R Predictor



Interactor Predictor



Kalman Filter Predictor

Figure 3.6c PI Control of Top Composition using KFP, IP and ORP (Deterministic Case).

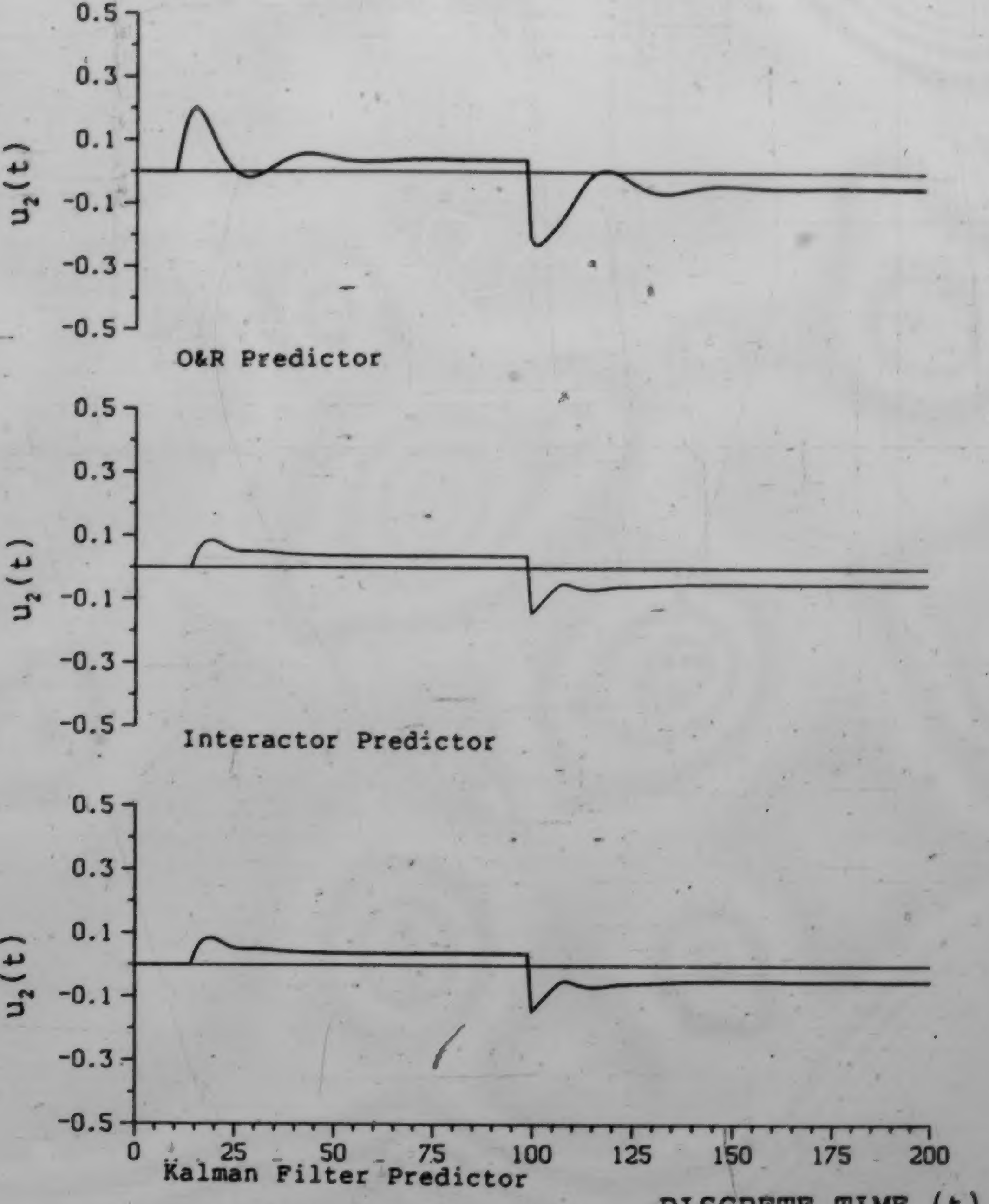
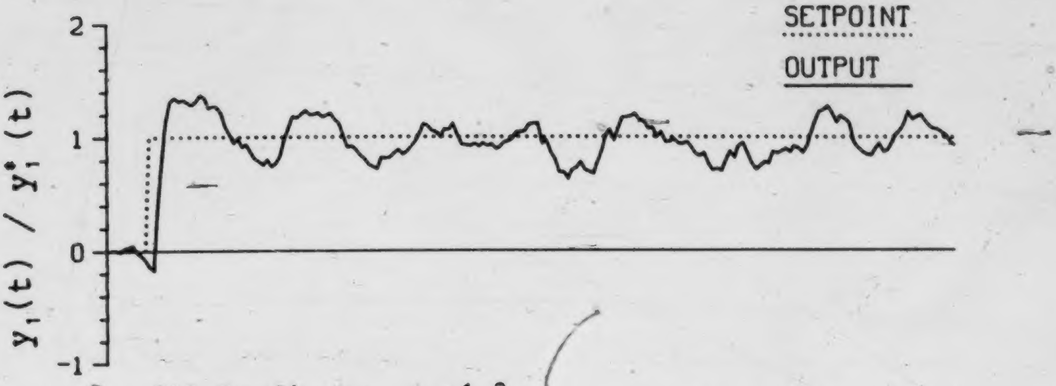
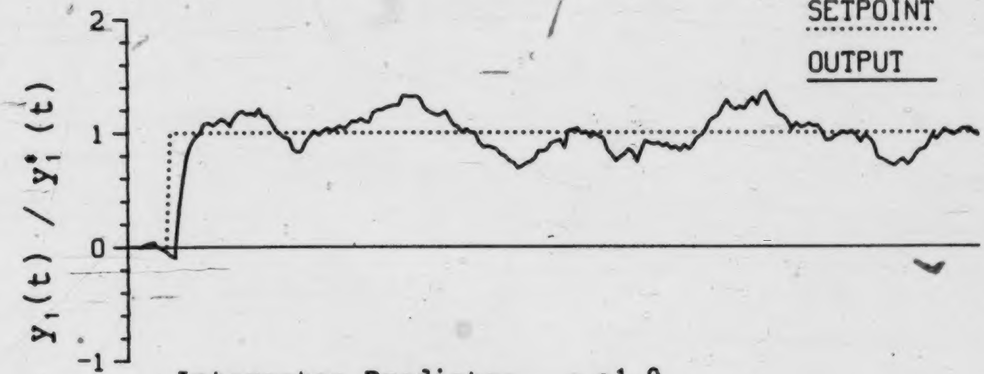


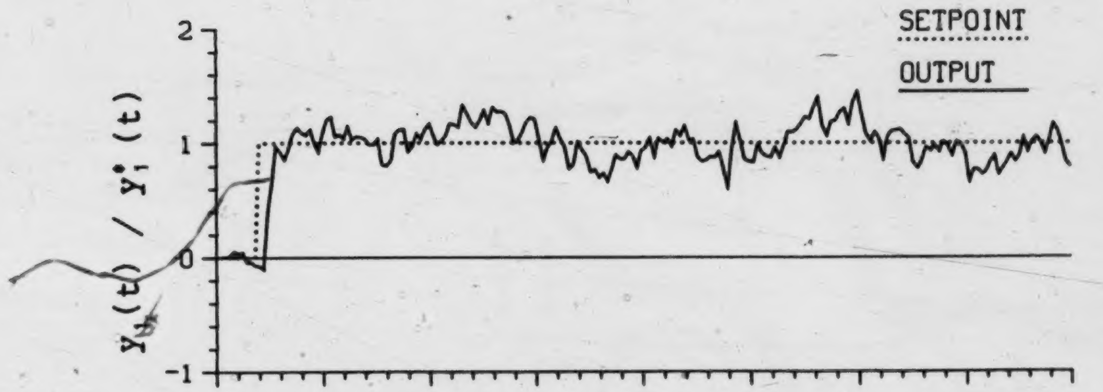
Figure 3.6d PI Control of Bottom Composition using KFP, IP and ORP (Deterministic Case).



O&R Predictor α_1=1.0

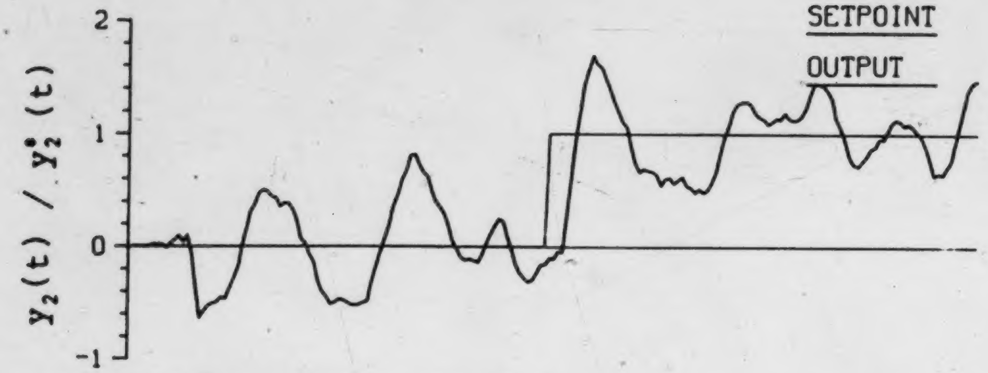


Interactor Predictor α_1=1.0

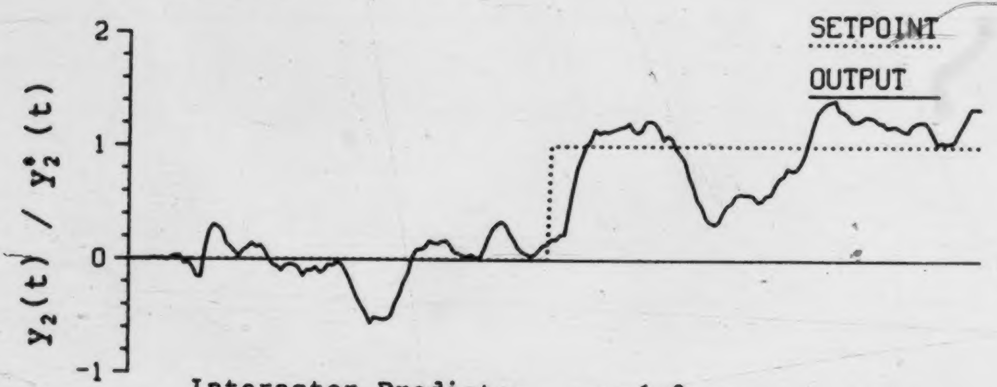


Kalman Filter Predictor
R_{v_1}/R_{v_1}=0.05

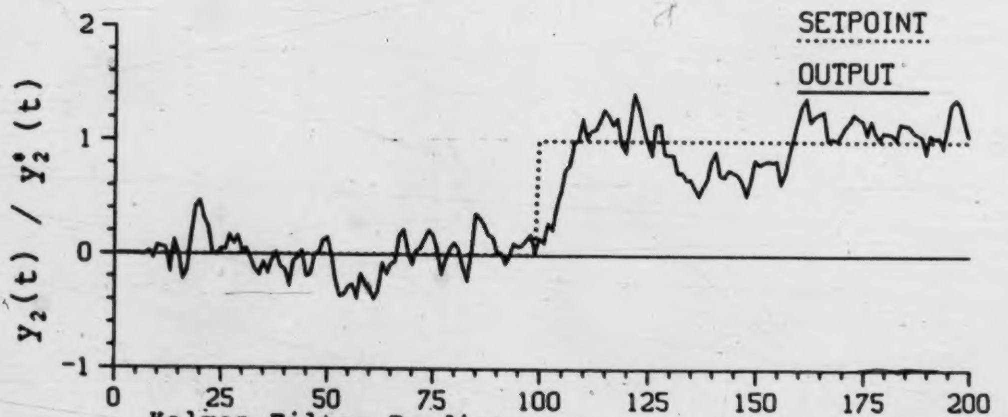
Figure 3.7a PI Control of Top Composition using KFP, IP and ORP (Stochastic Case).



O&R Predictor $\alpha_2=1.0$

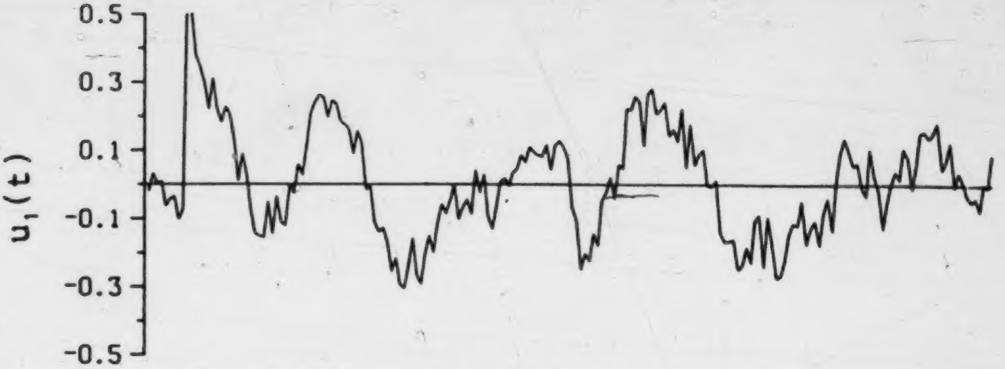


Interactor Predictor $\alpha_2=1.0$

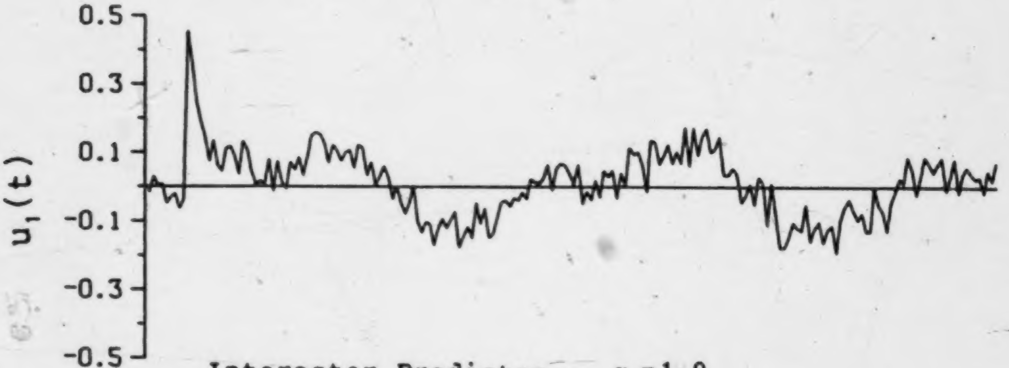


Kalman Filter Predictor
$R_{v_2}/R_{v_2}=0.05$

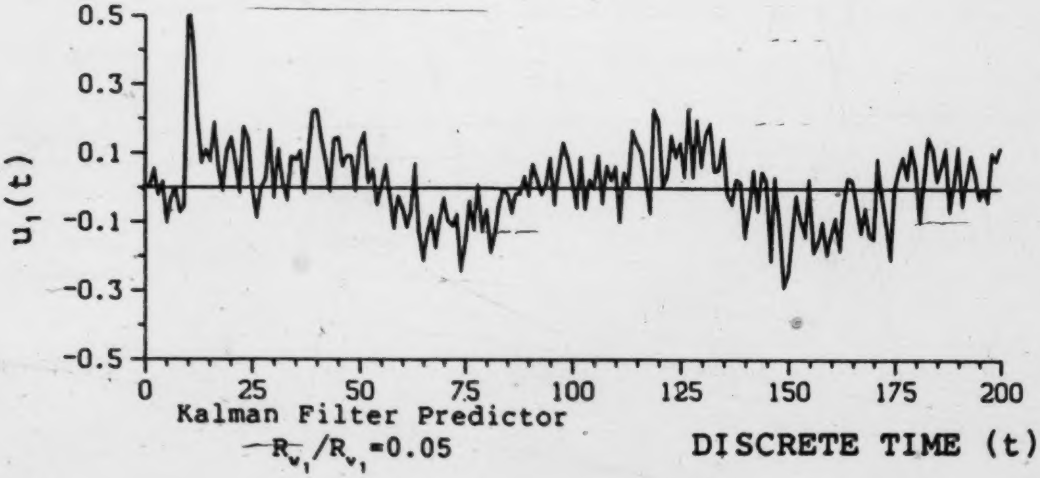
Figure 3.7b PI Control of Bottom Composition using KFP, IP and ORP (Stochastic Case).



O&R Predictor $\alpha_1 = 1.0$



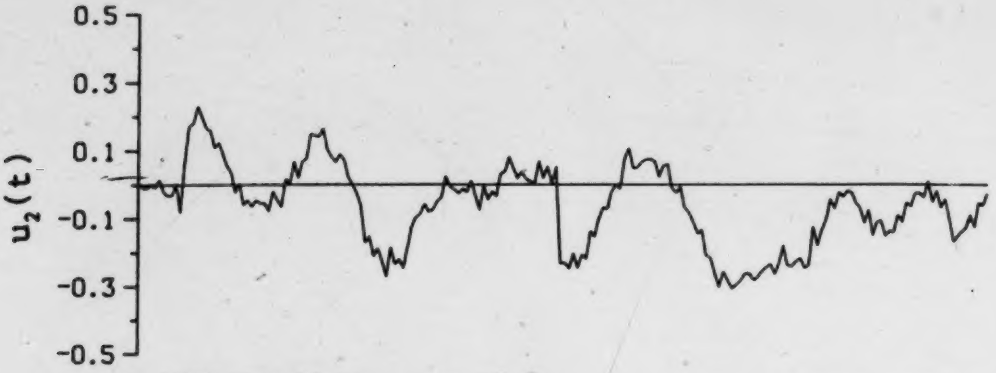
Interactor Predictor $\alpha_1 = 1.0$



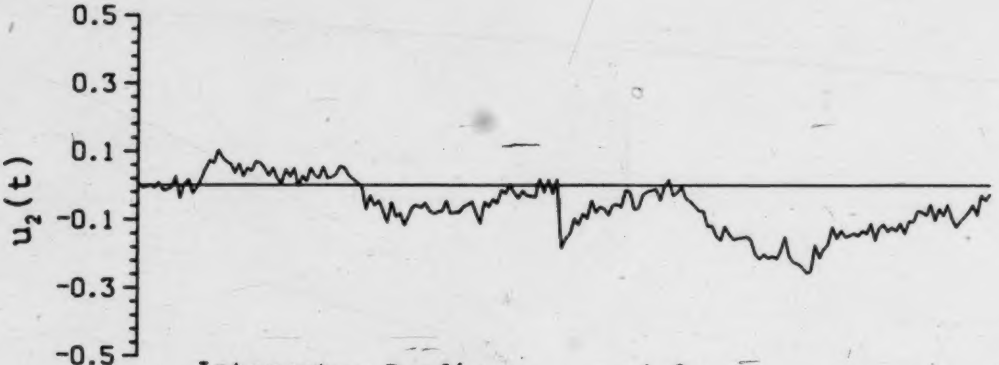
Kalman Filter Predictor

$R_{v_1}/R_{v_1} = 0.05$

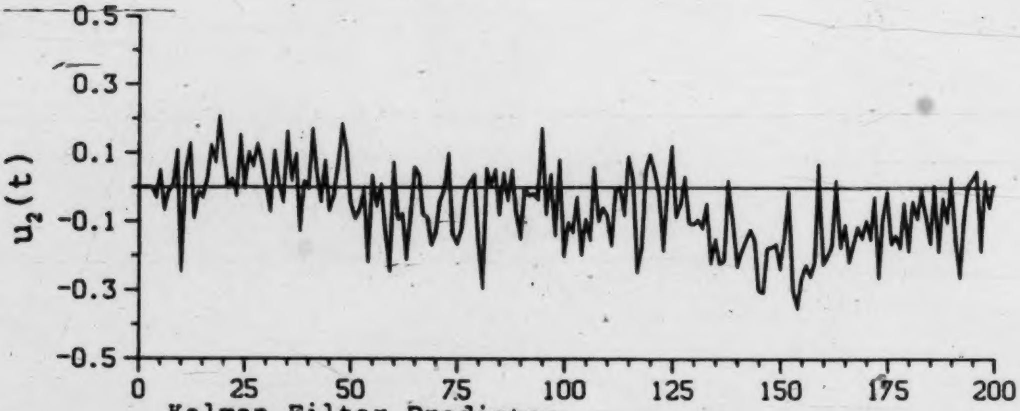
Figure 3.7c PI Control of Top Composition using KFP, IP and ORP.



O&R Predictor α_2=1.0



Interactor Predictor α_2=1.0



Kalman Filter Predictor
R_{v_2}/R_{v_2}=0.05

Figure 3.7d PI Control of Bottom Composition using KFP, IP and ORP.

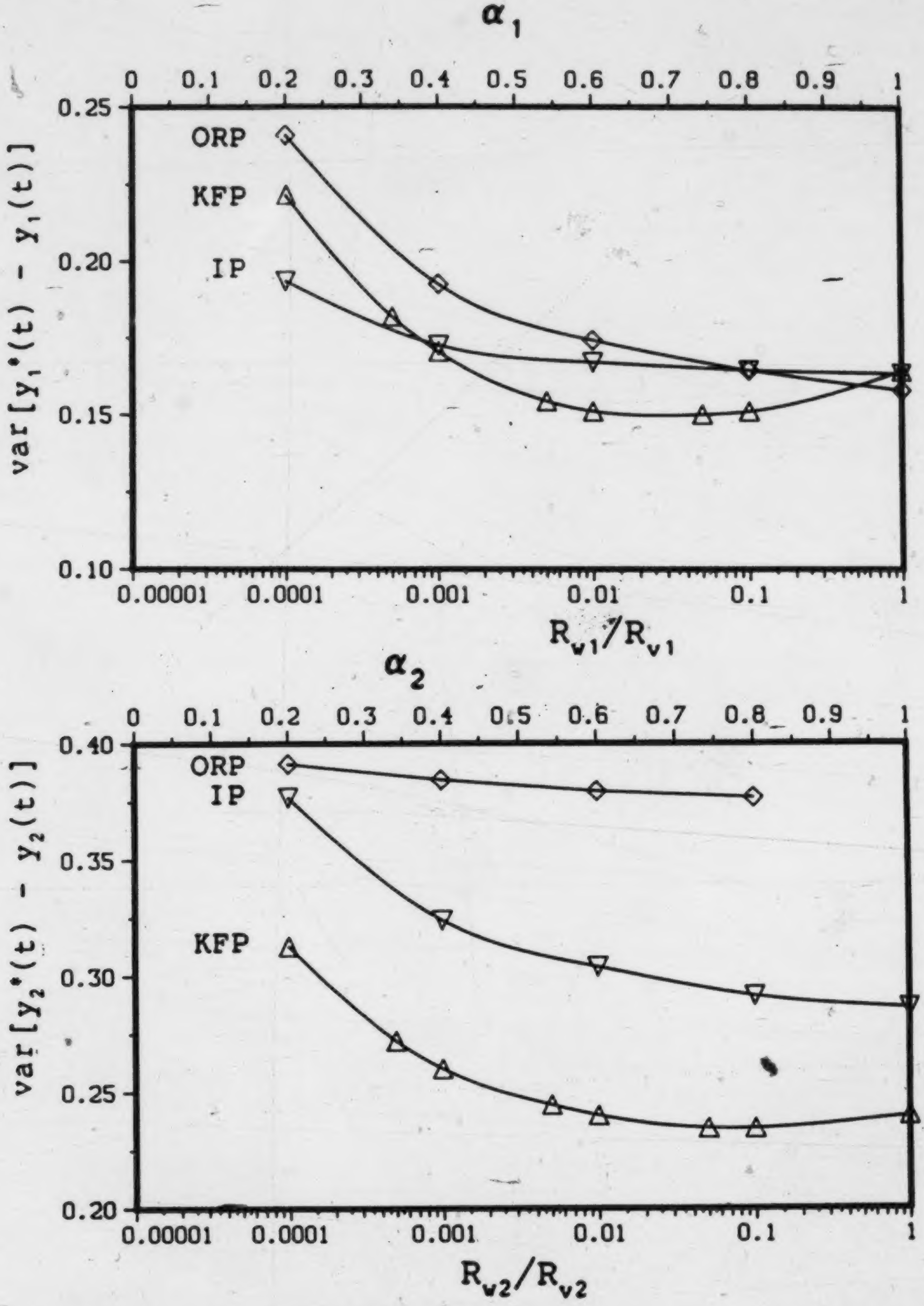
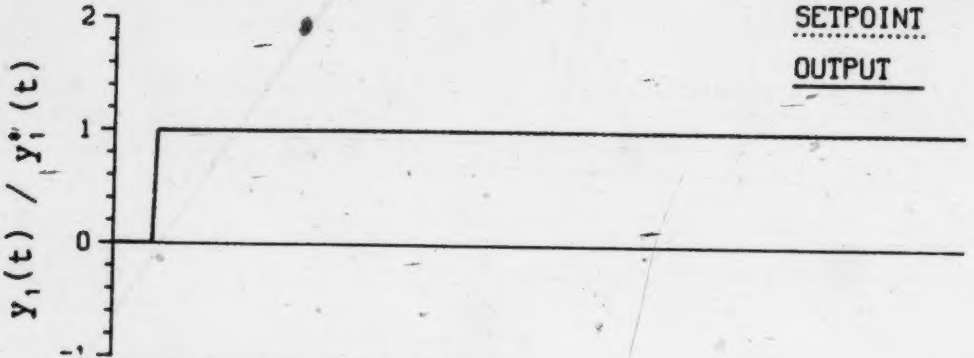
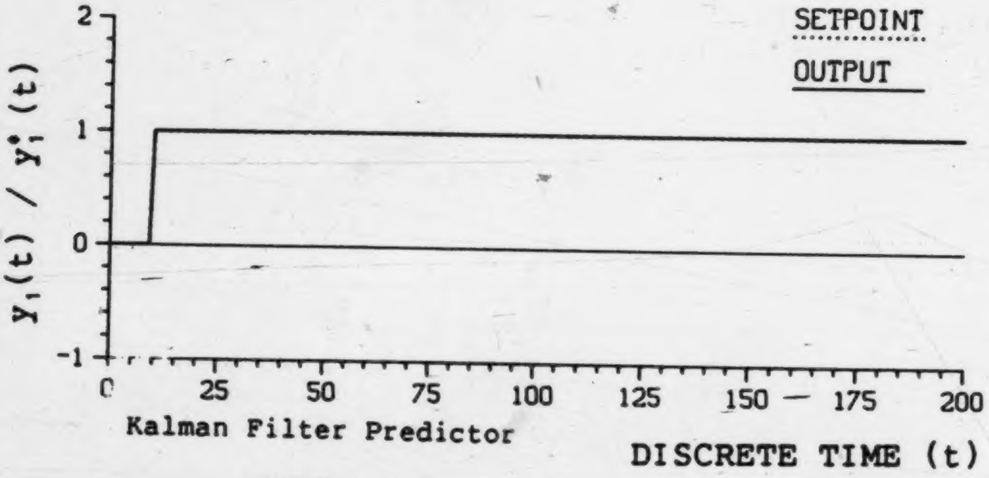


Figure 3.7e Variance of Tracking Error for KFP, IP and ORP.



Interactor Predictor



Kalman Filter Predictor

Figure 3.8a Predictive Control of Top Composition using KFP and IP (Deterministic Case).

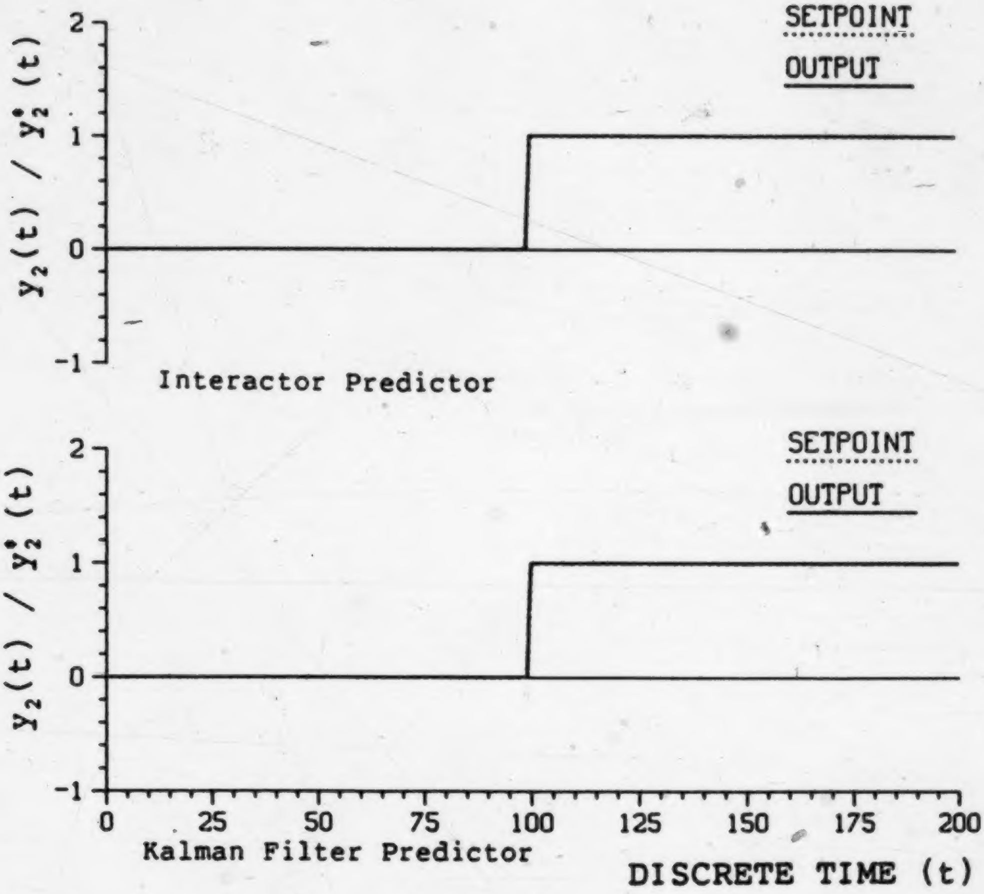
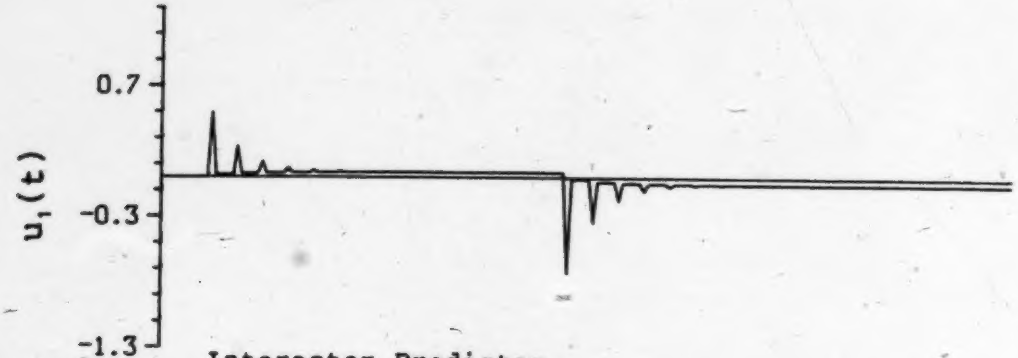
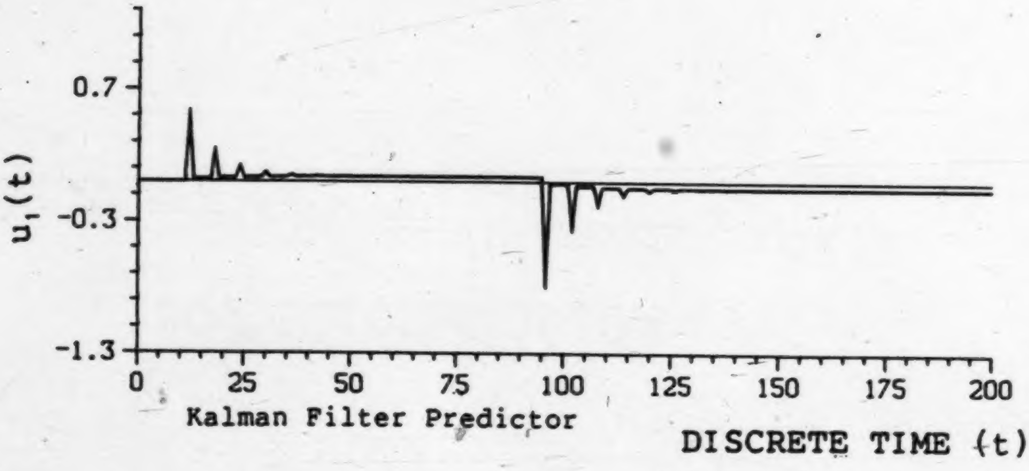


Figure 3.8b Predictive Control of Bottom Composition using KFP and IP (Deterministic Case).

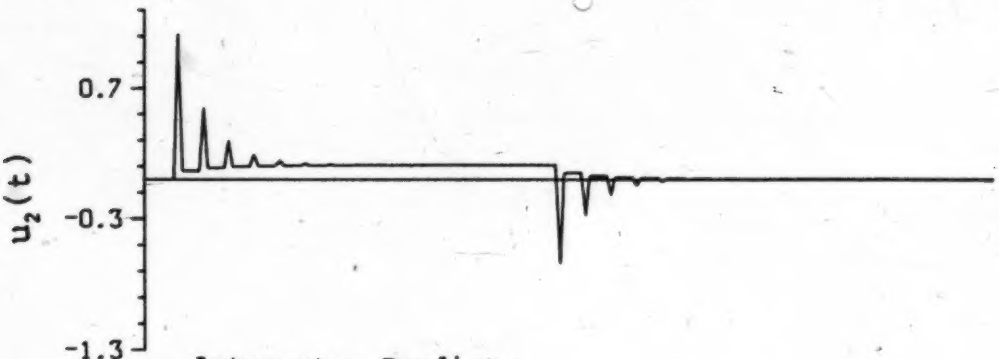


Interactor Predictor

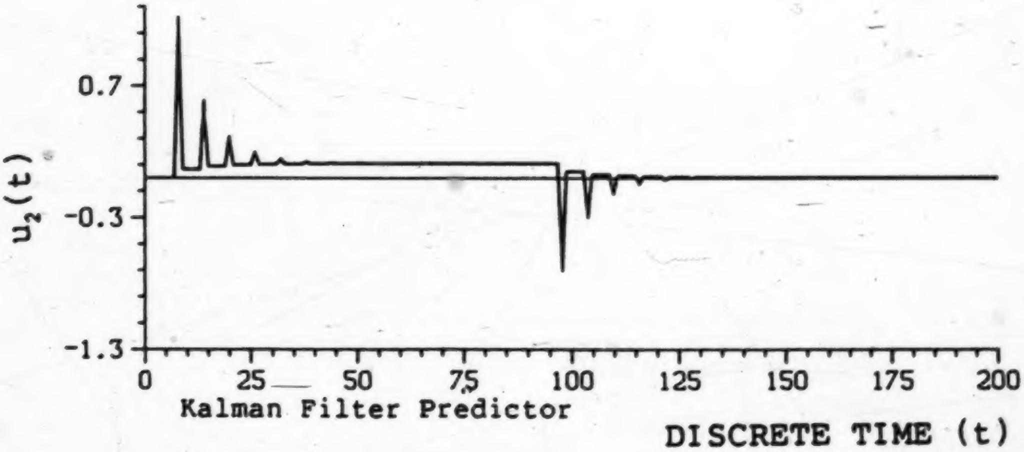


Kalman Filter Predictor

Figure 3.8c Predictive Control of Top Composition using KFP and IP (Deterministic Case).

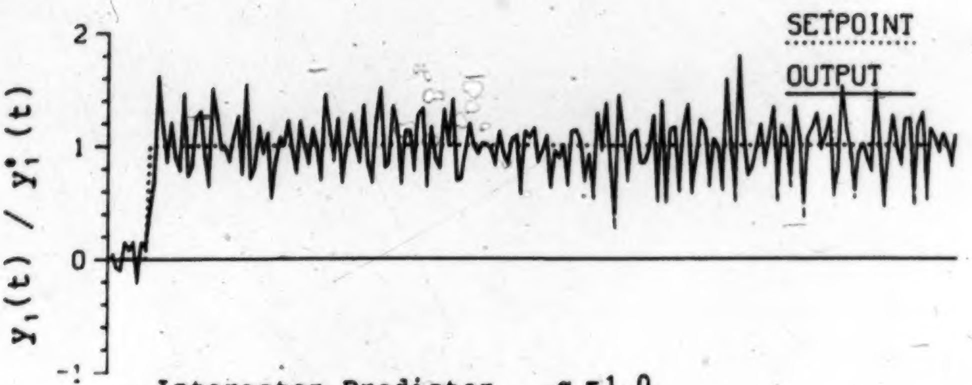


Interactor Predictor

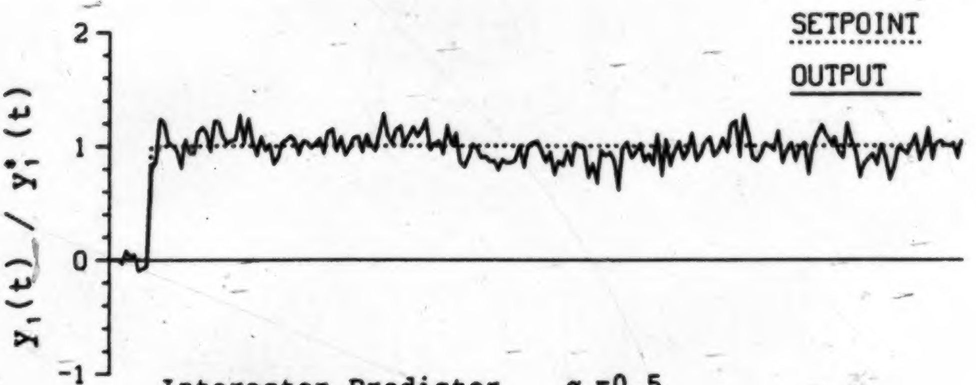


Kalman Filter Predictor

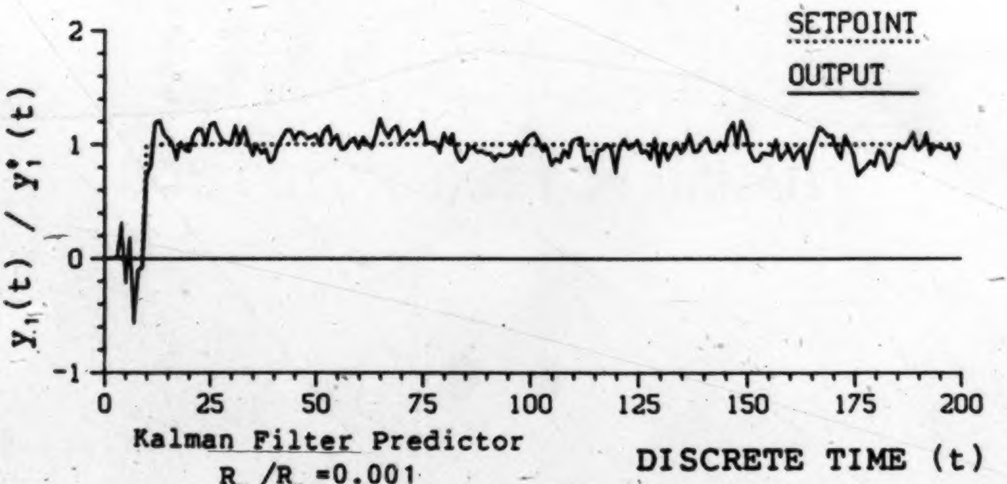
Figure 3.8d Predictive Control of the Bottom Composition using KFP and IP (Deterministic Case).



Interactor Predictor α_1=1.0

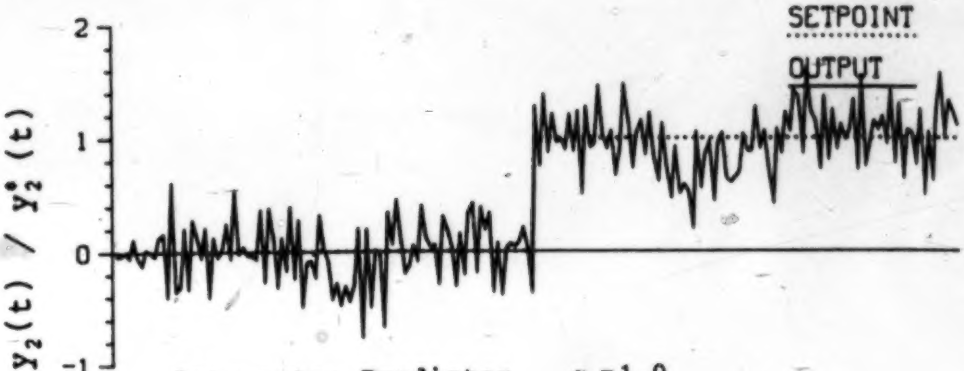


Interactor Predictor α_1=0.5

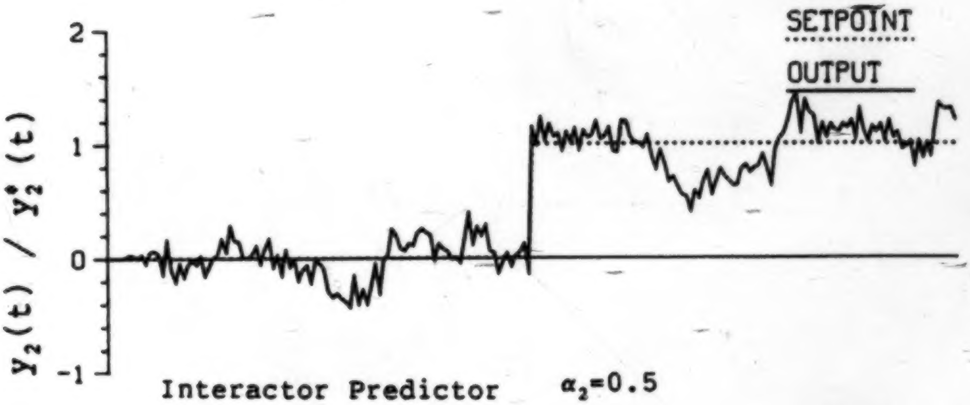


Kalman Filter Predictor
R_{v_1}/R_{v_1}=0.001

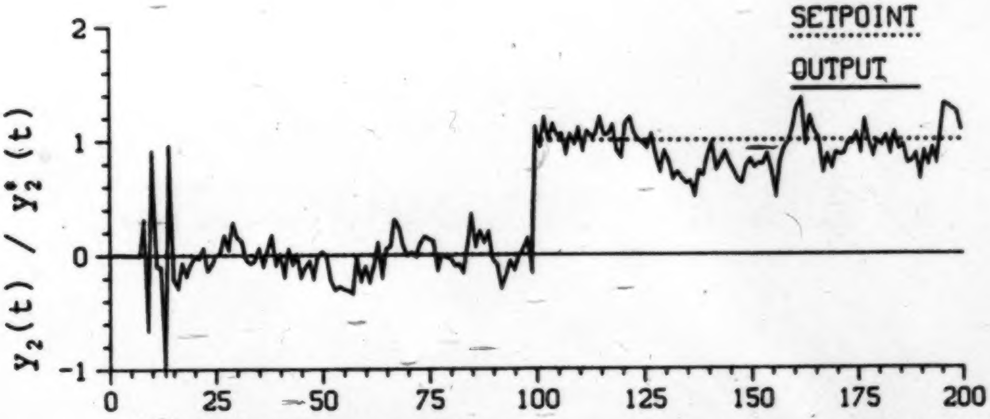
Figure 3.9a Predictive Control of Top Composition using KFP and IP (Stochastic Case).



Interactor Predictor α_2=1.0

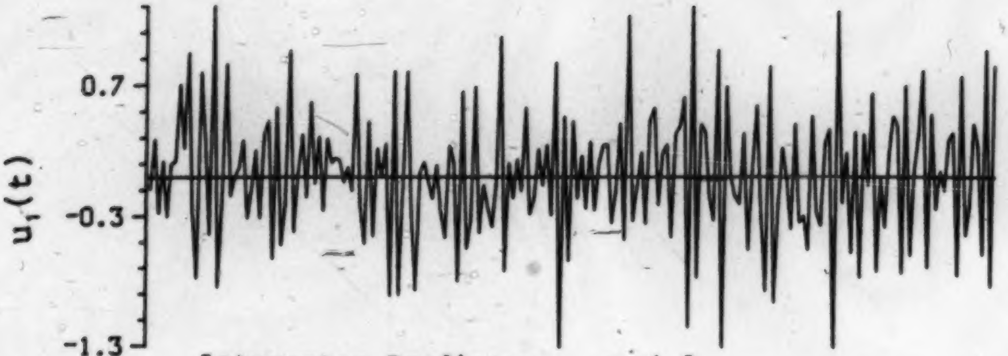


Interactor Predictor α_2=0.5

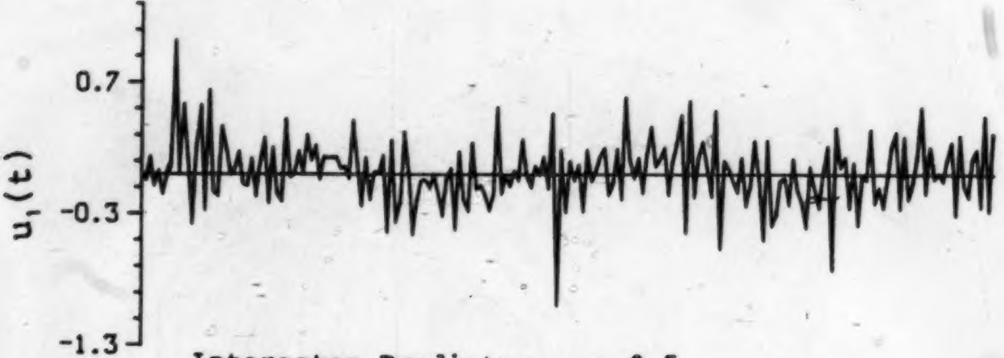


Kalman Filter Predictor
R_{v_2}/R_{v_2}=0.001

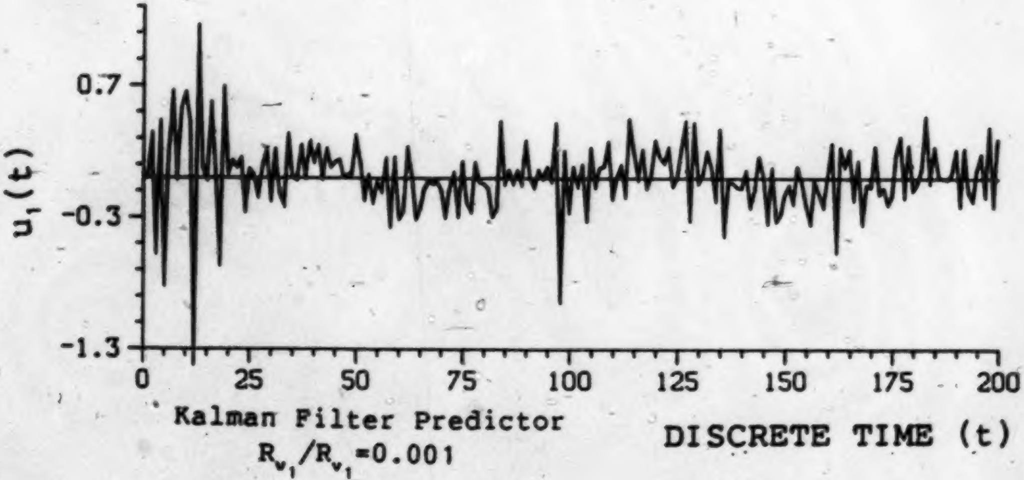
Figure 3.9b Predictive Control of Bottom Composition using KFP and IP (Stochastic Case).



Interactor Predictor $\alpha_1=1.0$

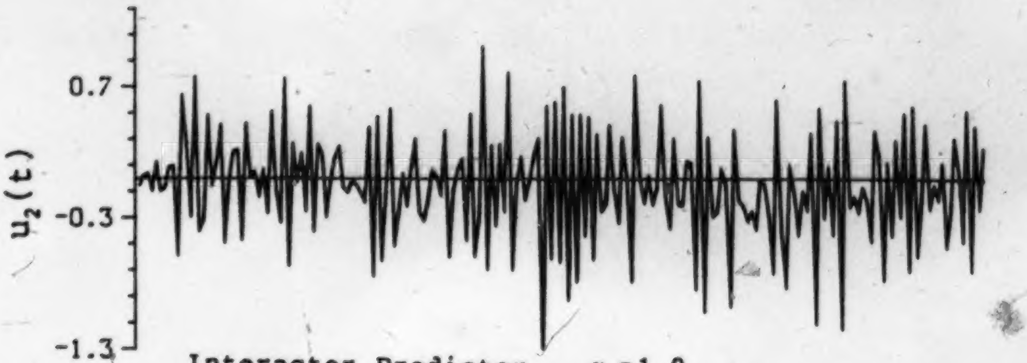


Interactor Predictor $\alpha_1=0.5$

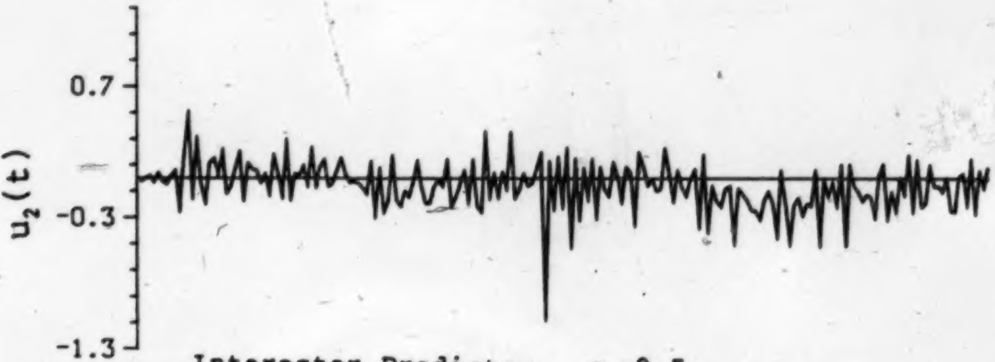


Kalman Filter Predictor
$R_{v_1}/R_{v_1}=0.001$

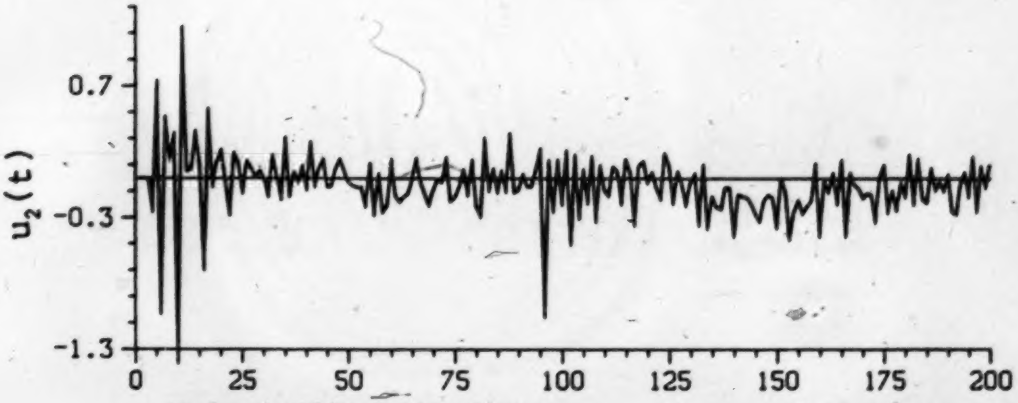
Figure 3.9c Predictive Control of Top Composition using KFP and IP (Stochastic Case).



Interactor Predictor $\alpha_2=1.0$



Interactor Predictor $\alpha_2=0.5$



Kalman Filter Predictor
$R_{v_2}/R_{v_2}=0.001$

Figure 3.9d Predictive Control of Bottom Composition using KFP and IP (Stochastic Case).

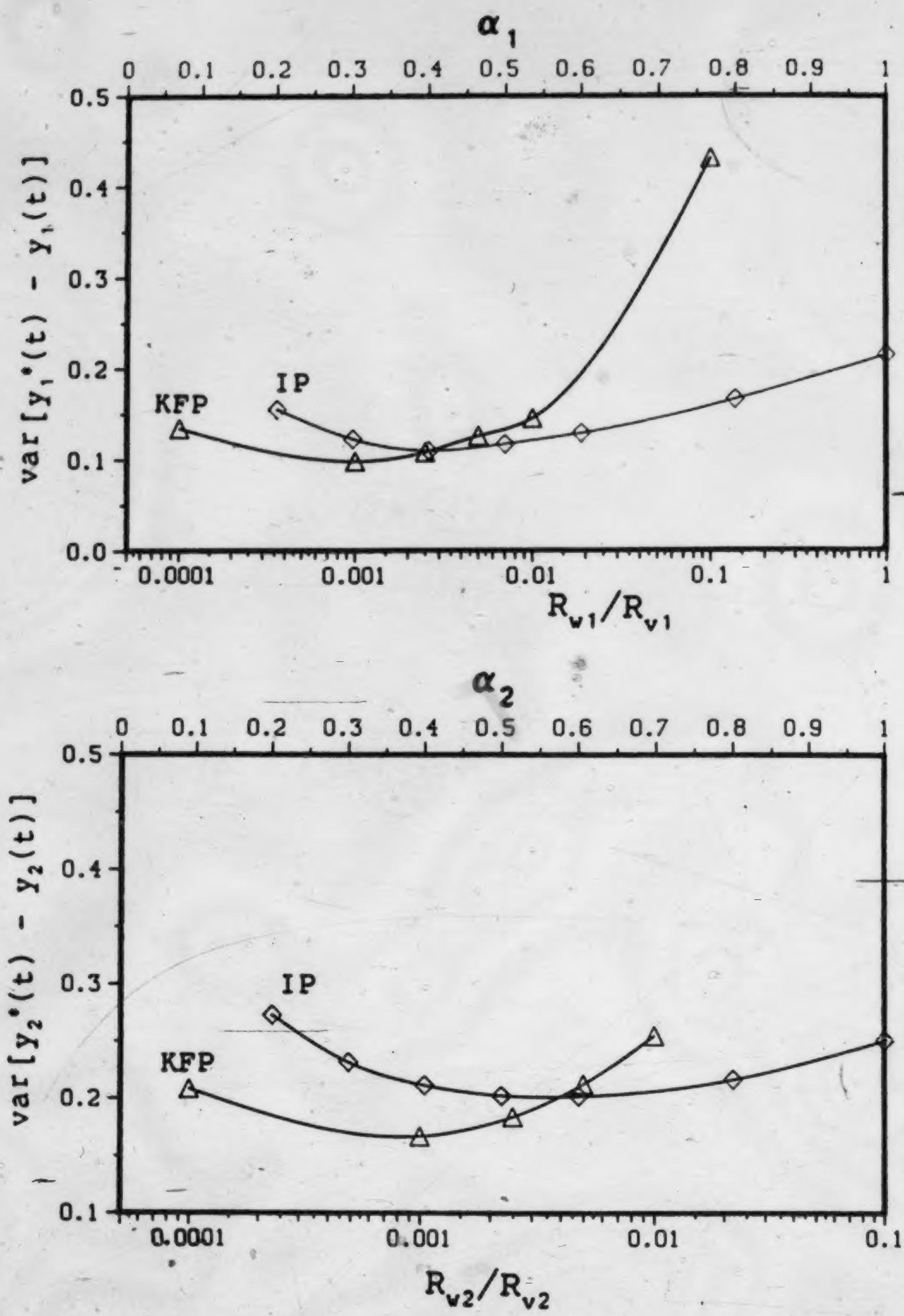
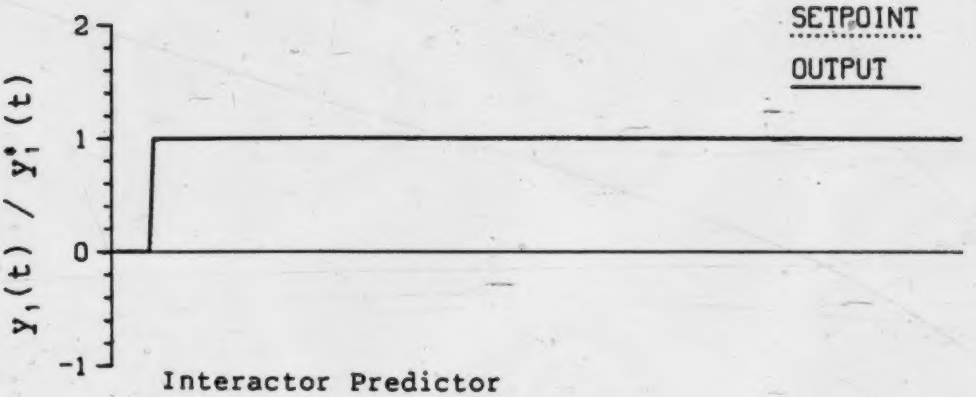
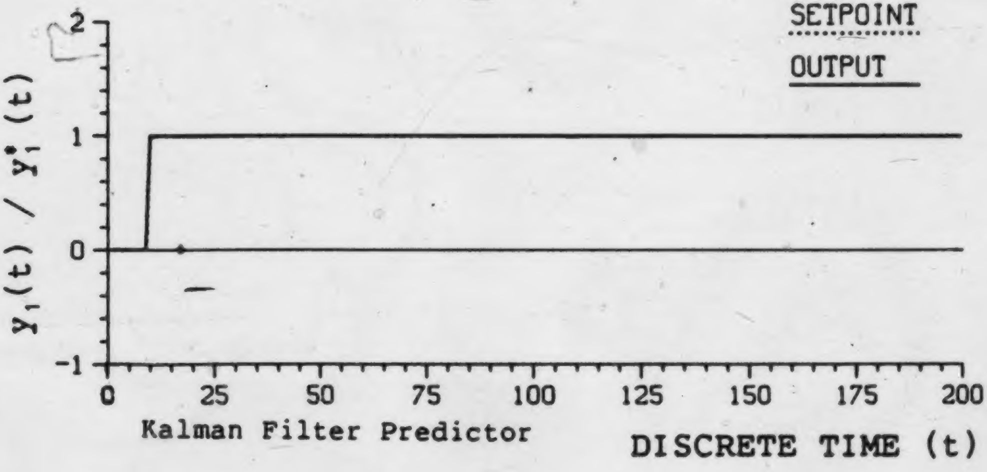


Figure 3.9e Variance of Tracking error for KFP and IP with an Exponential Filter.

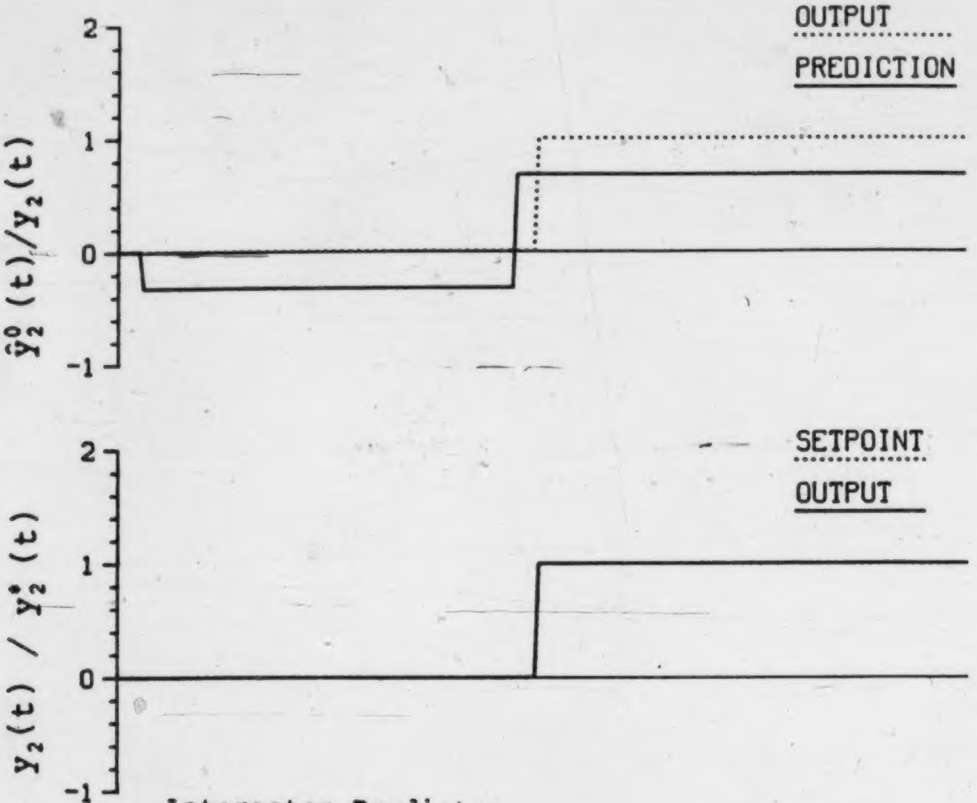


Interactor Predictor

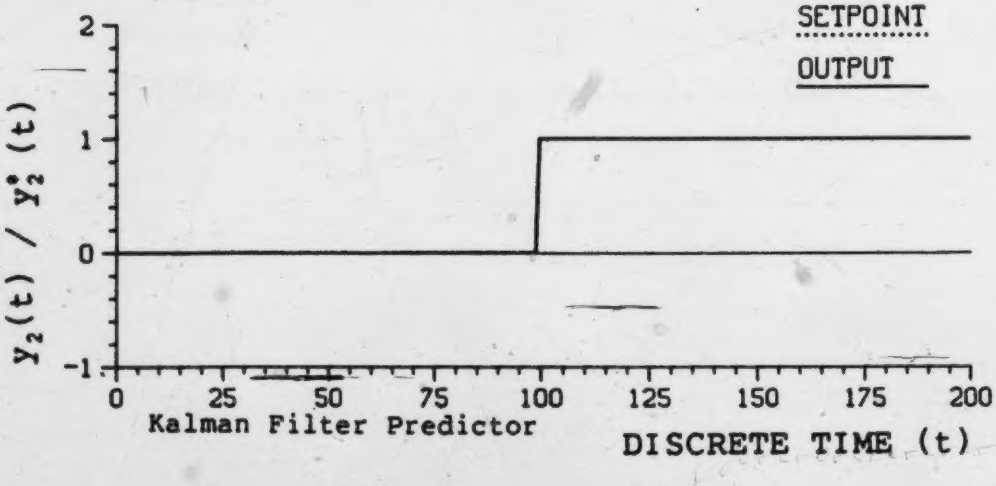


Kalman Filter Predictor

Figure 3.10a Predictive Control of y_1 using KFP and IP (Deterministic Process with a Triangular Interactor).

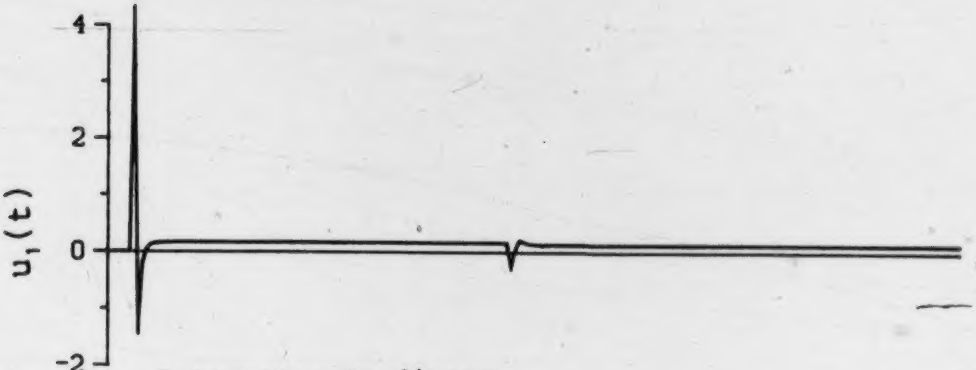


Interactor Predictor

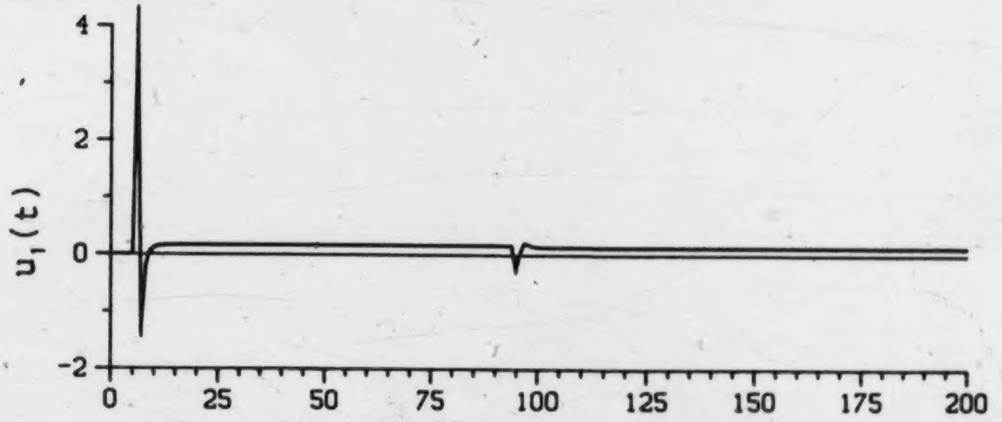


Kalman Filter Predictor

Figure 3.10b Predictive Control of y_2 using KFP and IP (Deterministic process with a Triangular Interactor).

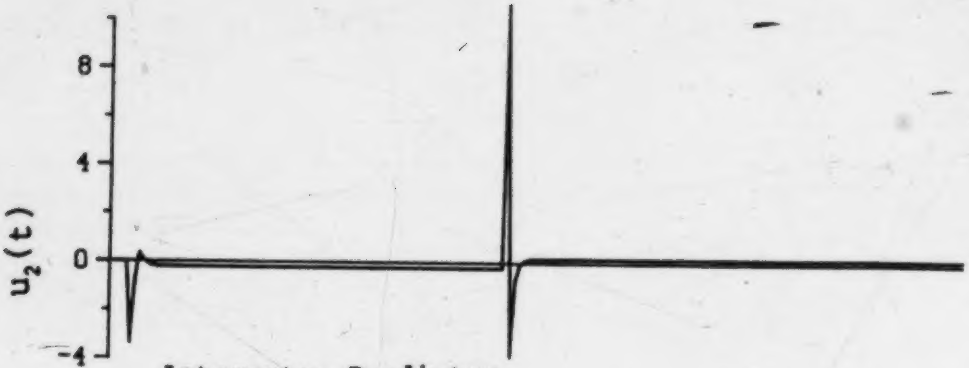


Interactor Predictor

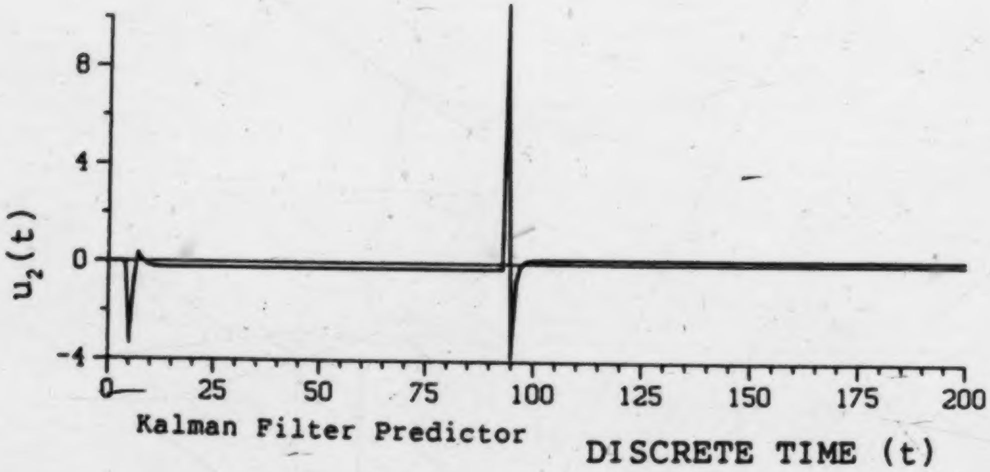


Kalman Filter Predictor

Figure 3.10c Predictive Control of a Deterministic process with a Triangular Interactor, using KFP and IP.

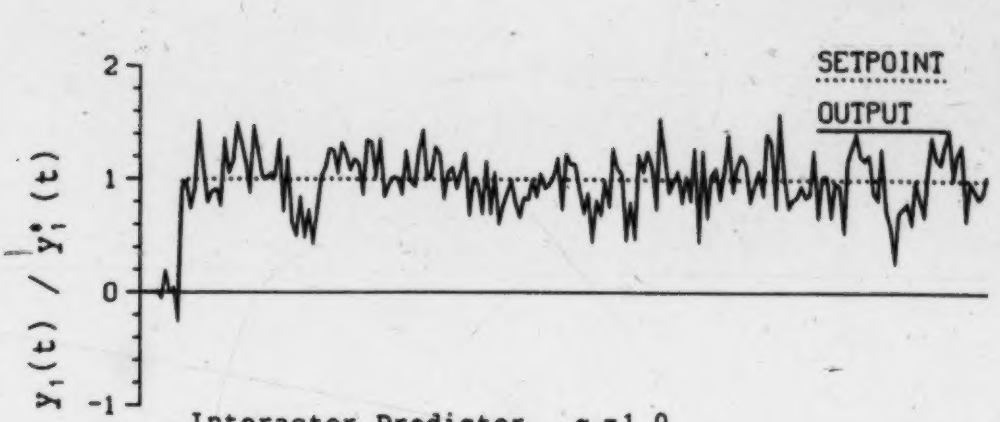


Interactor Predictor

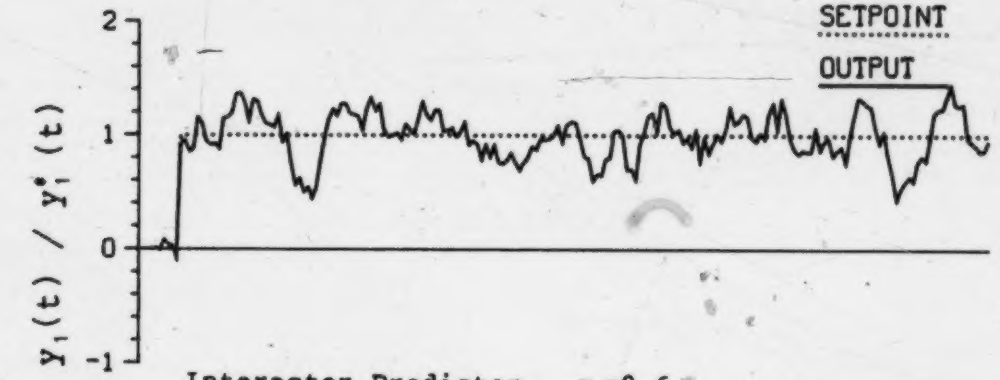


Kalman Filter Predictor

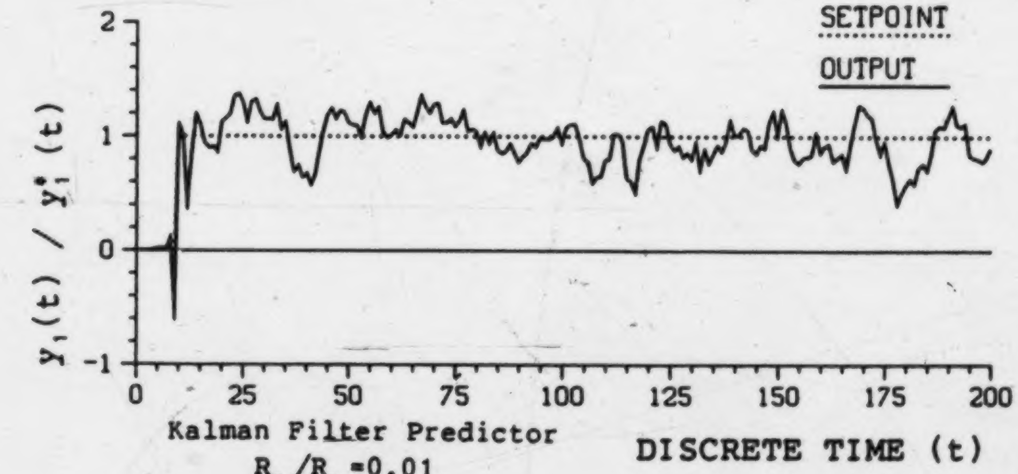
Figure 3.10d Predictive Control of a Deterministic process with a Triangular Interactor, using KFP and IP.



Interactor Predictor $\alpha_1 = 1.0$

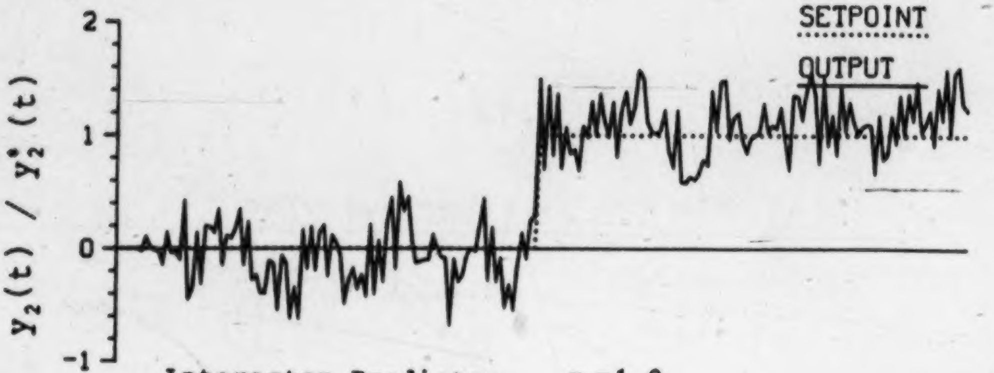


Interactor Predictor $\alpha_1 = 0.6$

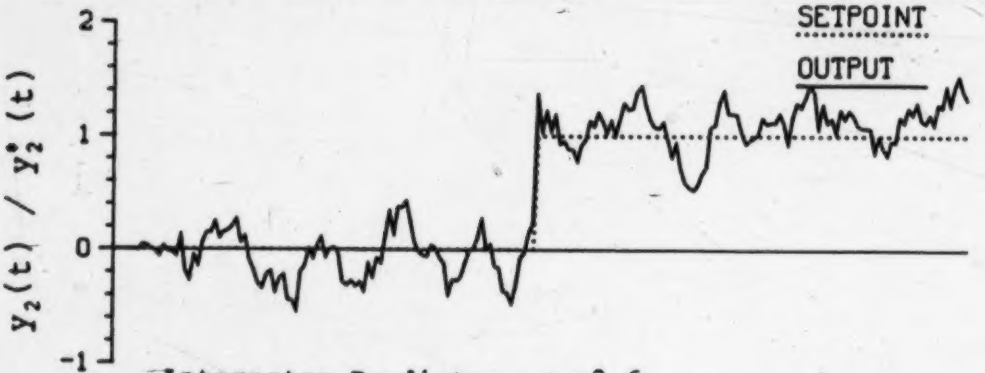


Kalman Filter Predictor
$R_{v_1}/R_{v_1} = 0.01$

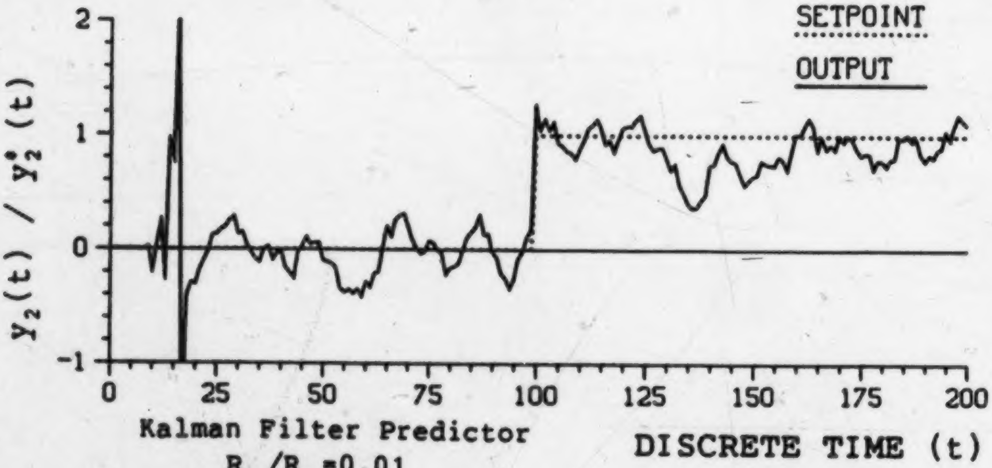
Figure 3.11a Predictive Control of y_1 using KFP and IP (Stochastic process with Triangular Interactor).



Interactor Predictor α_2=1.0

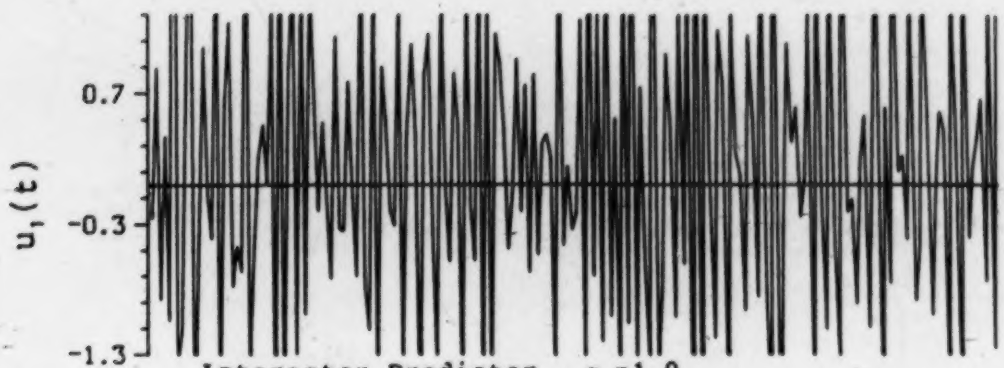


Interactor Predictor α_2=0.6

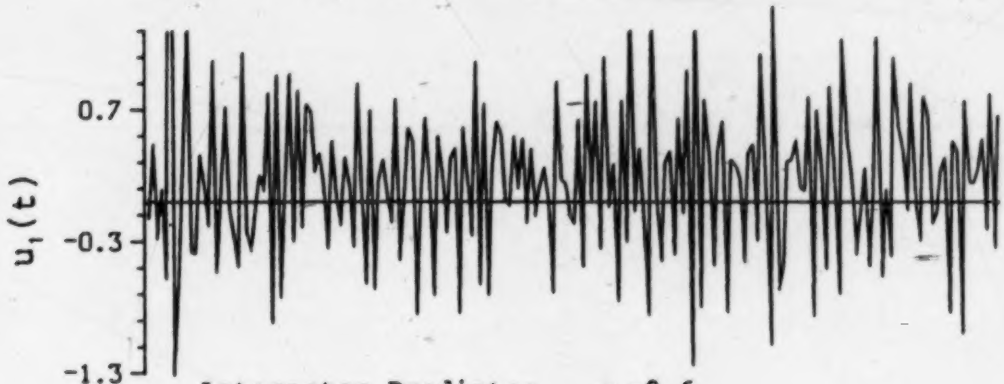


Kalman Filter Predictor
R_{v_2}/R_{v_2}=0.01

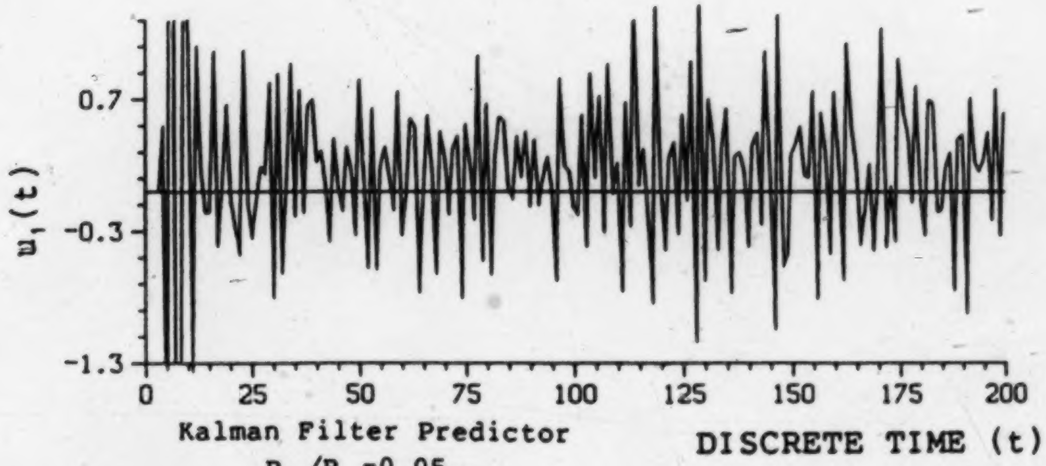
Figure 3.11b Predictive Control of y_2 using KFP and IP (Stochastic process with Triangular Interactor).



Interactor Predictor α_1=1.0

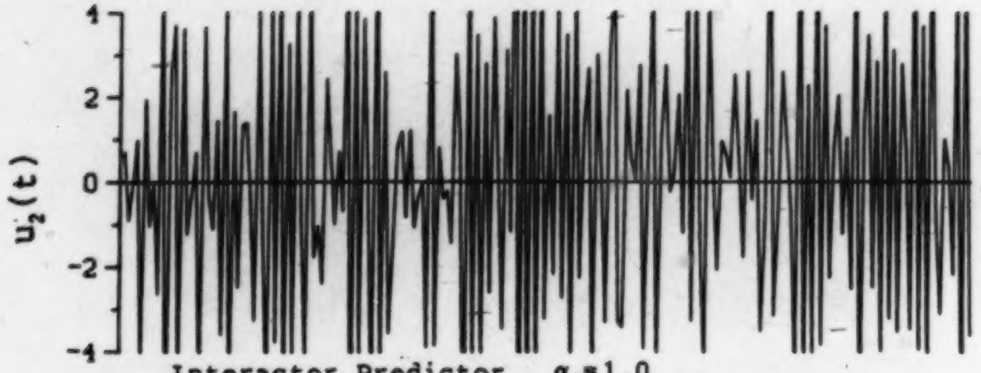


Interactor Predictor α_1=0.6

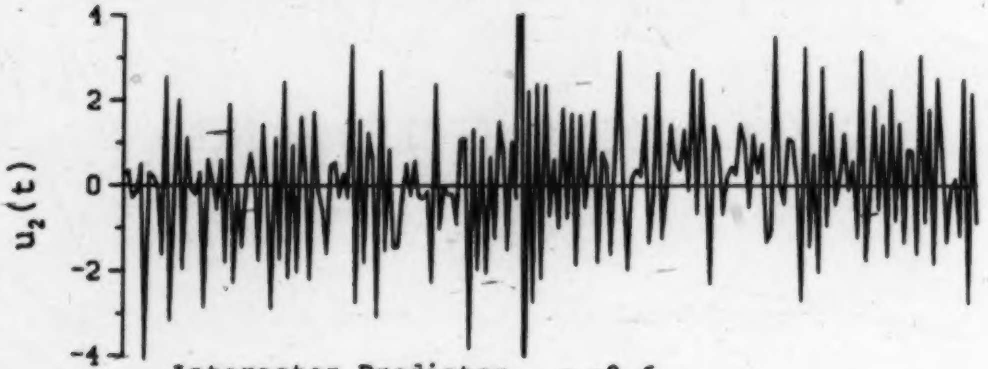


Kalman Filter Predictor
R_{v_1}/R_{v_1}=0.05

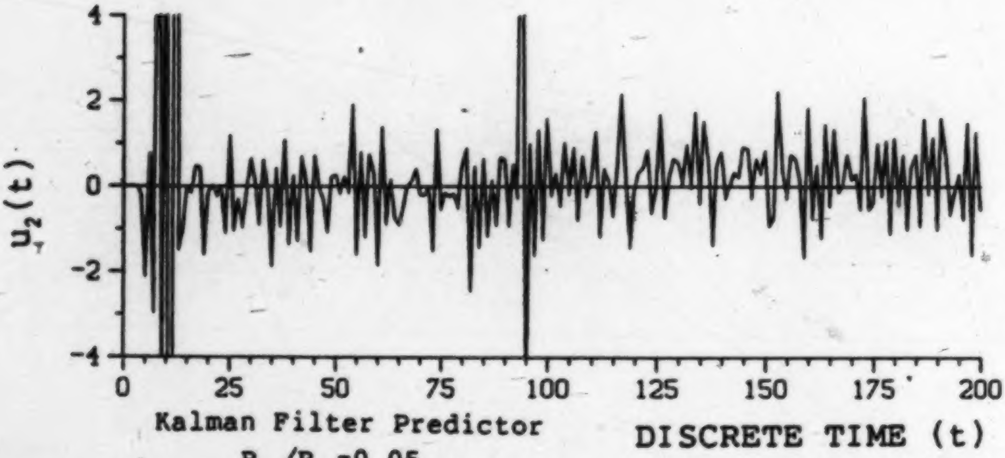
Figure 3.11c Predictive Control of the stochastic process with Triangular Interactor, using KFP and IP.



Interactor Predictor α_2=1.0



Interactor Predictor α_2=0.6



Kalman Filter Predictor
R_{v_2}/R_{v_2}=0.05

Figure 3.11c Predictive Control of the stochastic process with Triangular Interactor using KFP and IP.

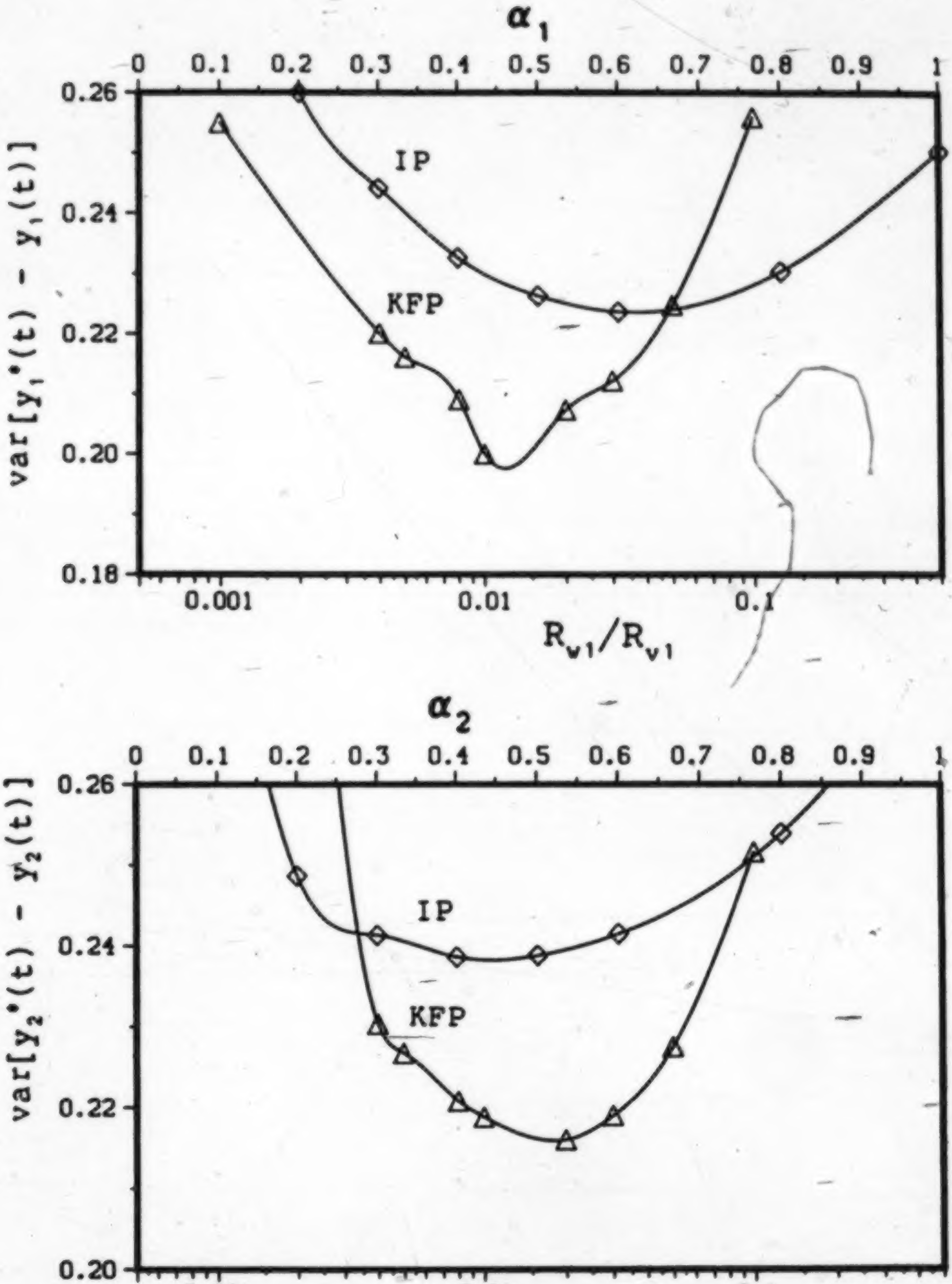


Figure 3.11e Variance of Tracking error for kFP and IP with an Exponential Filter.

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4. An Adaptive Kalman Filter Predictor

4.1 Introduction and Overview

The difficulties encountered in controlling processes with time delays have been documented extensively, and a large number of different schemes are available in the literature. Almost all of these schemes use some predictor technique as the key component, which in turn demands a priori knowledge of the parameters of the model. To overcome this difficulty, the parameters of the process model can be estimated on-line, using a suitable identification method, and the predictor is updated with the latest estimates of the parameters.

Most of the available dead time compensation schemes such as the Smith predictor (Smith, 1957, 1959) can be easily extended to adaptive versions if the order of the process and the time delays are accurately known. Other adaptive schemes which handle time delays are the self-tuning regulator of Astrom et al (1973) and the self-tuning controller of Clarke & Gawthrop (1975, 1979).

A dead time compensation scheme based on, a specific, discrete state space formulation to represent a process with time delays, and a Kalman filter (Kalman, 1960) was developed in Chapter 2. In this scheme the future outputs are treated as states of the process, and are estimated using the Kalman filter. The Kalman Filter Predictor (KFP), like all the other predictor schemes, requires a process model, which in turn demands a priori knowledge of the process parameters.

An adaptive extension of the above KFP, based on a Recursive Least Squares (RLS) identification scheme for parameter estimation and a Kalman filter for state estimation is given in Walgama,Shah and Fisher(1985). The state space model of the Kalman filter is updated at every sampling instant, with the current parameter estimates. A dead time compensation scheme, using the Adaptive Kalman Filter Predictor (AKFP) and a PID controller, for processes with uncorrelated output noise is presented in Walgama,Shah and Fisher(1985). This AKFP is extended to deal with the process noise and measurement noise. An Extended Least Squares (ELS) identification is used in the Adaptive Kalman Filter (AKF) to obtain the converged unbiased parameters of the process. The minimum variance predictive control scheme based on the Kalman Filter Predictor (KFP) as given in Chapter 2, is extended to an adaptive minimum variance predictive control scheme.

Using the innovation model, the adaptive Kalman filter predictor is interpreted as an adaptive Smith predictor with an additional time varying filter that filters the error between the process output and the model output. To implement the AKFP it is necessary to know the noise statistics a priori or to tune the Kalman filter using the ratio of the noise covariances as a tuning parameter. An approach that does not need the noise statistics or tuning is developed using the innovation model. The innovation model leads to an ARMAX representation of the Kalman filter predictor. By estimating the parameters of the ARMAX model, the Kalman gains are evaluated indirectly. These gains are then used to implement the time varying adaptive filter. This approach leads to a 'self-tuning' Kalman filter predictor for stochastioc processes with time delays.

The Kalman filter predictor assumes that the process noise and the measurement noise present in the system are uncorrelated. It is shown in Chapter 2 that the KFP may give biased estimates of the predicted outputs in presence of deterministic disturbances. This problem can be overcome by introducing an integrator into the already existing Proportional and Derivative (PD) estimation configuration. As suggested by Balchan et al (1970, 1973) and Bialkowski (1983), integral action is incorporated in the state space model, by augmenting the state vector with an additional state that corresponds to noise with integrator dynamics. A detailed analysis of this scheme is given in chapter 2. The above modified KFP is extended to an adaptive version by incorporating an incremental RLS identification scheme, based on an ARIMA representation of the process, (e.g. Tuff et al, 1985). The incremental adaptive KFP can be used with any feedback controller such as the conventional PID controller or incremental adaptive predictive controller.

4.2 Adaptive Kalman Filter Predictor (AKFP)

4.2.1 State Space Formulation

The adaptive Kalman filter predictor is developed assuming a linear discrete model for the process with time delays, as shown in figure 4.1, and assigning a state to each output of the unit delay blocks. A detailed discussion on the state space formulation is given in Chapter 2. Only the final state space formulation is presented here.

Let the n^{th} order SISO process with time delays be described by the ARMA representation.

$$y(t) = A^{-1}(q^{-1})\, B(q^{-1})\, u(t) \tag{4.2.1}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \cdot\ \cdot\ \cdot + a_n q^{-n} \tag{4.2.2}$$

$$B(q^{-1}) = q^{-(1+d)}[b_1 + b_1 q^{-1} + \cdot\ \cdot\ \cdot\ b_n q^{-n+1}] \tag{4.2.3}$$

d- is the time delay in the process excluding the discretization delay.

The state space representation obtained by augmenting the two observable state space representations, i.e. for the process without time delays and for the time delays, is given by,

$$x(t+1) = \Phi x(t) + \Lambda\, u(t) + \Gamma\, w(t) \tag{4.2.4}$$

$$y(t) = \Theta x(t) + v(t) \tag{4.2.5}$$

where

$$\Phi = \begin{bmatrix} \Phi_1 & 0 \\ \Theta_x & \Phi_2 \end{bmatrix}_{(n+d)x(n+d)} \qquad \Lambda = \begin{bmatrix} \Lambda_1 \\ 0 \end{bmatrix}_{1x(n+d)} \qquad \Gamma = \begin{bmatrix} \Gamma_1 \\ 0 \end{bmatrix}$$

$$\Theta = [\ 0 : \Theta\ ]_{1x(n+d)}$$

with

$$\Phi_1 = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & -a_n \\ 1 & 0 & \cdot & \cdot & \cdot & -a_{n-1} \\ 0 & 1 & \cdot & \cdot & \cdot & -a_{n-2} \\ \cdot & & & & & \\ \cdot & & & & & \\ 0 & \cdot & \cdot & 1 & 0 & -a_2 \\ 0 & \cdot & \cdot & 0 & 1 & -a_1 \end{bmatrix}_{nxn}$$

$$\Phi_2 = \begin{bmatrix} 0 & 0 & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & 0 \\ \cdot & & & & \\ 0 & 0 & \cdot & 1 & 0 \end{bmatrix}_{dxd} \qquad \Lambda_1 = \begin{bmatrix} b_n \\ \cdot \\ \cdot \\ \\ b_1 \end{bmatrix}_{nx1} \qquad \Gamma_1 = \begin{bmatrix} \gamma_1 \\ \cdot \\ \cdot \\ \\ \gamma_n \end{bmatrix}_{nx1}$$

$$\Theta_x = \begin{matrix} \Theta_1 \\ 0 \end{matrix} \qquad \Theta_1 = [0 \;\cdot\; \cdot\; 1]_{1xn} \qquad \Theta_2 = [0 \; 0 \;\cdot\; \cdot\; 1]_{dxd}$$

$$x(t) = [x_1, x_2, \cdot \cdot, x_n, x_{n+1}, \cdot \cdot, x_{n+d}]^T$$

w(t) - process noise

v(t) - measurement noise

w(t) and v(t) are assumed to be white noise sequences with zero mean and covariances defined by $E[w(s)\, w(s)^T] = R_w$ and $E[v(s)\, v(s)^T] = R_v$ respectively.

The states x_{n+1} to x_{n+d} are the delayed versions of x_n, which is the output of the process without time delays.

The dimension of the state vector depends on the order of the process, and on the time-delays, and hence on the sampling time used for the discretization. Notice that the state transition matrix Φ is a singular matrix.

4.2.2 Adaptive Kalman Filter Predictor

The states $x_n(t)$ to $x_{n+d-1}(t)$ of the state space formulation (equations 4.2.4 and 5) are the future outputs y(t+d) to y(t+1) respectively. Thus by estimating the states of the formulation (equations 4.2.4 and 5), using any type of state estimator, the future outputs up to time t+d can be predicted at time t. However considering the stochastic disturbances present in the process, a stochastic estimation scheme such as the Kalman filter should be employed to obtain optimal estimates of the states, i.e. optimal predictions of the future outputs.

If the process parameters of the state space model are known the implementation of the above KFP is straight forward. A detail discussion on the KFP is given in chapter 2. When the process parameters are not known apriori, except for an upper bound on the order and the time delay of the process, then an identification scheme has to be employed to estimate the process parameters on-line. Thus it is necessary to use an AKF scheme that does both parameter and state estimation.

A commonly used AKF is the Extended Kalman Filter(EKF). In this scheme the state vector is augmented with the unknown parameter vector. This leads to a non-linear Kalman filtering problem due to the occurrence of products between parameters and states. It has been observed from simulations and practical applications that the EKF algorithm may give biased estimates, and may sometimes diverge. The convergence properties of the EKF has been investigated by Ljung (1979), who has demonstrated that the convergence difficulties may arise due to a combination of factors such as incorrect specification of noise covariances and the dependence of the

Kalman gains on the parameter estimates. An improved EKF algorithm that would give converged parameters was proposed by Ljung (1979).

Another approach to the AKF is to use sequential prediction error parameter estimation and the Kalman filter as given in Goodwin & Sin (1984). This parameter estimator uses the estimated output from the Kalman filter in its regressor vector. Estimated parameters are used in the KF for state estimation. A disadvantage of this approach is, that it does not guarantee good estimates of the complete state vector. The best one can expect is that the estimated output of the KF will approach the true system output asymptotically. Good estimates of the states can be obtained, only if the estimated parameters are a good estimates of the true parameters.

The adaptive Kalman filter used in this report consists of an independent identification scheme and a Kalman filter scheme that uses the updated parameters from the identification scheme. The schematic block diagram of the AKFP based on the above AKF is shown in figure 4.2. The parameter identification scheme uses only the process input and output data to estimate the parameters. The state space model used in the Kalman filter is updated using the current estimates of the parameters, at each sampling instant.

An adaptive Kalman filter as suggested above can be used to estimate the states, thus the future output of the process, i.e. $\hat{y}(t+d|t)$, · ·, $\hat{y}(t+1|t)$. It is important to note that the parameters of the ARMA model given by the equation (4.2.1) are explicitly present in state space model given by equation (4.2.4 and 5), and that they are the only non unity, non zero elements of the matrices Φ and Λ of the state space model. This facilitates the development of a direct AKF, without having to do any intermediate calculations or linear transformations.

The time varying Kalman filter can be implemented either by using the algorithm given in Appendix A or E. However the algorithm given in Appendix E, for the KFP, is more computationally efficient, and hence is more suitable for the adaptive Kalman filter predictor applications.

4.2.3 Parameter Estimation

To obtain good estimates of the states, i.e. future outputs, it is important to obtain converged, true estimates of the process parameters. For deterministic processes and for a class of stochastic processes a Recursive Least Squares (RLS) identification can be used to obtain the converged true parameters of the process i.e.

deterministic discrete processes represented by the DARMA model,

$$A\ y(t) = B\ u(t) \tag{4.2.6}$$

or stochastic discrete represented by the ARMAX model

$$A\ y(t) = B\ u(t) + \xi(t) \tag{4.2.7}$$

where $\xi(t)$ is a white noise sequence and A and B polynomials as defined in equation (4.2.2 and 3).

However, in the general case when there is process noise and/or measurement noise , i.e. when the $C(q^{-1})$ of the ARMAX representation,

$$A(q^{-1})\, y(t) = B(q^{-1})\, u(t) + C(q^{-1})\xi(t) \qquad (4.2.8)$$

is not unity, it is necessary to use a stochastic parameter estimation algorithm, to obtain the true parameters of the process. One such stochastic parameter estimation scheme is the sequential predictor error algorithm, which is commonly known in literature as the recursive maximum likelihood version 2 (RML2), (Goodwin and Sin, 1984). One of the problems with this method is that the input/output data is passed through a time varying filter having its denominator polynomial depend on the parameter estimation. If the roots of this polynomial are not assured to be within the unit circle divergence problems can occur. An additional test, such as Jury's test (see Kuo, 1980) has to be used to keep the roots of the polynomial inside the unit circle.

The identification method used in this work is the commonly known Recursive Extended Least Square (RELS) algorithm presented by Penuska (1968, 1969) and Young (1968). This is also called RML1 (Soderstrom, Ljung and Gustavsson, 1978), and extended matrix method (Talman and vander Boom, 1973). A theoretical analysis of RELS method along with other recursive identification methods is given in Soderstrom, Ljung and Gustavsson (1978), and a convergence analysis is given in Ljung (1978). The extended least squares method is not as robust as RLS. As shown by

Ljung (1978), a theoretical short coming of the ERLS is that it cannot be proved that the estimate of the C polynomial in (4.2.8) will converge for all cases.

Extended Least Squares algorithm:

Consider the ARMAX model given by equation (4.2.8),

$$A(q^{-1})\ y(t) = B(q^{-1})\ u(t) + C(q^{-1})\xi(t) \tag{4.2.8}$$

where

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdot \cdot + a_nq^{-n} \tag{4.2.9}$$

$$B(q^{-1} = q^{-(1+d)}[b_1+b_2q^{-1}+\cdot\cdot\cdot+b_nq^{-n+1}] \tag{4.2.10}$$

$$C(q^{-1}) = 1 + c_1q^{-1} + \cdot \cdot + c_lq^{-l} \tag{4.2.11}$$

$A(q^{-1})$ and $C(q^{-1})$ are monic by definition, and have their zeros inside the unit circle. The polynomials A and B or A and C do not have any common factors. A theoretical analysis would also need a persistently exciting input signal for the convergence of the parameters.

The regressor vector is,

$$\psi(t-1)^T = [y(t-1),\ y(t-2),\ \cdot\ \cdot,\ y(t-n),\ u(t-d-1),\ \cdot\ \cdot\ u(t-n-d),\ e(t-1)\ \cdot\ \cdot\ e(t-l)] \tag{4.2.12}$$

The parameter vector is,

$$\theta = [a_1,\ \cdot\ \cdot,\ a_n,\ b_1,\ \cdot\ \cdot,\ b_n,\ c_1,\ \cdot\ \cdot\ c_l] \tag{4.2.13}$$

and hence

$$y(t) = \psi^T(t-1)\theta + e(t) \tag{4.2.14}$$

If $\hat{y}(t|t-1)$ is the a priori predicted output and $\hat{\theta}(t)$ is the estimated parameter vector, then,

$$\hat{y}(t|t-1) = \psi^T(t-1)\ \hat{\theta}(t-1) \tag{4.2.15}$$

Gain Calculation

$$G(t) = \frac{P(t-2)\psi(t-1)}{\mu+\psi^T(t-1)P(t-2)\psi(t-1)} \qquad (4.2.16)$$

Parameter estimation

$$\hat{\theta}(t) = \hat{\theta}(t-1) + G(t)\,[y(t) - \hat{y}(t|t-1)] \qquad (4.2.17)$$

Covariance Update

$$P(t-1) = \frac{1}{\mu}\,[I - G(t)\psi^T(t-1)]\,P(t-2) \qquad (4.2.18)$$

The prediction error e(t) can be calculated in two ways: the a priori predicton error,

a. $$e(t) = y(t) - \hat{y}(t|t-1) = y(t) - \psi^T(t-1)\,\hat{\theta}(t-1) \qquad (4.2.19)$$

or the a posteriori prediction error.

b. $$e(t) = y(t) - \hat{y}(t|t) = y(t) - \psi(t-1)^T\hat{\theta}(t) \qquad (4.2.20)$$

This was first proposed by Young (1974). The idea is to use the most recent estimates of the parameters.

An important fact that has to be recognized is, that a guarantee of convergence of the process parameters for a stochastic process is obtained at the expense of estimating additional noise parameters. These estimated noise parameters are not used anywhere else in the scheme.

4.3 Innovation Model Approach for the Adaptive Kalman Filter Predictor

The configuration of the KFP and some analytical results were investigated using the innovation model approach, in Chapter 2. In this section the innovation model ideas are applied to the adaptive version of the KFP.

The results obtained in Chapter 2 can directly be applied to the AKFP. The only different is the parameters $a_1, \cdot \cdot, a_n$ and $b_1 \cdot \cdot b_n$ of Φ and Λ are now the estimated values from the parameter identification scheme. Thus, Φ and Λ are now time varying. However for convenience, the same time invariant notations as used in Chapter 2 are used here.

From Chapter 2, equation (2.4.1) the innovation model for the AKFP is given by,

$$\hat{x}(t+1) = \Phi\hat{x}(t) + \Lambda u(t) + L(t+1)\,[y(t+1) - \Theta\Phi\hat{x}(t) - \Theta\Lambda u(t)] \tag{4.3.1}$$

where

$$\Theta\Phi = [0\ 0\ \cdot\ \cdot\ 1\ 0]$$

$$\Theta\Lambda = [0] \qquad \text{if } d\geq 1$$

From (C.2) the predicted current output is,

$$\hat{y}(t) = \hat{y}(t|t-1) = \Theta\Phi\hat{x}(t-1) + \Theta\Lambda u(t-1) = \hat{x}_{n+d-1}(t-1) \tag{4.3.2}$$

And the innovation sequence $\omega(t)$ is defined as,

$$\omega(t) = y(t) - \hat{y}(t) \tag{4.3.3}$$

The predicted value of the output of the process without time delays, i.e. $\hat{y}_1(t) = \hat{y}(t+d|t) = \hat{x}_n(t)$, is given by equation (2.4.21).

$$\hat{y}(t+d|t) = q^{-1}A^{-1}(q^{-1})B^1(q^{-1})u(t) + \frac{K_1(t,q^{-1})}{C(t,q^{-1})}[y(t) - q^{-(d+1)}A^{-1}(q^{-1})B^1(q^{-1})u(t)] \tag{4.3.4}$$

where

$$K_1(t,q^{-1}) = L_n(t) + L_{n-1}(t-1)q^{-1} + \cdot\ \cdot + L_1(t-n+1)q^{-n+1} \tag{4.3.5}$$

$$K_2(t,q^{-1}) = L_{n+d-1}(t) + L_{n+d-2}(t-1)q^{-1} + \cdot\cdot$$
$$+ L_{n+1}(t-d+2)q^{-d+2} \qquad (4.3.6)$$

From equation (2.2.13),

$$A(q^{-1})\, y(t) = B(q^{-1})u(t) + C(t,q^{-1})w(t) \qquad (4.3.7)$$

where

$$C(t,q^{-1}) = A(q^{-1})[1+q^{-1}K_2(t-1,q^{-1})+q^{-d}A^{-1}(q^{-1})K_1(t-d,q^{-1})]$$
$$= 1+c_1(t)q^{-1}+c_2(t)q^{-2}+ \cdot\cdot + c_{n+d-1}(t)q^{-d-n+1}$$
$$(4.3.8)$$

Now define the following transfer functions,

$$G_m(t,q^{-1}) = q^{-(d+1)}\, A^{-1}(q^{-1})B'(q^{-1})$$
$$(4.3.9)$$

$$G_p(t,q^{-1}) = q^{-1}\, A^{-1}(q^{-1})\, B'(q^{-1})$$
$$(4.3.10)$$

and

$$G_r(t,q^{-1}) = \frac{K_1(t,q^{-1})}{C(t,q^{-1})} \qquad (4.3.11)$$

The transfer functions G_m and G_p are time varying because the A and B polynomials are time varying.

Substituting (4.3.9, 10 and 11) in (4.3.4) we get,

$$\hat{y}(t+d|t) = G_p(t,q^{-1})u(t)$$
$$+ G_r(t,q^{-1})[y(t) - G_m(t,q^{-1})u(t)] \qquad (4.3.12)$$

If we define the variables,

$$y_m(t) = G_m(t,q^{-1})\, u(t) \qquad (4.3.13)$$

$$y_p(t) = G_p(t,q^{-1})\, u(t) \qquad (4.3.14)$$

Equation (4.3.12) can be rewritten as,

$$\hat{y}(t+d|t) = y_p(t) + G_r(t,q^{-1})\, [y(t) - y_m(t)] \qquad (4.3.15)$$

Equations (4.3.13, 14 and 15) can be used to obtain an internal configuration for the AKFP. A schematic block diagram of the AKFP is shown in figure 4.3.

Since $G_m(t,q^{-1})$ is the model of the process, and $G_p(t,q^{-1})$ is the process model without time delays, it is clear from figure 4.3 that the internal structure of the AKFP has the same configuration as the Adaptive Smith predictor (ASP) , as shown in figure 4.4.However the AKFP has an additional time varying filter $G_F(t,q^{-1})$, which filters the error between the process output and the model output.

The process model G_m and predictor model G_p are time varying, because the parameters of these two models are updated at every sampling instant by the identification algorithm. The filter G_F is time varying due to the parameter identification and the time varying Kalman filter gains. The filter G_F can be interpreted as an adaptive filter based on the Kalman filter.

4.3.1 Comments on the stability and the convergence of the Adaptive Kalman Filter Predictor

At present we are interested in the convergence and stability of the AKFP, but not of the closed loop system that uses this predictor.

Convergence

Since the AKFP is implemented by combining an independent identification scheme with a Kalman filter, the convergence of the AKF depends mainly on the convergence of the identification scheme, especially the convergence of the polynomials $A(q^{-1})$ and $B(q^{-1})$ to their true values.

The convergence properties of the extended least squares identification method has been investigated by Ljung (1977), Solo (1979), and also a good account of their analysis is given in Goodwin and Sin (1984).

The parameters estimated by the extended least squares algorithm will converge to their true values, i.e. $\lim_{t \to \infty} \hat{\theta}(t) = \theta_0$, if the following conditions are satisfied:

1. Stability assumption

Both $A(q^{-1})$ and $C(q^{-1})$ are stable polynomials.

2. Persistent Excitation Condition

$$\lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} \psi(t-1)\psi(t-1)^T = R \qquad (4.3.16)$$

exists and is positive definite, where,

$$\psi(t-1)^T = [-y(t-1), \cdot \cdot, -y(t-n), u(t-d-1)$$
$$\cdot \cdot \, u(t-d-n), e(t-1), \cdot \cdot \, e(t-1)]$$

3. Passivity Condition

$[\frac{1}{C(q^{-1})} - \frac{1}{2}]$ is very strictly passive, i.e.

$$\{Re[\frac{1}{C(e^{jw})} - \frac{1}{2}] > 0; \; -\pi \le w \le \pi\} \qquad (4.3.17)$$

If the RELS gives converged true parameters, it is possible to guarantee that the KF algorithm will converge to the steady state Kalman filter. Since the parameter identification is independent of the state estimation in the KF, irrespective of the behaviour of the KF, the RELS will give converged true parameters, if the conditions for convergence are satisfied. Referring to figure 4.3,

$$\lim_{t \to \infty} \hat{\theta}(t) = \theta_0$$

will guarantee that,

$$\lim_{t \to \infty} G_m(t,q^{-1}) = G_m(q^{-1})$$

$$\lim_{t \to \infty} G_p(t,q^{-1}) = G_p(q^{-1})$$

$$\lim_{t \to \infty} G_r(t,q^{-1}) = G_r(q^{-1})$$

Stability

Stability of the AKFP is quite important because of the possibility that it could go unstable, simply as a predictor, not to mention closed loop stability. From figure 4.3, it is clear that the stability of the AKFP, i.e. boundedness of the predicted output, depends on the transfer functions G_m, G_p and G_r. The time varying adaptive filter G_r is important for the stability analysis. If the $C(q^{-1})$ (equation 4.3.8) polynomial, evaluated by the parameter estimation and the Kalman filter algorithm has its roots outside the unit circle, then G_r is a unstable system. The asymptotic convergence of G_r to its steady state filter is guaranteed, if the parameter identification converges to its true values and the stabilizability and detectability conditions as given in Chapter 2 are satisfied. This would also guarantee a stable filter G_r. But until the parameters have converged yet, the stability of the filter G_r and hence of the AKFP, is not guaranteed.

4.4 A 'Self-Tuning' Kalman Filter Predictor

The AKFP discussed in the previous section requires a priori knowledge of the noise statistics in order to tune the ratio of the noise covariances $R_v|R_v$ to satisfy a certain performance criterion. A 'self-tuning' KFP can be developed by using the innovation model approach.

Consider the equation (4.3.8), which was derived using the innovation model.

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})\omega(t) \tag{4.4.1}$$

where

$$C(q^{-1}) = 1+C_1q^{-1}+\cdot\cdot+C_{n+d-1}q^{-n-d+1} \tag{4.4.2}$$

$$A(q^{-1}) = 1+a_1q^{-1}+\cdot\cdot+a_nq^{-n} \tag{4.4.3}$$

$$B(q^{-1}) = q^{-(d+1)}[b+b_1q^{-1}+\cdot\cdot+b_{n-1}q^{-(n-1)}] \tag{4.4.4}$$

and the coefficients of C are given by,

$$C(q^{-1}) = A(q^{-1})+q^{-1}A(q^{-1})K_2(t-1,q^{-1})+q^{-d}K_1(t-d,q^{-1}) \tag{4.4.5}$$

From equation (4.4.5) we get:

Case 1. $n>(d-1)$

for $i\le d-1$

$$c_i(t) = \sum_{l=0}^{i} L_{n+d-l}(t-l)a_{i-l} \tag{4.4.6}$$

with $a_0=1$ and $L_{n+d}=1$

for $n\ge i>d-1$

$$c_i(t) = \sum_{l=0}^{d-1} [L_{n+d-l}(t-l)a_{i-l}]+L_{n+d-1}(t-i) \tag{4.4.7}$$

for $n+d-1 \geq l > n$

$$c_i(t) = \sum_{l=i-n}^{d-1} [L_{n+d-l}(t-1)a_{i-l}] + L_{n-i+d}(t-i) \tag{4.4.8}$$

Case 2. $n=d-1$

for $i \leq d-1$

$$c_i(t) = \sum_{l=0}^{i} L_{n+d-l}(t-1)a_{i-l} \tag{4.4.9}$$

for $n+d-1 \geq i > d-1$

$$c_i(t) = \sum_{l=0}^{n} [L_{n+d-l}(t-1)a_{i-l}] + L_{n-i+d}(t-i) \tag{4.4.10}$$

Case 3. $n<(d-1)$

for $i < n$

$$c_i(t) = \sum_{l=0}^{i} L_{n+d-l}(t-1)a_{i-l} \tag{4.4.11}$$

for $d-1 \geq i \geq n$

$$c_i(t) = \sum_{l=0}^{n} L_{n+d-l}(t-1)a_{i-l} \tag{4.4.12}$$

for $i \geq d$

$$c_i(t) = \sum_{l=i-d+1}^{n} [L_{n+d-l}(t-1)a_{i-l}] + L_{n-i+d}(t-1) \tag{4.4.13}$$

If the coefficients c_1 to c_{n+d-1} are known the Kalman gains L_n to L_1 can be calculated sequentially using equations (4.4.6-13). Thus it suggests a method of calculating the Kalman gains indirectly by estimating the C polynomial.

Since equation (4.4.1) is in ARMAX form, a recursive extended least squares algorithm can be used to estimate the parameters of the A, B and C. The estimated parameters are used in equation (4.4.6-13) to solve for the Kalman gains. Thus, instead of using the Kalman filter algorithm, which requires knowledge of the noise statistics, the Kalman gains are calculated directly from the parameter identification method. The evaluated Kalman gains are used to implement the adaptive filter G_F of the KFP, as given in figure 4.3. This will give the predicted output $\hat{y}(t+d|t)$. But if it is necessary to predict all the future outputs up to y(t+d) and other states, then the Kalman gains can be directly used in the innovation model given by equation (4.3.1).

$$\hat{x}(t+1) = \Phi\hat{x}(t) + \Lambda u(t) + L(t+1)[y(t) - \Theta\Phi\hat{x}(t) - \Theta\Lambda u(t)] \qquad (4.3.1)$$

This method leads to a "Self-Tuning" Kalman Filter Predictor (STKFP).

4.4.1 Comments on the "Self-Tuning" Kalman Filter Predictor

1. The order of the C polynomial in equation (4.4.1) is n+d-1, and is greater than the order of the A and B polynomials. The order of C polynomial depends on the time delay d. For very large dead times this method may not be suitable because of the large number of parameters that must be estimated. However, it is important to note that compared to other implicit optimal predictor methods (Goodwin and Sin, 1984),

which use the Diophantine identity to obtain the optimal predictor equation, this method still has less parameters to identify.

2. The time varying Kalman gains depend on the process input and output measurements, and on the convergence properties, especially the convergence rate of the identification method. In the AKFP scheme described in section 4.3, once the parameters of the A and B polynomials have converged, the KF converges exponentially fast to the steady state KF, but in the STKFP scheme the KF will converge only after the C polynomial has converged. The convergence rate of C polynomial is usually slower for a identification method like RELS, which uses a pseudo linear regressor vector $\psi(t)$ which depends on the parameter estimation $\hat{\theta}(t)$. If the parameter estimation converges to the true values then the STKFP will give minimum variance prediction.

3. The stability of the STKFP depends mainly on the stability of the time varying adaptive filter G_r. The denominator polynomial of G_r is the polynomial $C(q^{-1})$, and the stability of G_r thus depends on the roots of $C(q^{-1})$. Unlike in the AKFP, in the STKFP the C polynomial is obtained from the parameter identification algorithm. It may be necessary to impose constraints on the parameter estimation of the C polynomial such that C is always stable.

A condition that is necessary in parameter identification is that C is a stable polynomial. It is shown in Chapter 2, that $C(q^{-1})$ is a stable polynomial if the stabilizability and detectability is guaranteed. Although this assures that the STKFP is asymptotically stable, it does not guarantee the stability before the convergence. This might cause divergence in the predicted output. A way to get around this problem is to obtain the C polynomial coefficients off-line and use them as initial conditions.

4.5 Feedback Control using the Adaptive Kalman Filter Predictor

The predicted future outputs, other estimated states and the estimated parameters from the AKFP or STKFP can be used in any type of controller design. The AKFP provides an explicit identification of the process parameters. Therefore any controller design can be done assuming that the true parameters, states and the future outputs are known, and use the estimated values in the actual calculations. In the next two sections the PID and adaptive predictive feedback control schemes are discussed.

4.5.1 PID Control

A PID feedback controller can be easily implemented for the non-adaptive version of the KFP as given in Chapter 2. However it poses a number of questions when used in conjunction with the AKFP scheme. If the model parameters are totally unknown or time varying, it may be difficult to obtain an initial set of controller parameters . If a reasonable model of the process is available the initial controller parameters can be calculated for that model.

A schematic block diagram of the feedback control using AKFP is shown in figure 4.2.

The tracking error is given by,

$$e(t) = y^*(t) - \hat{y}(t+d|t) \qquad (4.5.1)$$

and PI control algorithm is given by,

$$u(t) = K_p[e(t) + \frac{T}{\tau_I}\sum_{i=0}^{t} e_i(t)] \qquad (4.5.2)$$

where

T - Sampling time

K_p - Proportional gain

τ_I- Integral reset time.

4.5.2 Adaptive Predictive Control

The adaptive Kalman filter predictor is a model based predictor, which uses a model that is being identified on-line. The knowledge of the process model parameters can be employed to design a controller. This leads to an adaptive control scheme.

It is shown in Chapter 2, that the control law that minimizes the variance of the tracking error, i.e. that minimizes the cost function:

$$J(t+d+1) = E\{[y(t+d+1)-y^*(t+d+1)]^2\}$$

$$(4.5.3)$$

where

$y^*(t+d+1)$ - desired output,

leads to a predictive control law given by the following equations:

$$u(t) = \frac{1}{b_1}\{y^*(t+d+1) - [1-A(q^{-1})]\hat{y}(t+d+1) - [B(q^{-1})- b_1]u(t)\} \tag{4.5.4}$$

where $\hat{y}(t+d+1)$ is the predicted output of the KFP.

Applying the certainity equivalence concept to this controller, the predictive control law given by equation (4.5.4) can be extended to its adaptive version by replacing the parameters of A and B by the estimated parameters $\hat{A}(q^{-1})$ and $\hat{B}(q^{-1})$. Thus the adaptive predictive control law based on the adaptive Kalman filter predictor is given by,

$$u(t) = \frac{1}{\hat{b}_1}\{y^*(t+d+1) - [1-\hat{A}(q^{-1})]\,\hat{y}(t+d+1) - [\hat{B}(q^{-1}) - \hat{b}_1]\}u(t) \tag{4.5.5}$$

This control law can be used with both AKFP and STKFP. It is important to note that in both cases the C polynomial estimated in the identification is not used for the control law calculations.

4.5.3 Comments on the Adaptive Kalman Filter Predictive Control (AKFPC) scheme

1. The number of parameters that has to be estimated in this scheme is less than in the implicit adaptive minimum variance controller, that uses the Diophantine identity to obtain the predictor. This can be a significant advantage when there are large time delays

present in the system.

2. In chapter 2 it is shown that the closed loop equation for the system using the Kalman filter predictive control system is given,

$$y(t) = \frac{1}{N_1(q^{-1})} y^*(t+d) + \frac{N_2(q^{-1})}{N_1(q^{-1})} \xi(t) \tag{4.5.6}$$

where

$$N_1(q^{-1}) = [A_x A^{-1} B_x^{-1} B' q^{-1} - (1-A) C^{-1} K_1 (A_x A^{-1} B_x^{-1} B' - 1)] \tag{4.5.7}$$

$$N_2(q^{-1}) = A_x B_x^{-1} [A^{-1} B' q^{-1} - (1-A) C^{-1} K_1 A^{-1} B] \tag{4.5.8}$$

A_x and B_x are the true ARMA polynomials of the process and $\xi(t)$ represents all the disturbances and noise present in the system.

The closed loop equation for the AKF predictive control is same as equation (4.5.6), but now the polynomials A, B', C and K_1 are obtained from the estimated parameters.

If the estimated parameters converge to their true values then,

$A \dashrightarrow A_x$

$q^{-(d+1)} B' \dashrightarrow B_x$

The closed loop equation (4.5.6) is now given by,

$$y^*(t) = y(t) + [(1-\hat{A}) \hat{C}^{-1} \hat{A}^{-1} \hat{K}_1 - 1] \xi(t) \tag{4.5.7}$$

If it is assumed the noise $\xi(t)$ is bounded then from equation (4.5.7) it is clear that the asymptotic stability of the AKF predictive control system depends on the roots of

A and C. Since A is a stable polynomial and the stability of C is guaranteed, by the KFP scheme for the AKFP and by the ERLS for the STKFP, the predictive controller given by equation (4.5.5) is asymptotically stable. However it does not guarantee the stability before the parameters are converged.

4.6 An Adaptive Kalman Filter Predictor for Deterministic Disturbances

If the AKFP or STKFP discussed in the previous sections are to be used in processes where deterministic disturbances or a bias (offset) is present, a number of difficulties will be encountered. They are:

1. As discussed in Chapter 2, due to the PD nature of the estimation scheme the output prediction will be biased.
2. The normal ARMAX model cannot be used for parameter identification under deterministic disturbances.
3. Even if unbiased predictions are made by the KFP the predictive control law cannot be directly used as given in section (4.5.2).

In the following sections each of these difficulties is discussed in detail, and a modified control scheme is developed.

4.6.1 Modified Kalman Filter Predictor

It was shown in Chapter 2 that due to the "PD estimation", especially the PD functional block in the feedback path of the state estimation scheme, the predicted output could be biased, since it does not pass the full information about the disturbances. The intuitive modification is to introduce integrator(s), thus obtaining a 'PID' estimation scheme which in turn will give an unbiased disturbance prediction. The introduction of the integrators was achieved by augmenting the state space formulation given by (4.2.4 and 5) with an additional state corresponding to noise. This was first proposed by Balchan et al (1970, 1973) and Bialkowski (1983).

When the process noise is not white noise, the state space representation of the process can be augmented by the states corresponding to the noise dynamics. If the step disturbances are interpreted as a random signal which introduces the steps at random time instants, by random step sizes, the dynamics of the step disturbances can be represented by integrated random noise (Balchan et al, 1970, 1973 and Tuff et al. 1985), i.e.

$$\xi(t) = \frac{w(t)}{\Delta} \qquad (4.6.1)$$

where

$\Delta = 1-q^{-1}$, differencing operator;

$w(t)$ = random signal, generally zero but may attain values p_i at arbitrary time instants i;

$\xi(t)$ = series of steps of height h_i starting at time i.

The augmented state space formulation of the KFP is given by,

$$x(t+1) = \begin{bmatrix} 1 & 0 & 0 \\ r & \Phi_1 & 0 \\ 0 & 0 & \Phi_2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \Lambda \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w(t) \qquad (4.6.2)$$

$$y(t) = [0:0:\Theta_1]x(t) + v(t) \qquad (4.6.3)$$

where

$$x(t) = [x_\rho, x_1, x_2, \cdot\cdot, x_n, x_{n+1}, \cdot\cdot x_{n+d}]$$

A detailed discussion and an interpretation of the modified KFP is given in Chapter 2.

4.6.2 Modified Adaptive Kalman Filter Predictor

The modified KFP given in the above section can be extended to its adaptive version by defining a suitable model and an identification scheme to handle the step disturbances. Obviously if the ARMAX model is defined as,

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})\omega(t) \qquad (4.6.4)$$

it would not model the disturbances correctly, and the identification would not guarantee converged true parameters.

Consider the ARIMA model (Tuff et al, 1985) for the process with deterministic disturbances,

$$A(q^{-1})y(t) = q^{-(d+1)}B'u(t) + \frac{w(t)}{\Delta} \qquad (4.6.5)$$

$$A(q^{-1})\Delta y(t) = q^{-(d+1)}B'\Delta u(t) + w(t) \qquad (4.6.6)$$

If we define the regressor vector $\psi(t-1)$ as,

$$\psi(t-1) = [\Delta y(t-1), \Delta y(t-2), \cdot\ \cdot, \Delta y(t-n), \\ \Delta u(t-d-1), \cdot\ \cdot, \Delta u(t-d-n)]^T \qquad (4.6.7)$$

and the parameter vector as,

$$\hat{\theta}(t) = [a_1, a_2 \cdot \cdot a_n, b_1, b_2, \cdot \cdot b_n]^T \tag{4.6.8}$$

then from (4.6.6)

$$y(t) = \psi(t-1)\hat{\theta}(t)^T + \xi(t)$$

Since $\xi(t)$ is an uncorrelated noise signal and it is also uncorrelated with $\psi(t-1)$, we could use a RLS algorithm to estimate the converged true parameters of $\hat{\theta}(t)$, i.e. the $A(q^{-1})$ and $B(q^{-1})$ polynomials. This is the RLS identification scheme using incremental variables. The identified parameters can be used in the KF and in any associated controller.

4.6.3 Incremental Adaptive Predictive Control for the modified Adaptive Kalman Filter Predictor

If the predictive control law given by equation (4.5.4) is used, when there are deterministic disturbances the output control will be biased. This can be corrected by introducing a bias term into the predictive control law. This bias term could be identified on-line by introducing an additional parameter to the $\hat{\theta}(t)$ in a normal RLS estimation scheme, or we can use the incremental predictive control as given below:

Consider the ARIMA model (4.6.5)

$$A(q^{-1})y(t) = q^{-(d+1)}B'u(t) + \frac{w(t)}{\Delta} \tag{4.6.5}$$

from (4.6.5)

$$\Delta y(t+d+1) + [A(q^{-1})-1]\Delta y(t)$$
$$= B'\Delta u(t) + \xi(t+d+1) \qquad (4.6.9)$$

Taking the conditional expectation of (4.6.9) we get,

$$\Delta\hat{y}(t+d+1|t) + a_1\Delta\hat{y}(t+d|t) + \cdot\cdot + a_n\Delta\hat{y}(t|t)$$
$$= B'\Delta u(t) \qquad (4.6.10)$$

The minimum variance estimates for $\Delta\hat{y}(t+d|t)$, $\Delta\hat{y}(t+d-1|t)$, $\cdot\cdot$, $\Delta\hat{y}(t|t)$ can be obtained from the KFP:

$$\Delta\hat{y}(t+d|t) = \hat{y}(t+d|t) - \hat{y}(t+d-1|t) \qquad (4.6.11)$$

If the control law is calculated such that the predicted output $\hat{y}(t+d|t)$ will reach the desired set point in one sample time instant, the incremental control law $\Delta u(t)$ is calculated by defining:

$$\Delta\hat{y}(t+d+1) = y^*(t+d+1) - \hat{y}(t+d|t)$$
$$= \Delta y^*(t+d+1) \qquad (4.6.12)$$

The incremental control law is given by,

$$\Delta u(t) = \frac{1}{b_1}[\Delta y^*(t+d+1) - [1-A(q^{-1})]\Delta\hat{y}(t+d|t)$$
$$- [\ B'(q^{-1})-b_1]\Delta u(t) \qquad (4.6.13)$$

The adaptive control of (4.6.13) is obtained simply by using the estimated parameters of $\hat{A}$ and $\hat{B}$.

4.7 Simulation Results and Discussion

The simulations were carried out for the 2nd order process given by the difference equation:

$$y(t) - 1.8954\ y(t-1) + 0.8987\ y(t-2) =$$
$$0.7975\ u(t-4) - 0.7758\ u(t-5) \qquad (4.7.1)$$

The process was simulated using the state space formulation given in Section 4.2. White noise sequences were added, to each process state to generate the process noise, and to the output to generate measurement noise.

Process noise $w(t) = [0,0.1]$

Measurement noise $v(t) = [0,0.1]$

1. Adaptive Predictive Control with AKFP:

1a. Deterministic Process

A periodic set point,

$y^*(t) = 1 \qquad 0 \le t \le 50$

$\quad\; = 0 \qquad 50 < t \le 100$

is used until the 100th time instant to obtain converged parameters. Figures 4.6a and b show the performance of the AKFP and ASP. As shown in theory both AKFP and ASP functions exactly the same under noise free conditions and if the parameters have converged to the true parameters. In the deterministic case the converged true parameters are obtained very fast.

1b. Stochastic Case

Figure 4.7a shows the output control using AKFP and ASP. As in the fixed parameter case, the AKFP can be tuned to obtain minimum variance performance. The ASP is quite noisy. It can be improved by introducing an exponential filter. However as shown in figure 4.7c the minimum value of the output error variance achieved by AKFP is less than that of the ASP plus exponential filter. Thus the AKFP performs as an adaptive minimum variance controller.

The parameter estimation in both AKFP and ASP are almost the same. Figure 4.7d shows the parameter estimation in the AKFP. Two C coefficients are estimated to obtain the converged true parameters of A and B polynomials. Converged values of the parameters after 800 sample instants are given in table 4.1.

2. Adaptive Predictive Control with STKFP

The Kalman gains L_1 to L_4 were calculated using the algorithm given in section 4.4. Four C coefficients are estimated in order to calculate the Kalman gains. The performance of the STKFP is shown in figure 4.8a. Although it shows satisfactory performance the error variance in this case is not the minimum variance achieved in AKFP. However it shows less variance than in the ASP. See figure 4.8b.

The convergence of the A and B parameters is faster than in the AKFP. But the convergence of the four c coefficients is quite slow. Parameter estimation of the STKFP is given in figure 4.8c. The estimated parameters at the 800th sample instants are given in table 4.2.

It is clear that the converged values of the parameters in A and B almost equal the true values. The difference in the convergence in AKFP and STKFP is due to the number of c coefficients that are estimated. The reason for the slow convergence of A and B in AKFP may be due to the fact that two c coefficients are not sufficient to represent the noise present in the system. Another reason could be that these was very rich excitation at the beginning of the response due to the large fluctuations in the Kalman gains etc.

Figure 4.8d shows the values of the Kalman gains. The gains fluctuates wildly at the beginning, and consequently poor performance is shown in this interval. Since the Kalman gains are calculated from C coefficients, which are slow in convergence, the Kalman gains are also slow in convergence.

Compared to AKFP there are more parameters to be estimated in the STKFP, and the number will increase with the time delay, because the number of c coefficients in the C polynomial depends on the time delay (equation 4.4.2). However it is still important to note that the number parameters estimated in the STKFP is less than in the implicit self-tuning controllers (Clarke and Gawthrop, 1979), or in implicit adaptive minimum variance control system (Goodwin and Sin, 1984).

3. Adaptive Incremental Predictive Control with Modified Adaptive Kalman Filter Predictor

The performance of the modified AKFP and ASP, in the presence of deterministic disturbances is investigated by simulating a process with the following difference equation:

$$y(t) - 1.8954\, y(t-1) + 0.8981\, y(t-2) =$$

$$0.7975\, u(t-4) - 0.7758\, u(t-5) + \xi(t) \qquad (4.7.2)$$

$\xi(t)$ is a periodic signal given by,

$$\begin{aligned} \xi(t) &= 0 \quad && 0 \le t \le 40 \\ &= 0.1 && 40 < t < 80 \end{aligned}$$

The steady state value of the disturbance expressed in the same units as y is $\xi_{ss}=37.037$.

Figure 4.9a shows the AMKFP and ASP under deterministic disturbances. Clearly the AMKFP shows significantly better disturbance rejection. This is due to the disturbance prediction property of the MKFP.

The parameter estimation in the incremental RLS identification is shown in figure 4.9c. The system is perturbed by a periodic disturbance to obtain converged parameters.

4.8 Conclusions

1. The adaptive Kalman filter predictor based on the specific state space formulation, Kalman filter and recursive extended least squares identification gives nearly minimum variance control performance for processes with time delays, and in the presence of process and measurement noise.
2. The adaptive Kalman filter predictor and the adaptive Smith predictor have the same configuration except for an additional filter in the AKFP, which performs adaptive minimum variance filtering.
3. Under noise free conditions, once the parameters have converged to their true values, the AKFP and the ASP are functionally the same.
4. A 'self-tuning' Kalman filter predictor that has the KF implemented indirectly in the identification scheme is presented. This avoids the need of a priori knowledge of the noise covariances or the tuning of the AKFP by the ratios of the covariances.
5. The number of parameters estimated in AKFP and STKFP are less than in the implicit self-tuning control systems or in adaptive minimum variance control systems.
6. By employing a computationally efficient Kalman filter algorithm, the heavy computation involved in implementing the AKFP is removed, and thus a practical AKFP is obtained.

7. An adaptive modified Kalman filter predictor based on the incremental identification and a modified state space formulation is presented for deterministic disturbances. The incremental adaptive predictive control law based on the adaptive modified Kalman filter predictor shows significantly better disturbance rejection.

Table 4.1 Parameter Estimation in the AKFP after 800 sampling instants (Stochastic Process)

Table 4.2 Parameter Estimation in the STKFP after 800 sampling instants (Stochastic Process)

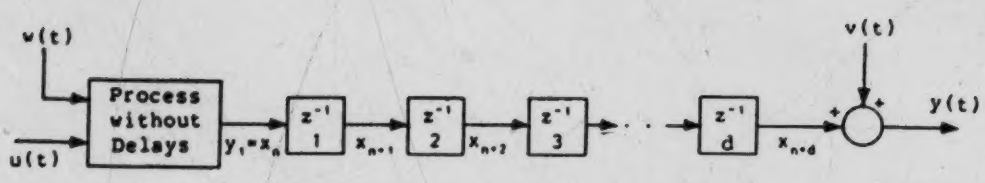

w(t)
u(t)
Process without Delays
$y_1=x_n$
z^{-1} 1
x_{n+1}
z^{-1} 2
x_{n+2}
z^{-1} 3
z^{-1} d
x_{n+d}
v(t)
+
+
y(t)

Figure 4.1 Schematic Block Diagram of the State Space Representation for a process with time delays.

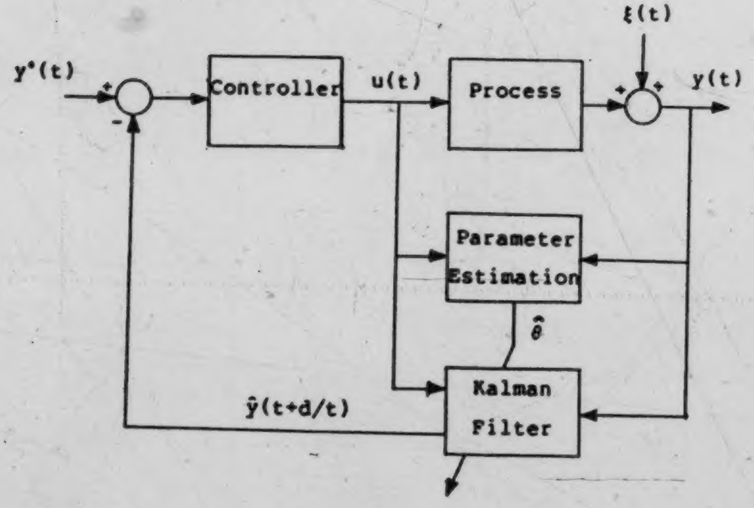

ξ(t)
y*(t)
+
-
Controller
u(t)
Process
+
+
y(t)
Parameter Estimation
$\hat{\theta}$
Kalman Filter
$\hat{y}(t+d/t)$

Figure 4.2 The Schematic Block Diagram of the Adaptive Kalman Filter Predictor.

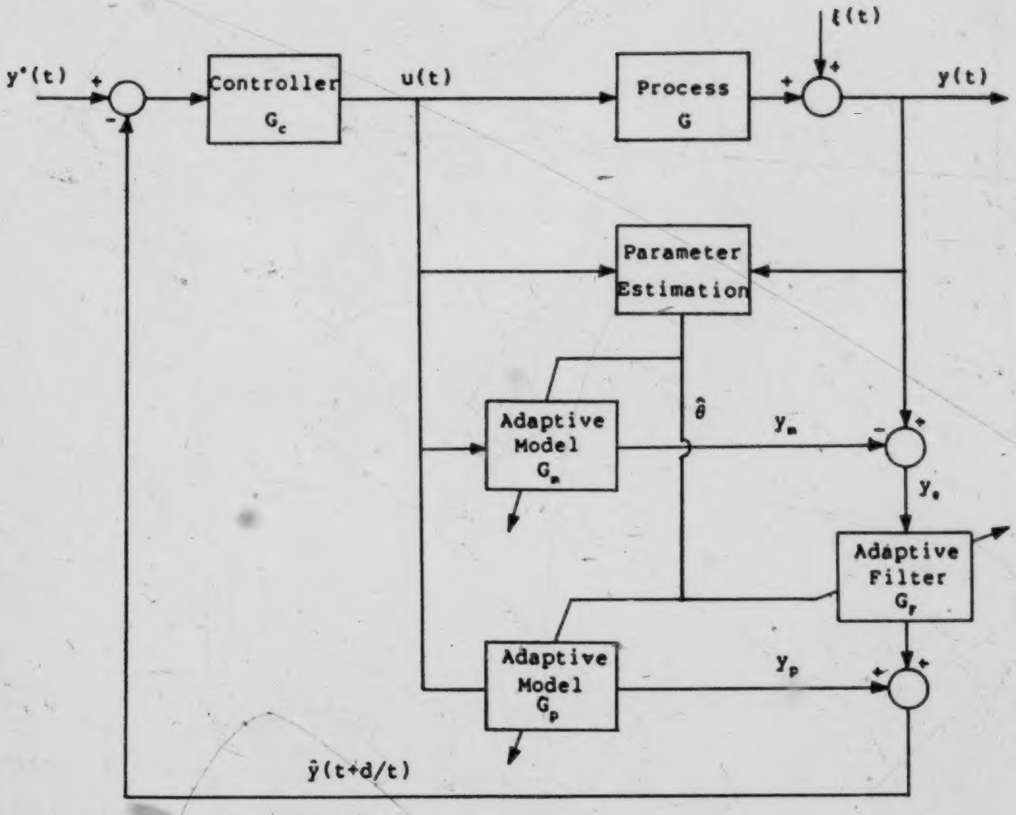

ξ(t)
y*(t)
+
-
Controller
G_c
u(t)
Process
G
+
+
y(t)
Parameter
Estimation
$\hat{\theta}$
Adaptive
Model
G_m
y_m
-
+
y_e
Adaptive
Filter
G_f
Adaptive
Model
G_p
y_p
+
+
$\hat{y}(t+d/t)$

Figure 4.3 The structure of the Adaptive Kalman Filter Predictor.

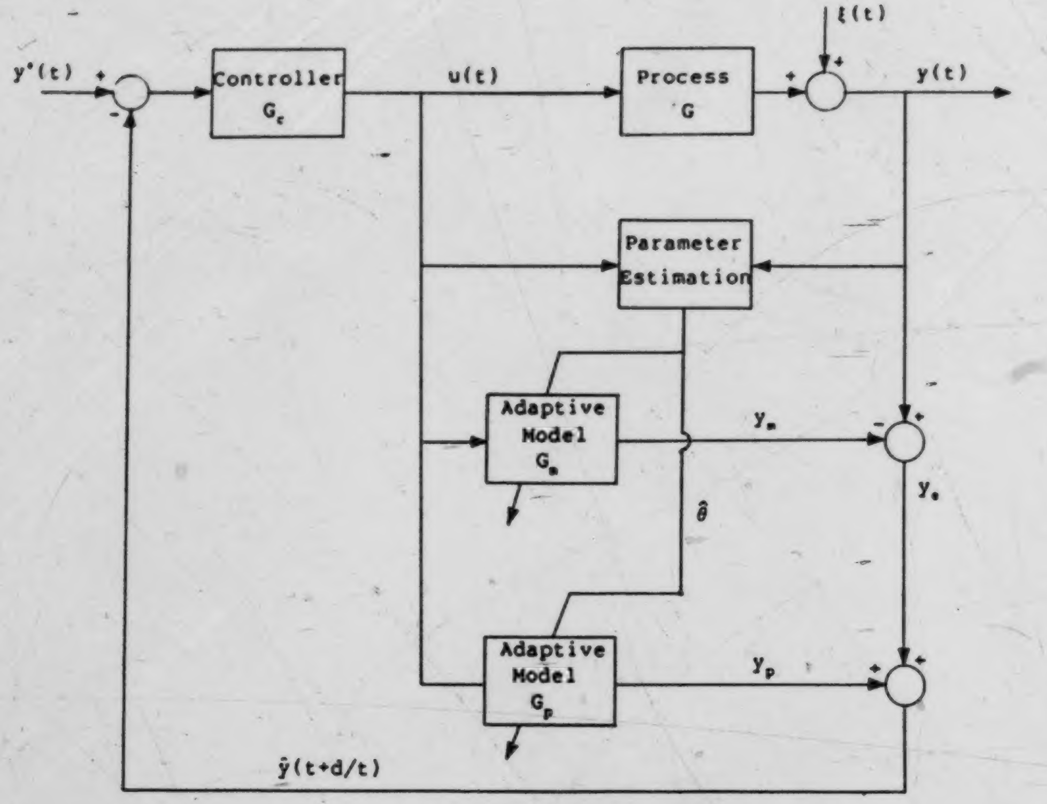

ξ(t)
y*(t)
+
-
Controller
G_c
u(t)
Process
G
+
+
y(t)
Parameter
Estimation
Adaptive
Model
G_m
y_m
-
+
y_e
$\hat{\theta}$
Adaptive
Model
G_p
y_p
+
+
$\hat{y}(t+d/t)$

Figure 4.4 The structure of the Adaptive Smith Predictor.

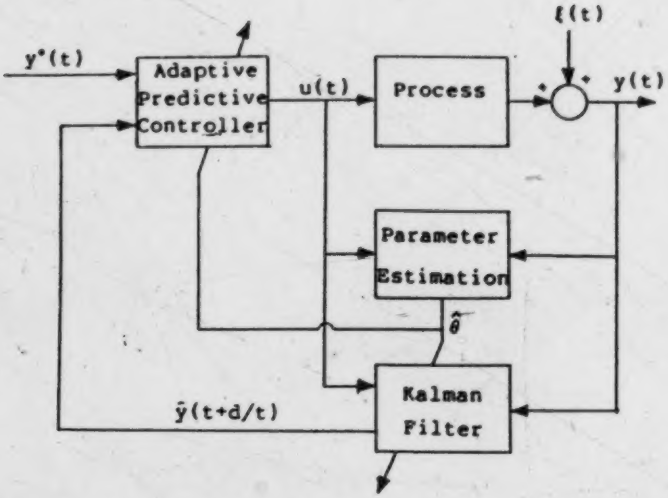

y*(t)
Adaptive Predictive Controller
u(t)
Process
ξ(t)
+
+
y(t)
Parameter Estimation
$\hat{\theta}$
Kalman Filter
ŷ(t+d/t)

Figure 4.5 The Schematic Block diagram of the Adaptive Predictive Control system based on the AKFP.

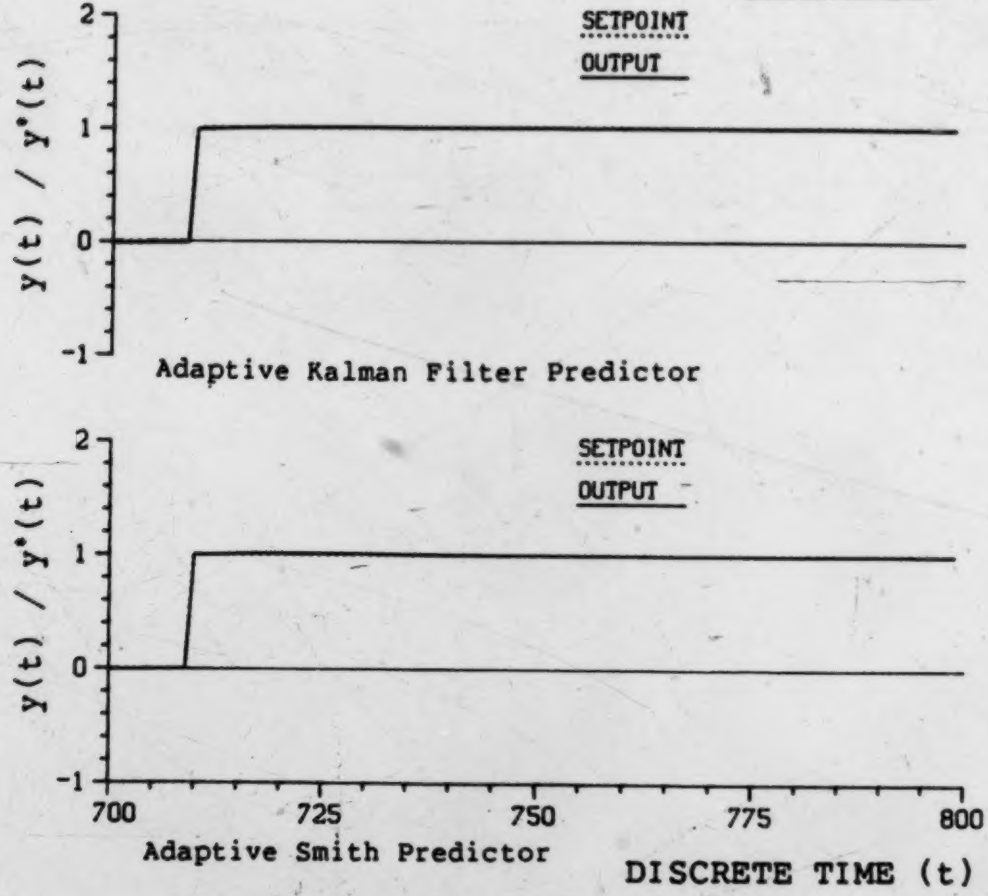
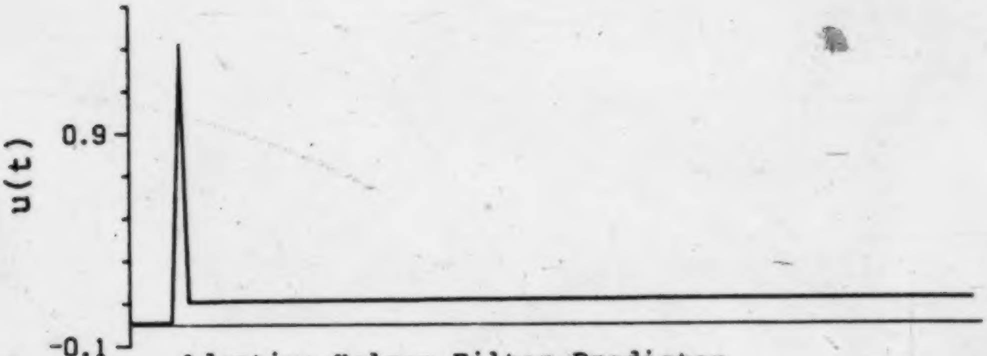
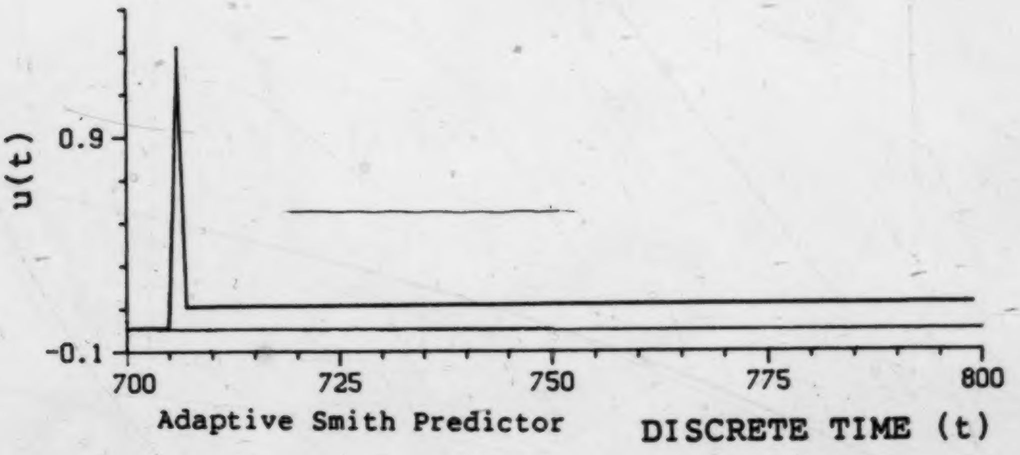


Figure 4.6a Adaptive predictive Control using AKFP and ASP (Deterministic Process).

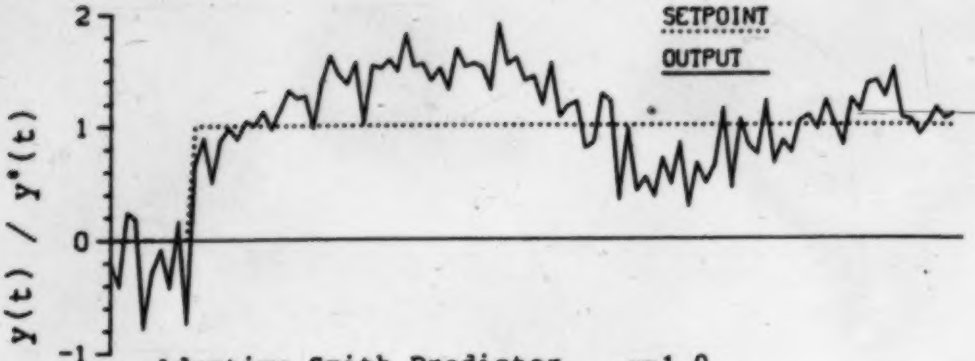


Adaptive Kalman Filter Predictor

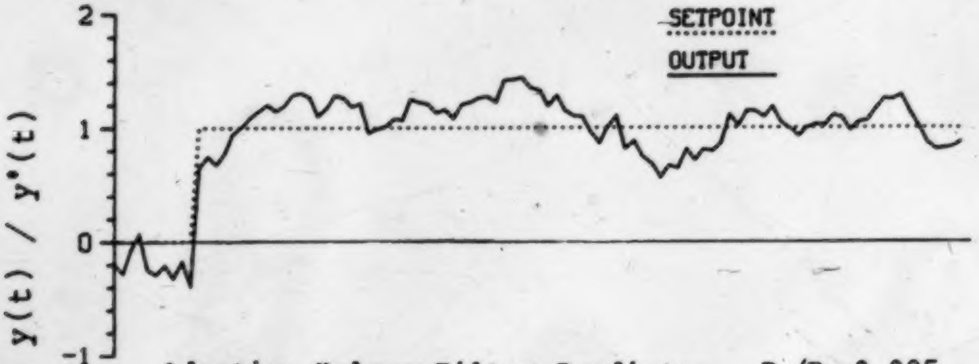


Adaptive Smith Predictor

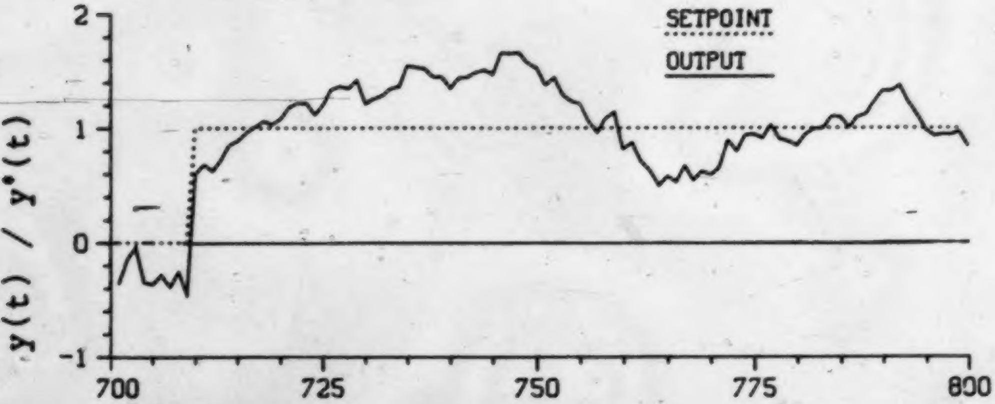
Figure 4.6b Adaptive predictive Control using AKFP and ASP (Deterministic Process).



Adaptive Smith Predictor $\alpha=1.0$



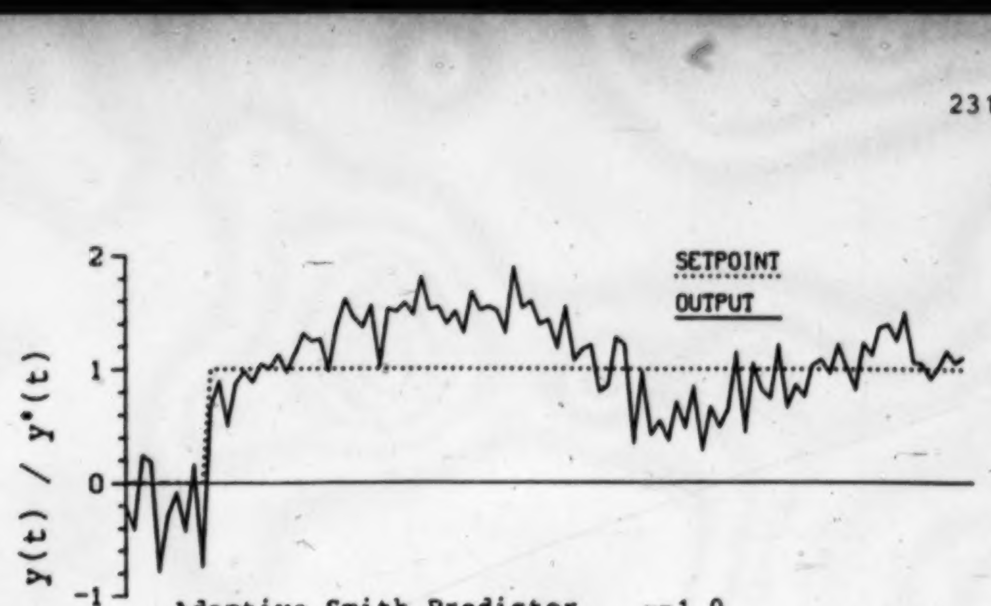
Adaptive Kalman Filter Predictor $R_v/R_v=0.005$



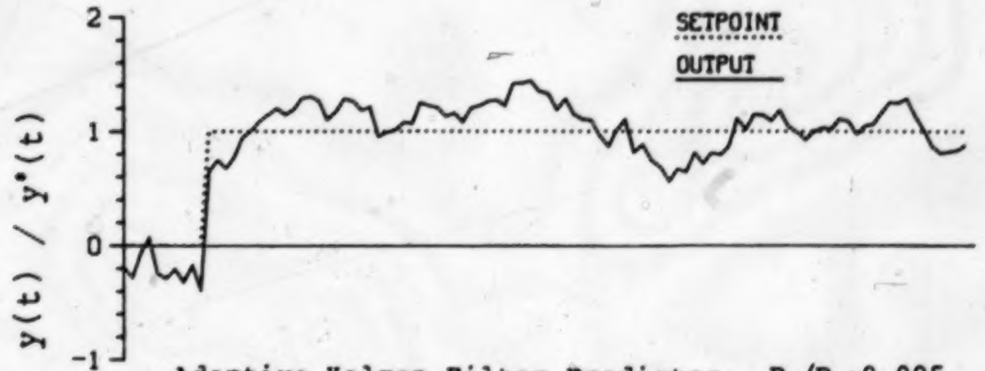
Adaptive Smith Predictor

$\alpha=0.4$

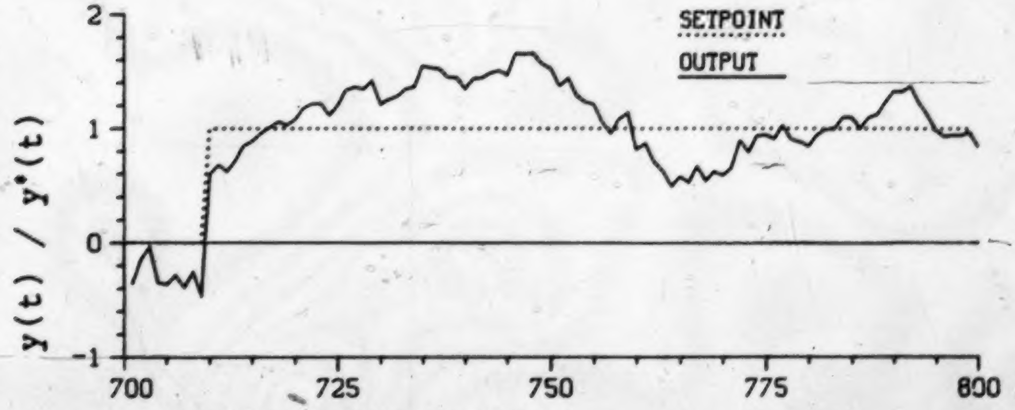
Figure 4.7a Adaptive Predictive Control using AKFP, and ASP with an Exponential Filter(Stochastic Process).



Adaptive Smith Predictor $\alpha=1.0$



Adaptive Kalman Filter Predictor $R_w/R_v=0.005$



Adaptive Smith Predictor
$\alpha=0.4$

Figure 4.7a Adaptive Predictive Control using AKFP, and ASP with an Exponential Filter(Stochastic Process).

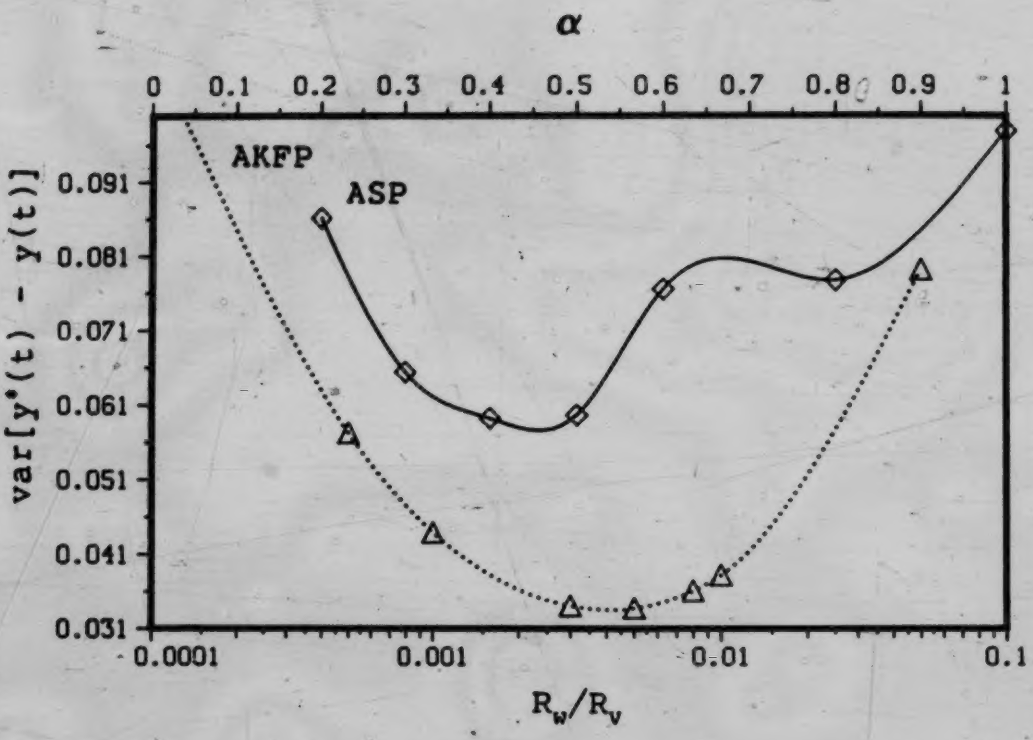


Figure 4.7c Variance of the Output Tracking Error for AKFP, and ASP with an Exponential Filter.

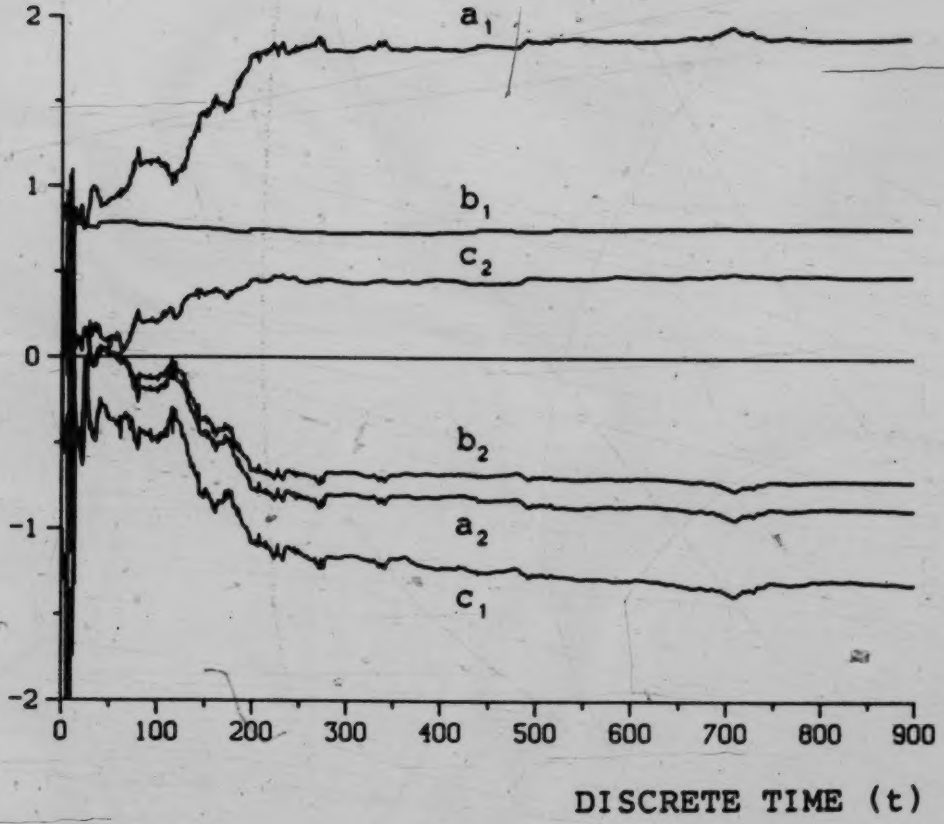


Figure 4.7d Parameter Estimation in AKFP (using Extended Least Squares Identification).

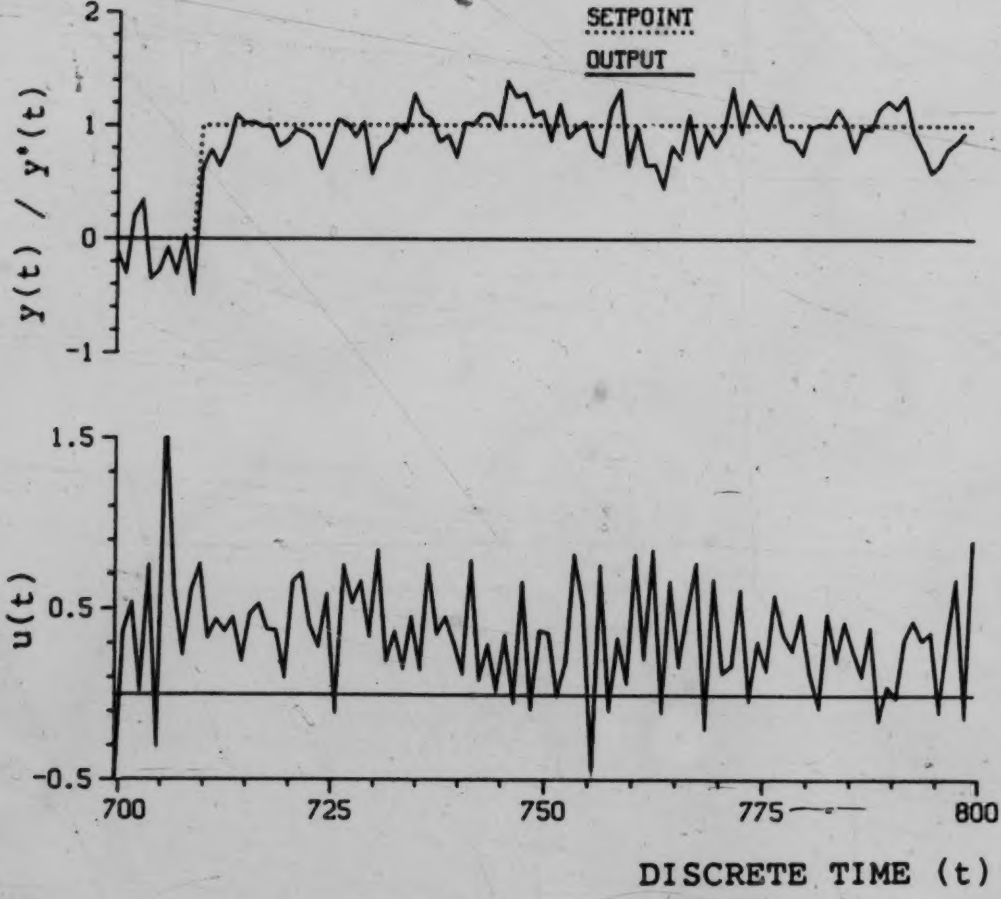


Figure 4.8a Adaptive Predictive Control with STKFP (Stochastic Process).

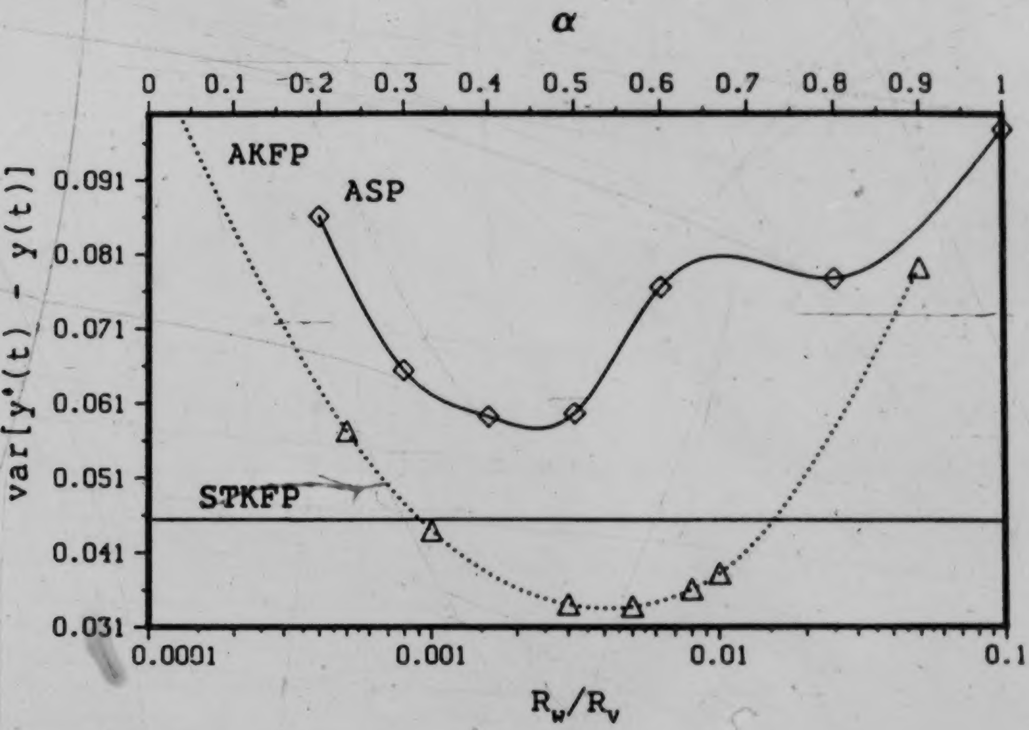


Figure 4.8b Variance of the Output Tracking Error for AKFP, STKFP, and ASP with an Exponential Filter.

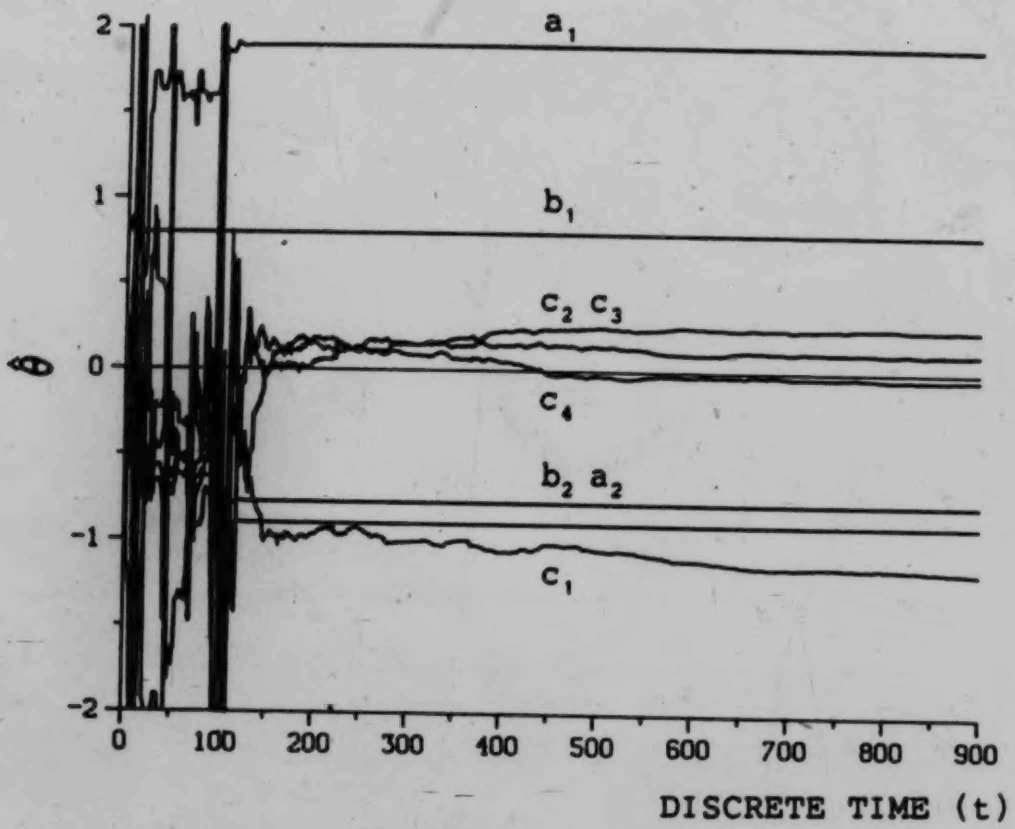


Figure 4.8c Parameter Estimation in STKFP using Extended Least Squares Identification.

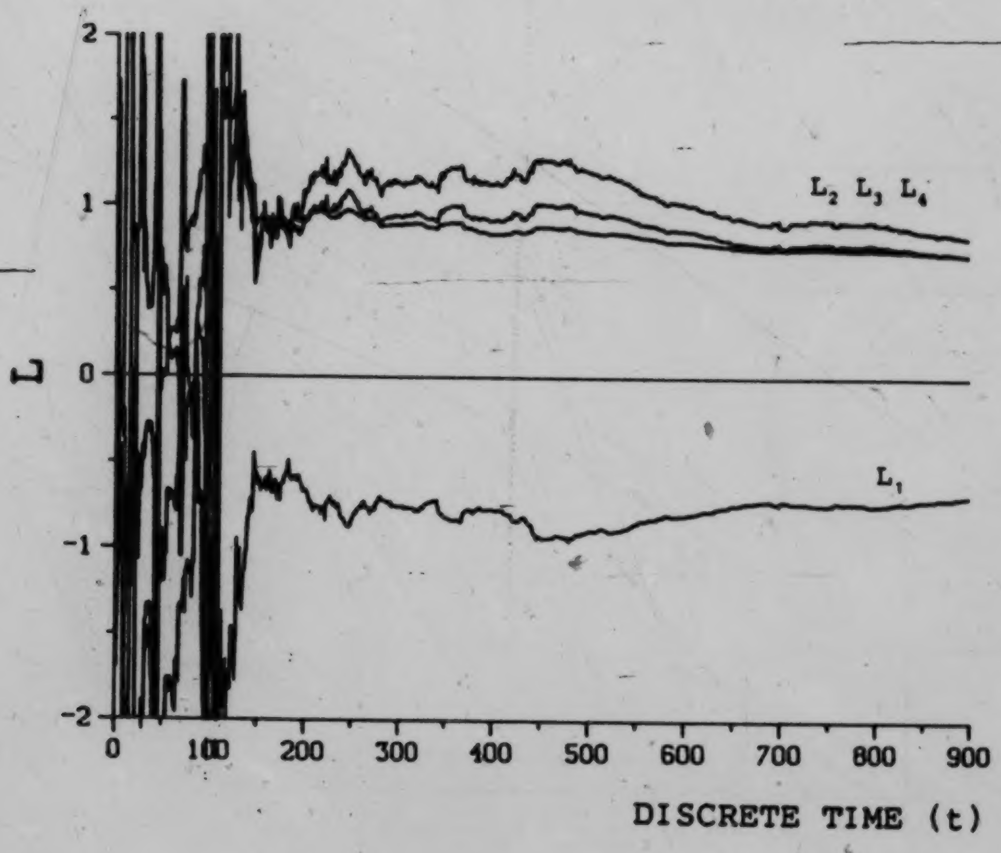
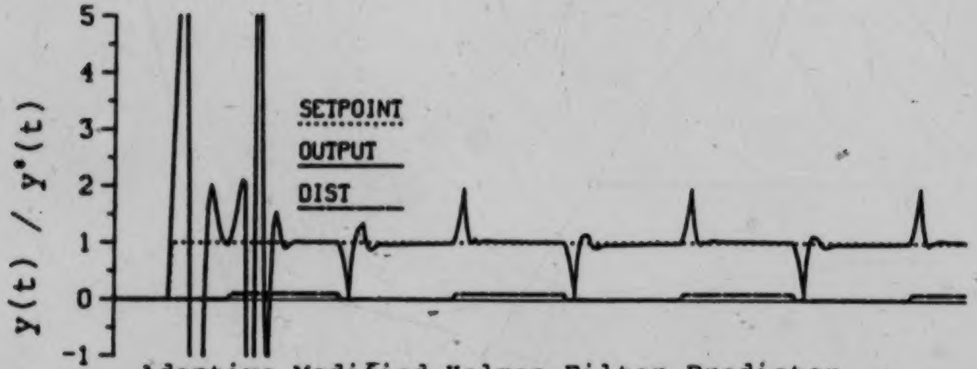
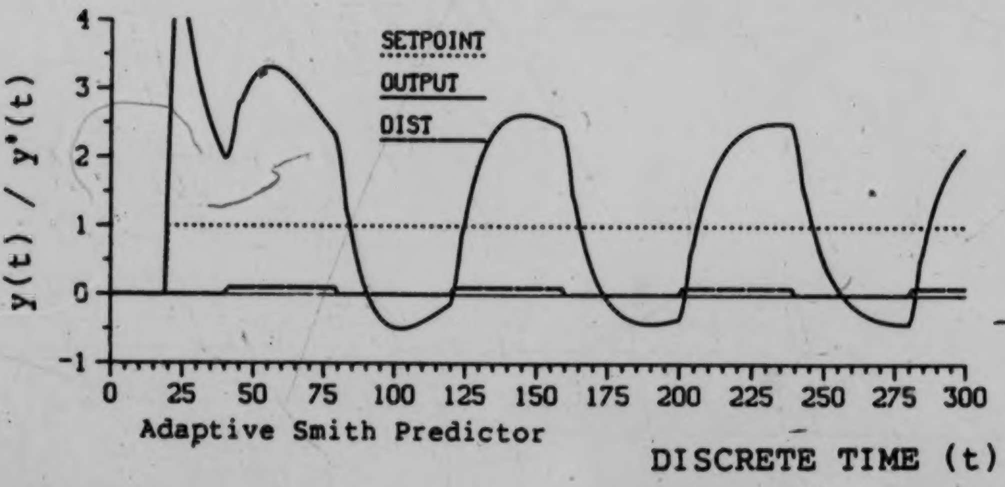


Figure 4.8d Kalman Gains in the STKFP.

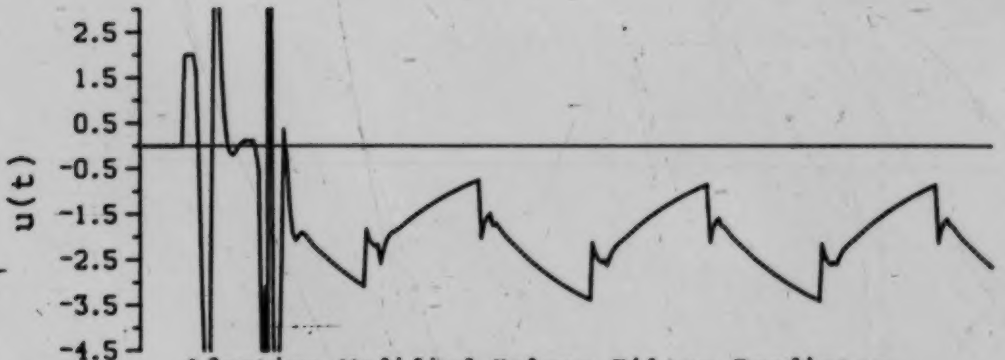


Adaptive Modified Kalman Filter Predictor

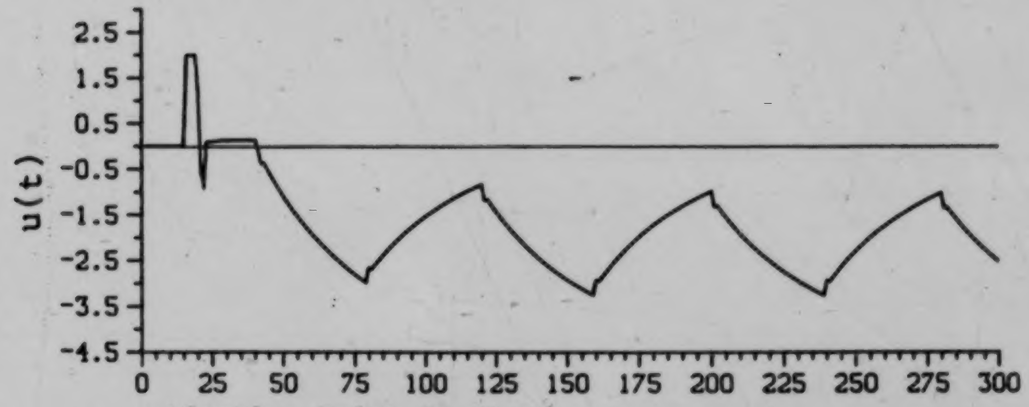


Adaptive Smith Predictor

Figure 4.9a Incremental Adaptive Predictive Control using AMKFP and ASP in presence of deterministic disturbances.



Adaptive Modified Kalman Filter Predictor



Adaptive Smith Predictor

Figure 4.9b Incremental Adaptive Predictive Control using AMKFP and ASP in presence of deterministic disturbances.

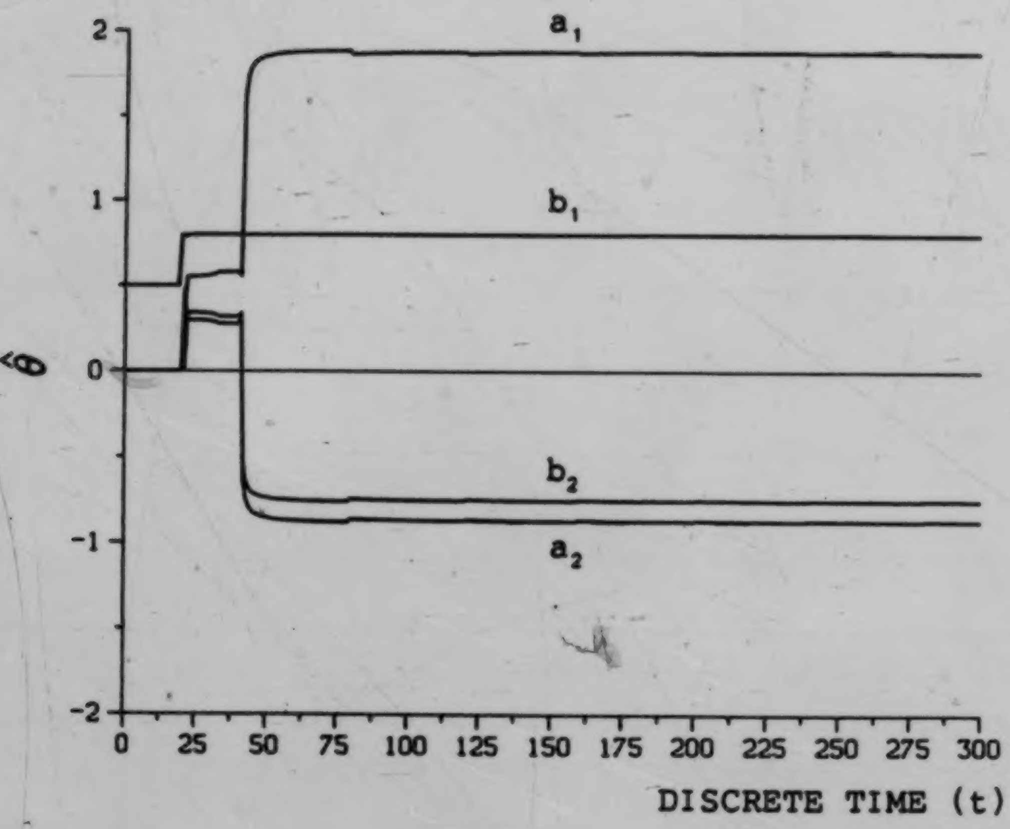


Figure 4.9c Parameter Estimation in the Incremental RLS Identification Scheme.

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5. Multivariable Adaptive Kalman Filter Predictor

5.1 Introduction and Overview

The dead time compensation schemes for Single Input Single Output (SISO) processes with time delays, i.e. the Smith Predictor (SP), and the Kalman Filter Predictor (KFP) (Chapter 2), can be extended to Multi-Input-Multi-Output systems. The extention to MIMO systems uses the Interactor matrix (Wolovich and Falb, 1976) to represent the time delays of the process, as illustrated by the Interactor Predictor (IP) (Sripada et al, 1985), and the multivariable KFP discussed in Chapter 3. Both IP and KFP are developed assuming apriori knowledge of the process Transfer Function Matrix (TFM), which implies apriori knowledge of the interactor. If the process model is not known apriori, it can be obtained by employing a suitable identification scheme to estimate the process parameters, and this leads to the Adaptive Kalman Filter Predictor (AKFP) and the Adaptive Interactor Predictor, (AIP).

For the class of MIMO processes where the interactor is diagonal, formulation of AKFP and AIP is quite straight forward, because the interactor contains only the time delays. However in the general triangular interactor case it is necessary to identify the parameters of the dynamic terms in the interactor matrix and this causes difficulties in the adaptive extensions of IP and KFP.

Other adaptive control schemes which also use the interactor matrix approach are the MIMO extensions of the Minimum variance controllers. These are presented in Goodwin and Sin (1984) and Goodwin and Dugard (1984). In these schemes the interactor matrix is identified on line.

The approach used in this chapter to formulate the AKFP and AIP for processes with triangular interactor is to introduce a suitable precompensator as suggested by Singh and Narendra (1984), to diagonalize the interactor. Once the interactor is diagonal, it is necessary only to estimate the parameters of the p MISO subsystems (where p=number of outputs) of the MIMO process TFM. Separate recursive extended least squares estimators are used to estimate the parameters of the p MISO subsystems which are then used directly in p independent MISO KFPs. Since the diagonalization of the interactor leads to p independent MISO adaptive Kalman filter predictors, the results obtained for the SISO AKFP in chapter 4 are directly applied to each MISO AKFP.

The minimum variance predictive control scheme based on the KFP as given in Chapter 3 is extended to an adaptive minimum variance predictive control scheme.

Using the innovation model, the adaptive Kalman filter predictor is interpreted as an adaptive interactor predictor with an additional time varying filter TFM that filters the error between process output and the model output.

To implement the AKFP, it is necessary to know the noise statistics apriori, or to tune the KF using the ratio of the noise covariances as a tuning parameter. An approach for SISO systems that does not need noise statistics or tuning was presented in Chapter 4. This 'self-tuning' KFP can be extended to the multivariable case by implementing p MISO 'self-tuning' KFPs.

It was shown in Chapter 2 that the SISO KFP gives biased predictions in the presence of deterministic disturbances or offset. A modified Kalman filter predictor was formulated to overcome this problem by augmenting the state space model with a state corresponding to the disturbance , as proposed by Balchan et al (1970, 1973). A multivariable incremental predictive control scheme based on the multivariable modified Kalman filter predictor is proposed in this chapter.

5.2 Interactor Factorization

The concept of interactor factorization which provides a means to factor out the 'natural' delay associated with a MIMO system, was first proposed by Wolovich and Falb (1976). They showed that for every pxm proper, rational transfer function matrix in q, T(q), there is a unique non-singular pxp lower left triangular matrix $\xi_T(q)$ called the interactor of T(q) such that,

1. $\det \xi_T(q) = q^g$ (5.2.1)

where g is an integer.

2. $$\lim_{q \to \infty} \xi_T(q) T(q) = K \tag{5.2.2}$$

where K is a non-singular matrix of full rank (= min [p,m]).

Since $\xi_T(q)$ is a non-singular matrix, there exists a stable operator matrix called the inverse interactor matrix $\xi_T^{-1}(q)$, which represents a left divisor of T(q). Physically $\xi_T^{-1}(q)$ represents a measure of the natural delay associated with a discretized MIMO system.

Thus T(q) can be factored out as follows:

$$T(q) = \xi_T^{-1}(q) R_T(q) \tag{5.2.3}$$

where

ξ_T^{-1} - is a pxp matrix

R_T - is a pxm matrix called the residual.

Note that as in the case of a SISO system $\xi_T^{-1}(q)$ factors out the delay due to ZOH also. This will remove all the delays from at least one element of each row of T(q), and hence in the corresponding elements of the $R_T(q)$. This would result in a TFM $R_T(q)$ that is not strictly proper, and would cause difficulties in the state space formulation given in section 5.3. To retain the delay due to ZOH, in $R_T(q)$, a modified factorization is defined as follows:

$$T(q) = \xi^{-1}(q) R(q) \tag{5.2.4}$$

where

$$R(q) = q^{-1} R_T(q) \tag{5.2.5}$$

$$\xi^{-1}(q) = q\, \xi_T^{-1}(q) \tag{5.2.6}$$

Now the residual R(q) is strictly proper, but this might remove the strictly proper property of the inverse interactor $\xi^{-1}(q)$ (see Chapter 3).

5.3 Predictor Concept for MIMO Systems

The predictor concept for MIMO systems based on the interactor matrix was presented in section 3.3.2.

Consider the model of the MIMO process,

$$y(t) = \xi_T^{-1}(q)R_T(q)\ u(t) \tag{5.3.1}$$

define a filtered output:

$$\bar{y}(t) = \xi_T(q)\ y(t) = q\xi(t)y(t) \tag{5.3.2}$$

$\bar{y}(t)$ is defined as the prediction in a MIMO system.

A schematic block diagram of the MIMO system, based on the interactor factorization is given in figure 5.1. The first block in figure 5.1 represents the MIMO system dynamics after necessary time delays have been factored as in equation (5.2.4). The vector $y^0(t)$ of dimension p, which is the output of block 1 is given by,

$$\begin{aligned} y_0(t) &= q^{-1}R_T(q)\ u(t) = R(q)\ u(t) \\ &= q^{-1}\ \bar{y}(t) \end{aligned} \tag{5.3.3}$$

From figure 5.1, the second block

$$\begin{aligned} y(t) &= q\ \xi_T^{-1}\ (q)\ y_0(t) \\ &= \xi^{-1}(q)\ y_0(t) \end{aligned} \tag{5.3.4}$$

The quantity we are interested in is $\bar{y}(t)$. Since the discretization delay is included in R(q), the quantity we are interested is $\bar{y}(t-1)$. From equation (5.3.3) it is clear that the output of the first block, $y^0(t)$ would give the desired prediction $\bar{y}(t-1)$. The Interactor predictor due to Sripada et al (1985) is a deterministic predictor that provides the estimates of $y^0(t)$, based on equation (5.3.3). The KFP formulated in Chapter 3 is based on the same equation, but produces the optimal prediction of $y^0(t)$ in a stochastic environment.

5.4 ARMA model for a MIMO system using interactor factorization

Let the $(i,j)^{th}$ element of the transfer function matrix $R(q)$ be expressed as

$$R_{ij}(q) = \frac{b^{ij}(q)}{a^i(q)} q^{-(d^{ij}+1)} \tag{5.4.1}$$

where

$$a^i(q) = q^{ni} + a_1^i \; q^{ni-1} + \cdot \; \cdot + a_{ni}^i \tag{5.4.2}$$

$$b^{ij}(q) = b_1^{ij} \; q^{ni} + \cdot \; \cdot \; \cdot + b_{ni-1}^{ij} q + b_{ni}^{ij} \tag{5.4.3}$$

d_{ij} is the time delay, and at least one, $d_{ij}=0$, where $j=1, m$ for each $i=1, p$.

The TFM $R(q)$ can be considered as p MISO systems. From equation (5.4.1) the ARMA model for the j^{th} MISO system can be written as,

$$\alpha^j(q^{-1}) y_j^0 (t) = q^{-1} \beta^j(q^{-1}) u(t) \tag{5.4.4}$$

where

$$\alpha^j(q^{-1}) = 1 + a_1^j q^{-1} + \cdot \cdot \cdot + a_{nj}^j q^{-nj} \tag{5.4.5}$$

$$\begin{aligned} \beta^j(q^{-1}) u(t) &= [\beta^{j1}(q^{-1}) \; \beta^{j2}(q^{-1}) \; \cdot \; \cdot \; \beta^{jm}(q^{-1})] u(t) \\ &= (\beta_1^j + \beta_2^j q^{-1} + \cdot \cdot + \beta_{nj} q^{-nj}) u(t) \end{aligned} \tag{5.4.6}$$

with

$$\beta^{jk}(q^{-1}) = q^{-d_{jk}}(b_1^{jk} + b_2^{jk}\, q^{-1} + \cdots + b_{nj}^{jk}\, q^{-nj}) \qquad (5.4.7)$$

and

$$n_j' = n_j + \max\,[d_{jk}] \text{ for } k=1,\ m.$$

Note that coefficients of α^j and β^{jk} are same as that of $a^j(q)$ and $b^{jk}(q)$ respectively.

The ARMA model for the TFM R(q) can be written as,

$$A(q^{-1})\, y_0(t) = q^{-1}\, B(q^{-1})\, u(t) \qquad (5.4.8)$$

where

$$\begin{aligned} A(q^{-1})\, y_0(t) &= [\alpha^1(q^{-1})\ \alpha^2(q^{-1})\ \cdots\ \alpha^p(q^{-1})]y_0(t) \\ &= (I + A_1 q^{-1} + A_2 q^{-2} + \cdots + A_n q^{-n})y_0(t) \end{aligned} \qquad (5.4.9)$$

with

$$A_j = \text{diag}\,[a_j^1\ \ a_j^2\ \cdots\ a_j^p\,] \qquad (5.4.10)$$

and

$$n = \max\,[nj] \text{ for } j=1,\ p$$

$$B(q^{-1})u(t) = \begin{bmatrix} \beta^{11}(q^{-1}) & \cdots & \beta^{1m}(q^{-1}) \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot\ \cdot & \cdot \\ \beta^{p1}(q^{-1}) & \cdots & \beta^{pm}(q^{-1}) \end{bmatrix} u(t) \qquad (5.4.11)$$

$$= B_1 + B_2 q^{-1} + B_3 q^{-2} + \cdots + B_{n}.q^{-n'+1}$$

where

$$n' = \max\,[n_j'\,]$$

As shown in Appendix I, B_1 is non-singular.

The inverse interactor $\xi^{-1}(q)$ can be written as,

$$\xi^{-1}(q) = D(q) + G_H(q) \qquad (5.4.12)$$

where

$$D(q) = \text{diag}\,[q^{-\lambda 1}\ q^{-\lambda 2} \cdot \cdot \cdot q^{-\lambda_p}] \qquad (5.4.13)$$

and

$$G_H(q^{-1}) = \begin{bmatrix} 0 & & & \\ g^{21}(q) & 0 & 0 & \\ \cdot & & \cdot & \\ \cdot & & & \cdot \\ g^{p1}(q) & \cdot\cdot\cdot & g^{p,p-1}(q) & 0 \end{bmatrix} \qquad (5.4.14)$$

with,

$$g^{ij}(q^{-1}) = g_1^{ij}\ q^{-1} + \cdot \cdot \cdot + g_{\lambda_i}^{ij} q^{-\lambda_i} \qquad (5.4.15)$$

The ARMA model for the TFM T(q) is given by,

$$y(t) = A_T^{-1}\ (q)\ B_T(q)\ u(t) \qquad (5.4.16)$$

where A_T and B_T are obtained in the same way as in (5.4.1).

Since $T(q) = \xi^{-1}(q)\ R(q)$

$$A_T^{-1}\ (q)\ B_T(q) = \xi_T^{-1}\ \ A^{-1}(q^{-1})\ B(q^{-1}) \qquad (5.4.17)$$

If the interactor is diagonal then the real coefficients in A and B are the same as A_T and B_T respectively. But when the interactor is triangular the coefficients are different. This is a crucial point in implementing the adaptive version of the Kalman filter predictor, and is discussed later in section (5.6).

5.5 State Space Formulation

The multivariable KFP depends on the specific state space formulation for the transfer function matrix T(q), which enables the estimation of the vector $y^0(t)$, as a state or a linear combination of states. This formulation is discussed in detail in Chapter 3. Instead of a single state formulation for T(q), state space formulations are developed for each of the p MISO subsystems. Each subsystem is obtained by augmenting a subsystem of R(q) with the corresponding subsystem of $\xi^{-1}(q)$.

The state space formulation of the j^{th} subsystem of T(q) is given by,

$$x^j(t+1) = \Phi_j x^j(t) + \Lambda_j u^j(t) + \Gamma^j w_j(t) \tag{5.5.1}$$

$$y_j(t) = \Theta_j x^j(t) + v_j(t) \tag{5.5.2}$$

and the predictor equation is given by,

$$\begin{aligned} y_0^j(t) &= [e_{nj}^T]_{nj+\lambda j} x^j(t) \\ &= x_{nj}^T(t) \end{aligned} \tag{5.5.3}$$

where

$$\Phi_j = \left[\begin{array}{c|c} \Phi_{1j} & 0 \\ \hline e_{nj}^T & \Phi_{2j} \\ & \\ & \\ 0 & \cdot \\ & \\ 0 & \end{array}\right] \quad (n_j+\lambda_j)x(n_j+\lambda_j)$$

with

$$\Phi_{1j} = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & -a^j_{nj} \\ 1 & 0 & \cdot & \cdot & \cdot & -a^j_{nj-1} \\ 0 & 1 & \cdot & \cdot & \cdot & -a^j_{nj-2} \\ 0 & 0 & \cdot & \cdot & 1 & -a^j_1 \end{bmatrix}_{nj\mathrm{x}nj} \qquad \Phi_{2j} = \begin{bmatrix} 0 & \cdot & \cdot & \cdot & 0 \\ 1 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & 1 & 0 \end{bmatrix}_{\lambda_j \mathrm{x} \lambda_j}$$

$$e^T_{nj} = [0 \;\cdot\;\cdot\; 0 \;\; 1]_{1\mathrm{x}nj}$$

$$\Lambda_j = \begin{matrix} \Lambda_{1j} & 0 \\ 0 & \Lambda_{2j} \end{matrix}\;\; (nj+\lambda j)\mathrm{x}(m+j-1)$$

with

$$\Lambda_{1j} = [\; \Lambda^{j1}_1 \;\; \Lambda^{j2}_1 \;\cdot\;\cdot\;\cdot\; \Lambda^{jm}_1 \;]_{nj\mathrm{x}m}$$

$$\Lambda^{ij}_1 = [b^{ij}_{ni} \;\cdot\;\cdot\;\cdot\; b^{ij}_1 \;]^T_{1\mathrm{x}nj}$$

$$\Lambda_{2j} = [\Lambda^{j1}_2 \;\; \Lambda^{j2}_2 \;\cdot\;\cdot\;\cdot\; \Lambda^{j,j-1}_2 \;]$$

$$\Lambda^{jk}_2 = [g^{jk}_{\lambda_j} \;\; g^{jk}_{\lambda_j-1} \;\cdot\;\cdot\;\cdot\; g^{jk}_1 \;]^T$$

$$x_j = [x^j_1 \;\; x^j_2 \;\cdot\;\cdot\; x^j_{nj+1} \;\cdot\; x^j_{nj+\lambda_j}]^T$$

$$u^j(t) = [u_1(t-d_{j1}),\; u_2(t-d_{j2}),\; \cdot\;\cdot\; u_m(t-d_{jm}),\; y^0_1,\; y^0_2,\; \cdot\;\cdot\; y^0_{j-1}]^T$$

Note that one or more of $d_{jk}=0$, $k=1,m$ and

$$\theta_j = [e^T_{nj+,\lambda_j}]$$

It is important to note that the process noise $w_j(t)$ and measurement noise $v_j(t)$ are included in the state space formulation. It is assumed that only the states x^j_1 to x^j_{nj}, i.e. states of R(q) are effected by $w_j(t)$

$$\Gamma^j = (\gamma^j_1 \;\; \gamma^j_2 \;\cdot\;\cdot\; \gamma^j_{nj} \;\; 0 \;\cdot\;\cdot\; 0)^T (nj\mathrm{x}\lambda_j)$$

w_j and v_j are sequence of uncorrelated random noise with mean zero and covariances given by $R_{vj} = \{w_j w^T_j\}$ and $R_{vj} = \{v_j v^T_j\}$ respectively.

Properties of the state space formulation:

1. Each subsystem is serially connected, when the

interactor is triangular. For a diagonal interactor all the p subsystems become independent.

2. In the triangular interactor case, the parameters of A, B and ξ^{-1} appear directly in the state space formulations, but they are not the same as A_T and B_T. When the interactor is diagonal, the parameters of A and B, and A_T and B_T are identical and appear directly in the state space formulations.
3. The State transition matrix θ_j of each subsystem is singular.
4. The state space formulation for each subsystem is observable, and is stabilizable if the system T(q) is stable. These properties are proved and discussed in detail in Chapter 2 and Chapter 3.

5.6 Adaptive Kalman Filter Predictor

When the ~~parameters~~ of the MIMO process given by the transfer function T(q) are known, any state estimator can be employed to obtain a estimates of $y^0(t)$, using the state space formulation given in section (5.5). Since the process we are concerned with is stochastic , a stochastic state estimator such as the Kalman filter can be employed to obtain the minimum variance estimates of $y^0(t)$, and other states.

The state space formulation given in the previous section consists of p subsystems which are connected in a serial manner. Thus a separate Kalman filter is used to estimate the states of each of the p subsystems. These Kalman filters are also connected serially. The non adaptive version of the KFP is discussed in details in Chapter 3.

When the parameters of the TFM T(q) are not known, the above KFP can be extended to its adaptive version by using a suitable adaptive Kalman filter Scheme, that includes simultaneous estimation of the parameters plus and the states as in the SISO case in Chapter 4.

The SISO AKF employed in Chapter 4 consists of an independent parameter estimation scheme and a Kalman filter which uses a state space equation whose parameters are updated by the identification scheme, at every sampling instant.

Before proceeding with further discussion of the multivariable AKF it is important to note the difficulties encountered in the identification.

The ARMA parameters of the process model T(q) given by,

$$y(t) = A_T^{-1} \; B_T \; u(t) \qquad (5.6.1)$$

can be estimated using the identification scheme presented in section (5.6.2).In the state space formulation of the j^{th} subsystem given in section (5.5) the matrices Φ_j and Λ_j have as their elements, the parameters of A, B and ξ^{-1}. To implement the adaptive Kalman filter it is necessary to estimate the parameters of A and B, plus the real coefficients in ξ^{-1}.

When ξ^{-1} is diagonal the parameters of A and B are the same as that of A_T and B_T, and there are no real parameters to be estimated in ξ^{-1}. In this case the implementation of the adaptive Kalman filter predictor becomes straight forward, because identification of the parameters A_T and B_T will give all the elements of the matrices Φ_j and Λ_j for j=1,p. For the diagonal interactor case all the p subsystems are independent and p independent Kalman filters can be employed to estimate the states. Figure 5.2 depicts this multivariable Kalman filter predictor configuration. However when the ξ^{-1} is triangular, the identification of A^{-1}, B and ξ^{-1} becomes difficult. If the interactor is known then the parameters of A and B can be identified. But the apriori knowledge of the complete interactor is almost equivalent to the knowledge of the complete system transfer function.To overcome the difficulties of having to know the interactor matrix, Johanson (1982) proposed a method to estimate the real variables of the interactor, knowing only the time delays associated with it. Further investigation of this scheme is presented in Dugard, Goodwin and de Soyza (1983), Goodwin and Sin (1984), and Goodwin and Dugard (1984). However, this formulation is not suitable for the implementation of the Kalman filter predictor. An alternative method suggested by Singh & Narendra (1984) is to introduce a suitably chosen precompensator to the system which would transform the interactor to a diagonal matrix. This technique works for a large class of multivariable systems, except for certain special cases.

5.6.1 Precompensator Design

When the interactor ξ^{-1} is diagonal, the identification and Kalman filter estimation become relatively straight forward. The precompensator proposed by Singh and Narendra (1984) can be used to convert a triangular interactor into an equivalent diagonal form. The formal proof of the precompensator theory and the design is given in Singh and Narendra (1984). The design procedure is demonstrated using an example, in Appendix K.

A diagonal precompensator W(q) is said to exist such that T(q) W(q) has a diagonal interactor $\xi_{TW}(q)$ for almost all T(q), under the following conditions:

1. The relative degree of each element of T(q) is known.
2. The high frequency gain matrix K_p defined by

   $$K_p = E(T(q)) \tag{5.6.2}$$

   where

   $$E(T(q)) = \lim_{q \longrightarrow \infty} \operatorname{diag}[q^{r_1}\ q^{r_2} \cdot \cdot\ q^{r_p}]T(q) \tag{5.6.3}$$

   and r_j is the minimum delay in the j^{th} row of T(q), which satisfies $\epsilon K_p(q) + K_p^T(q)\epsilon = Q > 0$ for same $\epsilon = \epsilon^T > 0$ (also referred to as the sign definiteness condition).
3. An upper bound ν on the observability index of T(q) is known.
4. The zeros of T(q) are located inside the unit disc.

To obtain the precompensator it is not necessary to know the plant parameters. The only required apriori information is the time delay associated with each element of the transfer function matrix T(q). However it is important to note an assumption made in this design. That is if E(T(q)) results in a matrix in which all the diagonal elements can be made non zero by permutations of rows/columns, then such a matrix is not singular. Such a matrix is singular only on an algebraic hypersurface in the parameter space of the elements of the matrix. Non singularity of such a matrix is a generic property.

The type of interactor, i.e. diagonal or triangular, can be determined by testing E(T(q)). The interactor is diagonal if and only if E(T(q)), is non-singular. When it is singular the interactor is triangular.

Properties of the precompensator:

1. It is a dynamic precompensator which adds time delays to the original system T(q).
2. Stable.
3. Diagonal.
4. Non singular.

Let the precompensated process be given by $T_p(q)$.

$$T_p(q) = T(q)\ W(q) \tag{5.6.4}$$

using the interactor factorization,

$$T_p(q) = \xi_T^{-1}\ R_T(q) = \xi^{-1}R(q) \tag{5.6.5}$$

where ξ_T^{-1} and ξ^{-1} are diagonal now.

$$\xi^{-1} = \text{diag}\ [q^{-\lambda_1}\ q^{-\lambda_2}\ \cdot\ \cdot\ \cdot\ q^{-\lambda_p}] \tag{5.6.6}$$

The predictor equation is,

$$y_0(t) = R(q)u(t) = q^{-1}A^{-1}B\ u(t) \tag{5.6.7}$$

$$y(t) = \xi^{-1} y_0(t) \qquad (5.6.8)$$

In the state space formulation given by (5.5.1) to (5.5.2),

$$u^j(t) = u(t) = [u_1 \; u_2 \; \cdot \; \cdot \; u_m]^T$$

and $$\Lambda_j = \Lambda_{1j}$$

$$0$$

Thus the MIMO system is broken down to p independent MISO systems.

5.6.2 Parameter Identification

5.6.2.1 Multivariable Identification Methods

The multivariable system identification methods available in literature can be catagorized according to the type of model, (e.g. transfer function matrix, impulse response, input output difference equation and state space formulation) and the type of identification method, (e.g. generalized and extended least squares, correlation, stochastic approximation, maximum likelihood, equation error and instrumental variables). A good survey of the multivariable system identification algorithms is given in El-Sherief (1984).

There are a number of schemes to identify a state space model, e.g. El-Sherief and Sinha (1979a, 1979b), Guydorzi (1975), Sinha and Kwong (1979). Also see El-Sherief (1984) for other relevant literature. However this method cannot be employed in the KFP, because it is necessary to have a certain canonical form for the identification.

If the precompensator is used for the KFP design the parameters of the state space formulation are the same as the ARMA parameters of T(q) or the transfer function coefficients of the MISO formulation of T(q). Thus a method is required for identification of the transfer function matrix parameters. The state space formulation is based on p MISO sub systems, which are obtained by considering the least common denominator of the each row of the transfer function. This model was considered for off line identification by Mital and Chen (1974). Some basic results on the identification of this model and other types of models are given in El-Sherief (1974). Lennard and Blair (1981) have proposed an instrumental variable algorithm to estimate the parameters of the transfer function of MISO systems. El-Sherief and Sinha (1979c) proposed an algorithm which identifies the transfer function from noisy data.

5.6.2.2 Parameter Identification for KFP

The identification method used in the KFP is based on the method proposed by El-Sherief and Sinha (1979). First the system is decomposed into p subsystems, each of which corresponds to one row of the TFM and can be considered as a MISO system. An extended least squares algorithm suggested by Penuska (1968 and 1969), and

Young (1968) is used for the parameter identification of each subsystem.

Let the ARMAX model for the j^{th} subsystem of T(q) or $T_p(q)$ be written including the noise model of the subsystem as follows:

$$\alpha^j(q^{-1})y_j(t) = q^{-(\lambda_j+1)}\beta^j(q^{-1})u(t) + C^j(q^{-1})\omega_j(t) \tag{5.6.9}$$

where

$$\alpha^j(q^{-1}) = 1+a_1^j q^{-1}+\cdots+a_{nj}^j q^{-nj} \tag{5.6.10}$$

$$\begin{aligned}\beta^j(q^{-1})u(t) &= [\beta^{j1}(q^{-1})\ \beta^{j2}(q^{-1})\ \cdots\ \beta^{jm}(q^{-1})]u(t)\\ &= (\beta_1^j + \beta_2^j q^{-1} + \cdots + \beta_{n'j}^j)u(t)\end{aligned} \tag{5.6.11}$$

with $$\beta^{jk}(q^{-1}) = q^{-djk}(b_1^{jk} + b_2^{jk} q^{-1} + \cdots b_{nj}^{jk} q^{-nj}) \tag{5.6.12}$$

and

$$\begin{aligned}n'_j &= n_j + \max\ [d_{jk}] \text{ for } k=1,m\\ C^j(q) &= 1+c_1^j q^{-1} + \cdots + c_{1j}^j q^{-1j}\end{aligned} \tag{5.6.13}$$

α_j and C^j are monic by definition and have their zeros inside the unit circle. They also do not have any common factors.

The theoretical analysis of ERLS, shows that a persistently exciting input signal is also necessary to guarantee convergence of the parameters.

The regressor vector is,

$$\begin{aligned}\psi_j(t-1)^T = [&y_j(t-1), \cdot\cdot\cdot, y_j(t-n_j),\\ &u_1(t-d_{j1}-\lambda_j-1) \cdot\cdot\cdot, u_1(t-d_{j1}-\lambda_j-n_j-1)\\ &\cdot\cdot\cdot,\\ &\cdot\cdot\cdot,\\ &u_m(t-d_{jm}-\lambda_j-1), \cdot\cdot, u_m(t-d_{jm}-\lambda_j-n_j-1)\\ &\omega_j(t-1), \cdot\cdot\cdot, \omega_j(t-l_j)]^T \end{aligned} \qquad (5.6.14)$$

The parameter vector is,

$$\begin{aligned}\theta_j^T = [&a_1^j , a_2^j \cdot\cdot, a_{nj}^j,\\ &b_1^{j1} , \cdot\cdot, b_{nj}^{j1} , \cdot\cdot, \cdot\cdot, b_1^{jm} ,\\ &\cdot\cdot\cdot, b_{nj}^{jm} , c_1^j , \cdot\cdot, c_{l_j}^j]^T\end{aligned} \qquad (5.6.15)$$

$$y_j(t) = \psi_j(t-1)^T\theta_j + \omega_j(t) \qquad (5.6.16)$$

If $\hat{y}_j(t/t-1)$ is the a priori predicted output and $\theta(t)$ is the estimated parameter vector,

$$\hat{y}_j(t/t-1) = \psi(t-1)^T \theta_j(t-1) \qquad (5.6.17)$$

Gain Calculation:

$$G_j(t) = \frac{P_j(t-2)\psi_j(t-1)}{\mu_j+\psi^T(t-1)P_j(t-2)\psi(t-1)} \qquad (5.6.18)$$

Parameter estimation

$$\theta_j(t) = \theta_j(t-1) + G_j(t)\ [y_j(t) - \hat{y}_j(t/t-1)] \qquad (5.6.19)$$

Covariance update:

$$P_j(t-1) = \frac{1}{\mu_j}\ [I-G_j(t)\psi(t-1)^T]P_j(t-2) \qquad (5.6.20)$$

The estimation error e_j can be calculated in two ways:

1. $e_j(t) = y_j(t) - \hat{y}_j(t/t-1) = y_j(t) - \psi_j(t-1)^T\ \theta_j(t-1)$ (5.6.21)

known as the a priori prediction error.

2. $$e_j(t) = y_j(t) - \hat{y}_j(t/t) = y_j(t) - \psi_j(t-1)^T \theta_j(t) \qquad (5.6.22)$$

known as the a posteriori prediction error, which was first proposed by Young (1974). The idea is to use the most recent estimates of the parameters.

An important fact that has to be recognized in using the above identification for the AKFP is, that the converged true parameters of the process are estimated at the expense of estimating additional noise parameters. However these estimated noise parameters are not used any where else in the scheme.

5.6.3 Kalman Filter

Since the precompensated process has a diagonal interactor, the state space formulation has p independent MISO subsystems. An independent filter can be employed to estimate the states of the each subsystem. Since the parameters of each state space formulation are given by the identification of each subsystem using a RELS, the configuration of the MIMO adaptive Kalman filter can be schematically shown as in figure 5.3. Each independent adaptive Kalman filter consists of an independent parameter identification scheme, and a time varying Kalman filter which uses a state space formulation that depends on the estimated parameters. The time varying Kalman filter algorithm used in this scheme is given in section (2.3.1.1).

5.7 A "Self-Tuning" Multivariable Kalman Filter Predictor

An innovation model for the SISO Kalman filter predictor was discussed in Chapter 2 and 4. A 'self-tuning' Kalman filter predictor for SISO systems based on the innovation model was proposed in Chapter 4. The innovation model approach for the multivariable KFP when the interactor is triangular is given in Chapter 3. Because of the diagonal nature of the interactor used in the AKFP for MIMO systems, the innovation model becomes simpler.

From the innovation model approach presented in Chapter 3, and considering a diagonal interactor the ARMAX model for the j^th^ MIMO system can be written from equation (3.6.17) as follows:

$$\alpha_j(q^{-1})y_j(t) = q^{-(\lambda_j+1)}\ \beta_j(q^{-1})\ u(t) + F_j(t)(q^{-1})\ \omega_j(t) \tag{5.7.1}$$

where $\alpha_j(q^{-1})$ and $\beta_j(q^{-1})$ are as given in equation (5.6.9)

$$F_j(q^{-1}) = 1+f^j_1\ (t)q^{-1}+\ \cdot\ \cdot\ \cdot\ +\ f^j_{n_j+\lambda_j-1}(t)q^{-\lambda_j-n_j+1} \tag{5.7.2}$$

and the coefficients of F_j are given by,

$$F_j(q^{-1})(t) = \alpha_j(q^{-1}) + q^{-1}\alpha_j(q^{-1})K^j_2\ (t-1)+q^{-\lambda_j}K^j_1\ (t-\lambda_j) \tag{5.7.3}$$

where

$$K^j_1\ (t)(q^{-1}) = L^j_{n_j}(t)q^{-1} + L^j_{n_j-1}(t-1)q^{-2}+\ \cdot\ \cdot\ \cdot\ + L^j_1\ (t-n_j+1)q^{-n_j}1> \tag{5.7.4}$$

$$K^j_2\ (t)(q^{-1}) = L_{n_j+\lambda_j-1}(t)q^{-1}+\cdot\ \cdot\ \cdot+ L_{n_j+1}(t-\lambda_j+2)q^{-(\lambda_j-1)} \tag{5.7.5}$$

From equation (5.7.2) the coefficients of F_j can be expressed as follows:

Case 1. $n_j > (\lambda_j - 1)$

for $i \leq \lambda_j - 1$

$$f_i^j(t) = \sum_{l=0}^{i} L^j_{n_j+\lambda_j-l}(t-l)a^j_{i-l} \qquad (5.7.6)$$

with $a_0^j = 1$ and $L_{n_j+\lambda_j} = 1$

for $n_j \geq i > \lambda_j - 1$

$$f_i{}^j(t) = \sum_{l=0}^{\lambda_j-1} [\, L^j_{n_j+\lambda_j-l}(t-l)a^j_{i-l}] + L^j_{n_j+\lambda_j-1}(t-i) \qquad (5.7.7)$$

for $n_j+\lambda_j-1 \geq l \geq n_j$

$$f_i{}^j(t) = \sum_{l=i-n_j}^{\lambda_j-1} [\, L^j_{n_j+\lambda_j-l}(t-l)a^j_{i-l}] + L^j_{n_j+\lambda_j-1}(t-i) \qquad (5.7.8)$$

Case 2. $n_j = \lambda_j - 1$

for $i \leq \lambda_j - 1$

$$f_i^j\,(t) = \sum_{l=0}^{i} L^j_{n_j+\lambda_j-l}(t-l)\; a^j_{i-l} \qquad (5.7.9)$$

for $n_j+\lambda_j-1 \geq i > \lambda_j - 1$

$$f_i^j\,(t) = \sum_{l=0}^{nj} [L^j_{n_j+\lambda_j-l}(t-l)\; a^j_{i-l}] + L^j_{n_j+\lambda_j-1}(t-i) \qquad (5.7.10)$$

Case 3. $n_j < (\lambda_j - 1)$

for $i < n_j$

$$f^j_i\,(t) = \sum_{l=0}^{i} L^j_{n_j+\lambda_j-l}(t-1)\; a^j_{i-l} \qquad (5.7.11)$$

for $\lambda_j - 1 \geq i \geq n_j$

$$f^j_i\,(t) = \sum_{l=0}^{nj} [L^j_{n_j+\lambda_j-l}(t-1)\; a^j_{i-l} \qquad (5.7.12)$$

for $i \geq d$

$$f^j_i\,(t) = \sum_{l=i-\lambda_j+1}^{n_j} [L^j_{n_j+\lambda_j-l}(t-1) a^j_{i-l}] + L_{n_j+\lambda_j-i}(t-1) \qquad (5.7.13)$$

If the coefficients of f^j_1 to $f^j_{n_j+\lambda_j-1}$ are known then the Kalman gains L^j_{nj} to L^j_1 can be calculated sequentially, using equations (5.7.6 - 13).

As discussed in detail in section (4.4) for SISO systems, this leads to a multivariable 'self-tuning' Kalman filter predictor. Since the multivariable 'self-tuning' KFP consists of independent, MISO, self-tuning, KFPs, all the results obtained in section 4.4 for SISO STKFP are applicable to the MIMO case.

5.8 Adaptive Predictive Control System using AKFP

A minimum variance predictive control scheme based on the model of the process and the predictions from the KFP was presented in Chapter 2 for the SISO case and in Chapter 3 for the MIMO case.

In the AKFP the model of the process is obtained from the parameter identification scheme. The results obtained in Chapter 3 are directly applicable to the AKFP and are summarized here for convenience.

The adaptive predictive control law that minimizes the covariance of the tracking error, i.e. that minimizes the cost function:

$$J(t) = E\,\{[\bar{y}(t) - \bar{y}^*(t)]\,[\bar{y}(t) - \bar{y}^*(t)]^T\} \tag{5.8.1}$$

is given by,

$$u(t) = B_1^{-1}\,\{\bar{y}^*(t) - [I-A]\hat{y}^0(t) - [B-B_1]u(t)\} \tag{5.8.2}$$

where B_1 is non-singular as shown in Appendix I. It is important to note that, the predictive control law does not always give p independent MISO controllers, even though the MISO AKFPs are independent. Independent predictive controllers can be obtained only if B_1 is diagonal. A schematic block diagram of the multivariable adaptive predictive control scheme based on the AKFP is shown in figure 5.4. A detail discussion on the fixed version of the predictive control scheme was given in Chapter 3.

5.9 Multivariable Modified Adaptive Kalman Filter Predictor

The behaviour of the SISO KFP in the presence of deterministic disturbances, e.g. steps, was discussed in detail in Chapter 2. Due to the nature of the KFP, the predictions are biased, i.e. do not carry the full information about the disturbances. To overcome this problem the state space formulation of the KFP is augmented with an additional state corresponding to the stochastic model of the noise. This was proposed by Balcan et al (1970, 1973) and Bialkowski (1983). This modification in effect results in a PID estimation scheme which gives unbiased prediction.

This modified KFP was extended to its adaptive version by incorporating an incremental identification scheme, as discussed in Chapter 4.

The adaptive Modified Kalman filter predictor presented in Chapter 4 can easily be extended to multivariable systems. Since the multivariable AKFP consists of p independent MISO AKFPs, the results obtained in Chapter 4 are reproduced here for each MISO subsystem.

5.9.1 State Space Formulation

The stochastic model for the deterministic disturbances is given by (see Balcan et al (1970, 1973), Tuff et al, 1985 and Chapter 2),

$$\epsilon_j(t) = \frac{w_j(t)}{\Delta} \qquad (5.9.1)$$

where $\Delta = 1-q^{-1}$ - differencing operator.

$w_j(t)$ - random signal, generally zero but may attain values pi at arbitrary time instants i.

$\epsilon_j(t)$ - series of steps of height pi starting at time i.

The augmented state space formulation of the j^{th} KFP is given by,

$$x^j(t+1) = \begin{bmatrix} 1 & 0 & 0 \\ \Gamma_j & \Phi_{j1} & 0 \\ 0 & 0 & \Phi_{j2} \end{bmatrix} x^j(t) + \begin{bmatrix} 0 \\ \Lambda_1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w_j(t) \qquad (5.9.2)$$

$$y(t) = [0 : 0 : \Theta_1^j]x^j(t) + v_j(t) \tag{5.9.3}$$

where

$$x^j(t) = [x_\rho^j, x_1^j, x_2^j, \cdot \cdot, x_{n_j}^j, x_{n_j-1}^j, \cdot \cdot, x_{n_j+\lambda_j}^j]$$

5.9.2 Modified Adaptive Kalman Filter Predictor

Consider the ARIMA model (Tuff et al, 1985) for the jth MISO system with deterministic disturbances.

$$\alpha_j(q^{-1})y_j(t) = q^{-(1+\lambda_j)}B_j(q^{-1})\ u(t) + \frac{w_j(t)}{\Delta} \tag{5.9.4}$$

$$\alpha_j(q^{-1})\Delta y_j(t) = q^{-(1+\lambda_j)}B_j(q^{-1})\ \Delta u(t) + w_j(t)$$

If we define the regressor vector $\psi_j(t-1)$ as,

$$\begin{aligned}\psi_j(t-1)^T = [&\Delta y_j(t-1), \cdot \cdot, \Delta y_j(t-n_j)\\ &\Delta u_1(t-d_{j1}-\lambda_j-1), \cdot \cdot, \Delta u_1(t-d_{j1}-\lambda_j-n_j-1)\\ &\quad, \cdot \cdot \cdot,\\ &\quad, \cdot \cdot \cdot,\\ &\Delta u_m(t-d_{jm}-\lambda_j-1), \cdot \cdot, \Delta u_m(t-d_{jm}-\lambda_j-n_j-1)]^T\end{aligned} \tag{5.9.5}$$

and the parameter vector as,

$$\begin{aligned}\theta_j(t) = [a_1^j, a_2^j, \cdot \cdot, a_{nj}^j, b_1^{j1}, \cdot \cdot, b_{nj}^{j1}, \cdot \cdot\\ b_1^{jm}, \cdot \cdot, b_{nj}^{jm}]^T\end{aligned} \tag{5.9.10}$$

From (5.9.4),

$$y_j(t) = \psi_j(t-1)\theta_j(t)^T + w_j(t) \tag{5.9.11}$$

Since $w_j(t)$ is an uncorrelated noise signal and it is also uncorrelated with $\psi_j(t-1)$, a RLS identification scheme is used to estimate the parameters in (5.9.10). This is a RLS algorithm using incremental variables. The identified parameters can be used in the AKFP and in the control system design.

5.9.3 Incremental Adaptive Predictive Control Scheme

If the unbiased prediction $\hat{y}^0(t)$ is used directly in the predictive control law given by equation (5.8.2) the output control will be biased in the presence of deterministic disturbances. This can be corrected by including a bias term in the predictive control law. This bias term could be identified on-line by introducing an additional parameter into the parameter vector $\bar{\theta}(t)$and using a normal RLS estimation scheme. Alternatively it is possible to use the incremental predictive controller discussed in Chapters 2 and 4 for SISO systems.

The incremental control law is given by,

$$\Delta u(t) = B_1^{-1}\ \{\delta\bar{y}^*(t) - [I-A]\Delta\hat{y}^0(t) - [B-B_1]u(t)\} \qquad (5.9.12)$$

where B_1 is non-singular and,

$$\delta\bar{y}^*(t) = \bar{y}^*(t) - \hat{y}^0(t) \qquad (5.9.13)$$

$$\Delta\hat{y}^0(t) = \hat{y}^0(t) - \hat{y}^0(t-1) \qquad (5.9.14)$$

5.10 Simulation Results and Discussion

Two processes were used for simulation studies. The distillation column model used in section 3.10 for the fixed KFP, has a diagonal interactor. Hence it is not necessary to use a precompensator.

To demonstrate the use of a precompensator the 2nd model used in section 3.10 was considered.

The discrete transfer function of the process model is given by,

$$T(q) = \begin{bmatrix} \dfrac{0.2658q^{-4}-0.061q^{-5}}{1-1.6q^{-1}+0.63q^{-2}} & \dfrac{0.012q^{-5}+0.0044q^{-6}}{1-1.6q^{-1}+0.63q^{-2}} \\[2ex] \dfrac{0.083q^{-5}+0.03q^{-6}}{1-1.6q^{-1}+0.63q^{-2}} & \dfrac{0.1063q^{-6}-0.0244q^{-7}}{1-1.6q^{-1}+0.63q^{-2}} \end{bmatrix}$$

The inverse interactor for this process is triangular,

$$\xi^{-1}(q) = \begin{bmatrix} q^{-3} & 0 \\ 0.3198q^{-4} & q^{-5} \end{bmatrix}$$

Using the precompensator design procedure given in Appendix K, the precompensator for this process is

$$W(q) = \begin{bmatrix} q^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

and the interactor for the precompensated process is given by,

$$\xi^{-1}(q) = \begin{bmatrix} q^{-4} & 0 \\ 0 & q^{-5} \end{bmatrix}$$

which is diagonal.

It is important to note that the coefficient matrix B_1 is not diagonal.

From Appendix I,

$$B_1 = \lim_{q \longrightarrow \infty} \xi(q)T_w(q) = \begin{bmatrix} 0.2658 & 0.012 \\ 0.083 & 0.1063 \end{bmatrix}$$

Clearly B_1 is non-singular.

The nature of B_1 can be found without knowing the actual parameters. However it is assumed that even if B_1 is not diagonal, it is always non-singular, due to the generic property of such matrices.

Case 1 . Distillation Column

1a. Deterministic case

Periodic setpoint changes are introduced into both y_1 and y_2 to obtain converged true parameters of the process. Output performance of the AKFP and AIP is shown in figures 5.5a and b. Under noise free conditions once the parameters have converged to their true values the AKFP and AIP perform exactly the same, and are same as the corresponding non-adaptive schemes.

Figures 5.5c and d show the manipulated variables. Figures 5.5e and f show the parameter estimates of the AKFP. All the parameters converged to their true values within 100 iterations.

1b. Stochastic Case

Figures 5.6a and b show the output performance of the AKFP and AIP. For the top composition y_1 the performance of AKFP and AIP are quite the similar. This is also seen in figure 5.6e which plots the variance of the output tracking error. The small difference in performance of the AKFP vs the AIP for y_1 may be due to the small time delay (λ_1=1) in the top composition. The minimum variance prediction in AKFP may not be significant for single time delay. However this results depends on the process dynamics too. The AKFP shows significantly better performance than the AIP for the bottom composition. Parameter estimates by the AKFP are shown in figures 5.6f and g. The estimated values of the parameters at the 800th iteration are given in the Table 5.1.

Case 2.. Process with the precompensator

2a. Deterministic Case

It is clear from figures 5.7a and b, that the AKFP and ASP show the same performance. Introducing the precompensator does not effect the performance under ideal conditions, but since it introduces additional time delays, it demands the knowledge of the future set points. Figures 5.7c and d show the manipulated variables. The parameter estimation is given in figures 5.7e and f. The parameter convergence is very fast in this case.

2b. Stochastic Case

Output performance of the AKFP and AIP is shown in figures 5.8a and b. As shown in figure 5.8e the AKFP gives significantly lower variance than the AIP. However it is important to note that, although introducing the precompensator does not effect the performance in the deterministic case it would effect the performance in the stochastic case. Introducing the time delays in the precompensator would increase the prediction error variance and thus the output tracking error variance. Parameter estimation in the AKFP is shown in figures 5.8f and g. The parameters of the A and B polynomials converges very fast. But the c coefficients are very slow to converge. The estimated value of the parameters at the 500th iteration is given in Table 5.2.

5.11 Conclusions

1. Due to the difficulties involved in the identification of the interactor matrix, a rigorous adaptive extension of the multivariable KFP is not possible when the interactor is triangular. However by introducing a suitably designed precompensator to diagonalize the interactor, the adaptive KFP can be developed for processes with triangular interactor. The resulting Multivariable adaptive Kalman filter predictor consists of p independent MISO adaptive Kalman filter predictors. Only the ARMA parameters of the MIMO process are needed and also the MISO Kalman filters can be implemented with a computationally fast algorithm. This gives a practical, AKFP and adaptive Predictive Control system for stochastic processes with delays.
2. The ratio of the noise covariances can be used as a tuning parameter for the AKFP to obtain minimum variance or nearly minimum variance control performance.
3. The multivariable extension of the SISO self-tuning KFP which provides a means of implementing the AKFP without having to know the covariances a priori, or tuning, is also derived.
4. A multivariable extension of the SISO adaptive modified KFP and the adaptive incremental predictive control scheme is presented for processes with deterministic disturbances.

Table 5.1 Parameter Estimation for the Distillation Column Model

Parameter	True Values	Estimated Values	
		Deterministic[1]	Stochastic[2]
$-a_1^1$	1.8954	1.895	1.891
$-a_2^1$	-0.8981	-0.893	-0.894
b_1^{11}	-0.8975	0.898	0.899
b_2^{11}	-0.8558	-0.852	-0.853
b_1^{12}	-0.8719	-0.872	-0.872
b_2^{12}	0.8212	0.816	0.816
c_1^1	?	.	-1.286
c_2^1	?	.	0.375
$-a_1^2$	1.845	1.843	1.834
$-a_2^2$	-0.8509	-0.85	-0.845
b_1^{21}	0.5786	0.579	0.578
b_2^{21}	-0.5398	-0.539	-0.538
b_1^{22}	-1.301	-1.301	-1.30
b_2^{22}	1.1865	1.184	1.180
c_1^2	?	.	-1.24
c_2^2	?	.	0.424

1 after 200 sampling instants

2 after 800 sampling instants

Table 5.2 Parameter Estimation for the Process with a Precompensator

Parameter	True Values	Estimated Values	
		Deterministic[1]	Stochastic[2]
$-a_1^1$	1.6	1.60	1.598
$-a_2^1$	-0.63	-0.629	-0.629
b_1^{11}	0.2658	0.2657	0.2656
b_2^{11}	-0.061	-0.0609	-0.0603
b_1^{12}	0.012	0.412	-0.012
b_2^{12}	0.0044	0.0044	0.0047
c_1^1	?	.	-0.8447
c_2^1	?	.	0.1149
$-a_1^2$	1.6	1.6	1.59
$-a_2^2$	-0.63	-0.629	-0.625
b_1^{21}	0.083	0.084	0.084
b_2^{21}	0.03	0.029	0.034
b_1^{22}	0.1063	0.10629	0.106
b_2^{22}	-0.0244	-0.02439	-0.0192
c_1^2	?	.	-0.98
c_2^2	?	.	0.3425

[1] after 200 sampling instants

[2] after 800 sampling instants

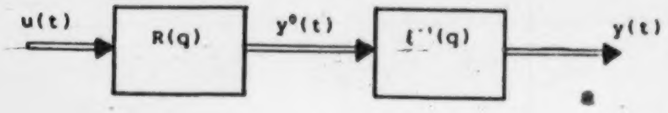

u(t)
R(q)
y°(t)
ξ⁻¹(q)
y(t)

Figure 5.1 Schematic block diagram of a MIMO process with time delays, based on the interactor factorization.

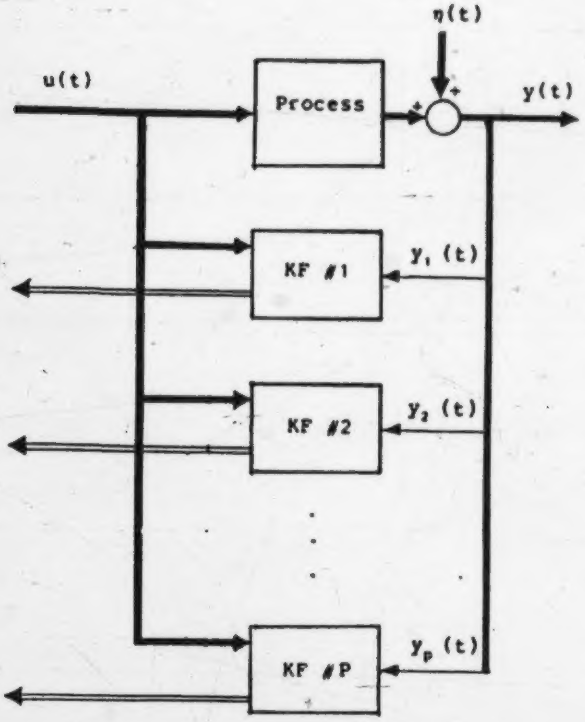

η(t)
u(t)
Process
+
+
y(t)
KF #1
y₁ (t)
x̂¹
KF #2
y₂ (t)
x̂²
KF #P
yₚ (t)
x̂ᵖ

Figure 5.2 Multivariable Kalman Filter Predictor Configuration for a Diagonal Interactor (p Independent MISO Kalman Filter Predictors).

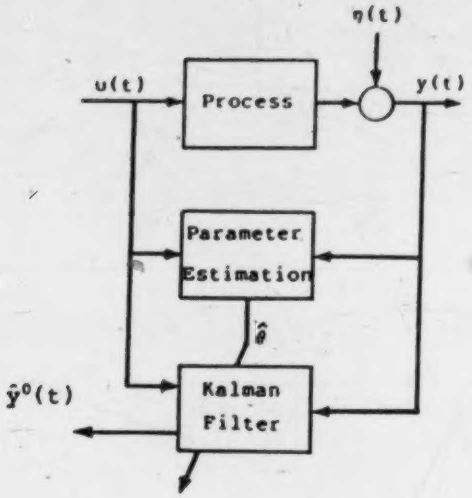

η(t)
u(t)
Process
y(t)
Parameter
Estimation
$\hat{\theta}$
$\hat{y}^0(t)$
Kalman
Filter

Figure 5.3 Schematic Block Diagram of the Multivariable Adaptive Kalman Filter Predictor.

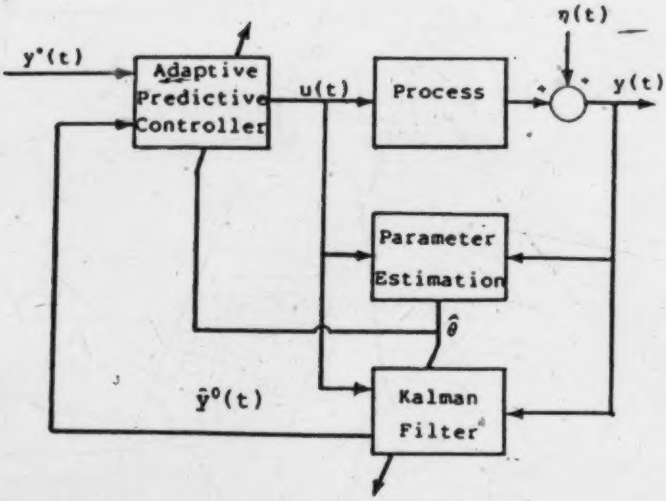
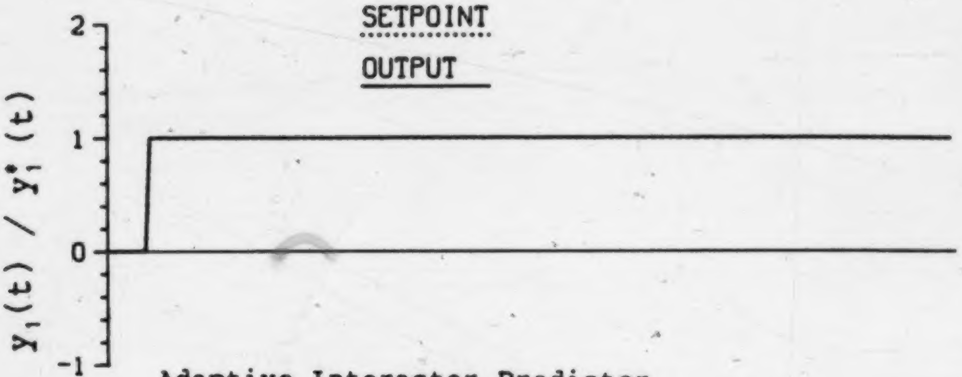
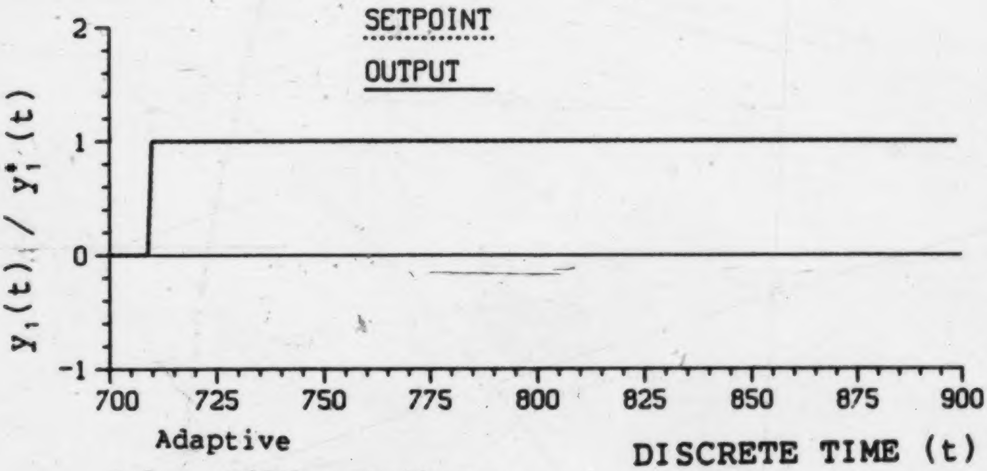

η(t)
$y^*(t)$
Adaptive
Predictive
Controller
u(t)
Process
+
+
y(t)
Parameter
Estimation
$\hat{\theta}$
$\hat{y}^0(t)$
Kalman
Filter

Figure 5.4 Schematic Block Diagram of the Multivariable Adaptive Predictive Control Scheme.

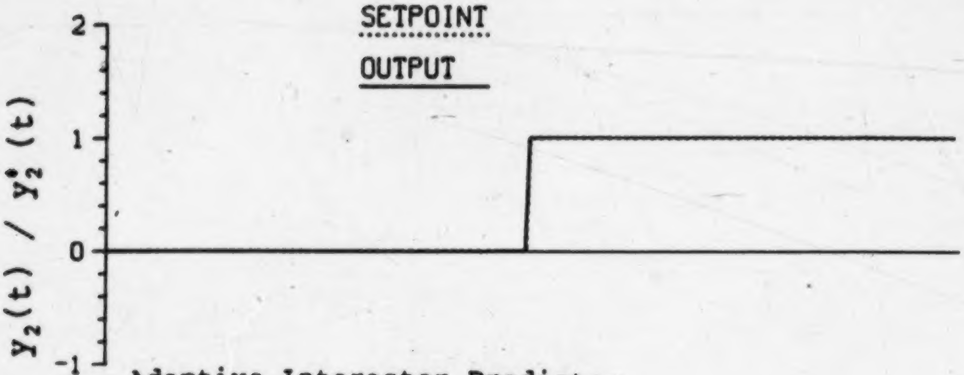


Adaptive Interactor Predictor

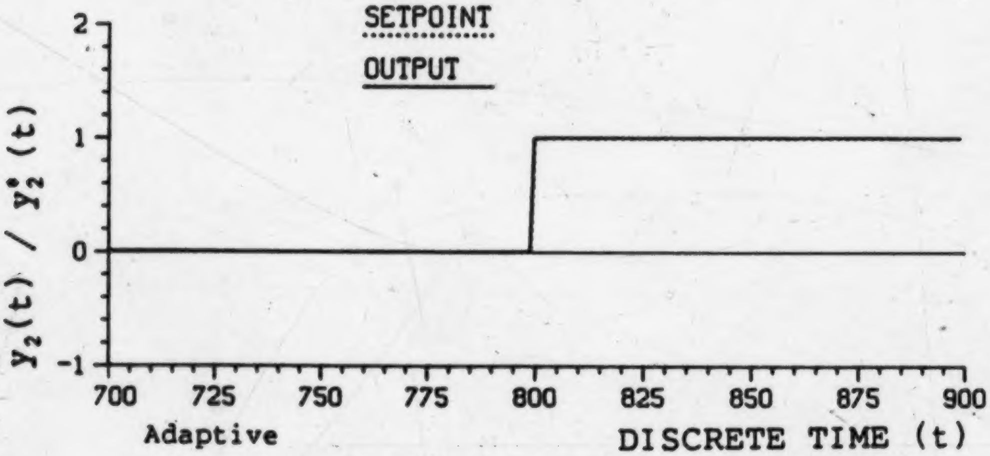


Adaptive
Kalman Filter Predictor

Figure 5.5a Adaptive Predictive Control of the Top Composition Using AKFP and AIP (Deterministic Case).

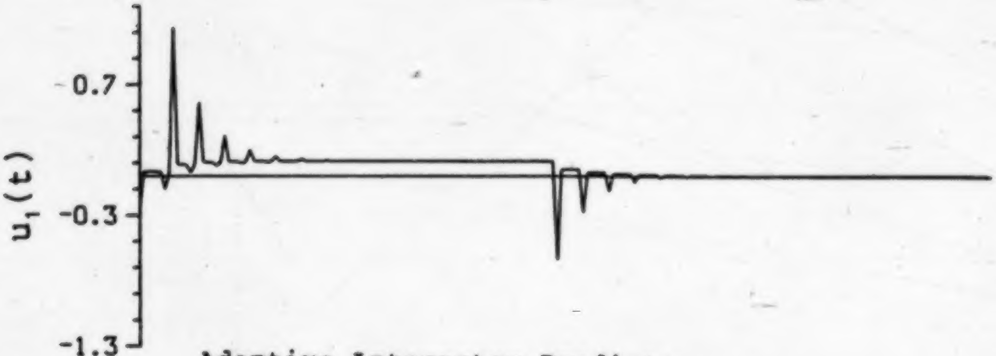


Adaptive Interactor Predictor

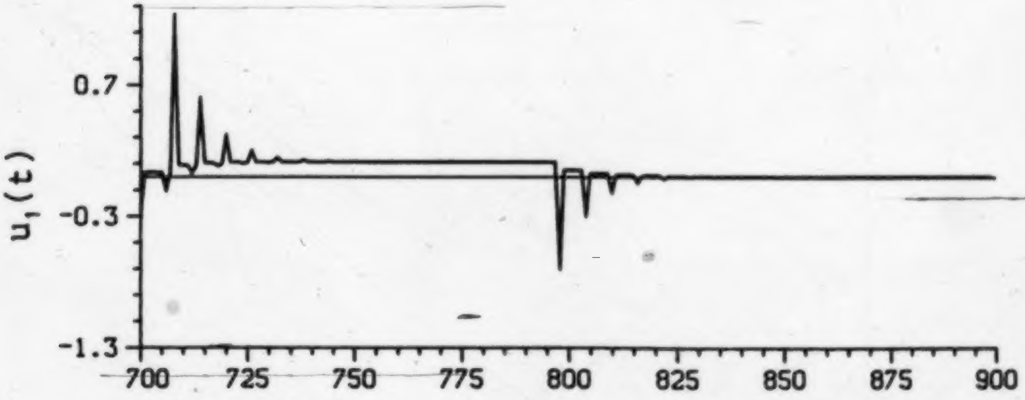


Adaptive Kalman Filter Predictor

Figure 5.5b Adaptive Predictive Control of the Bottom Composition Using AKFP and AIP (Deterministic Csae).

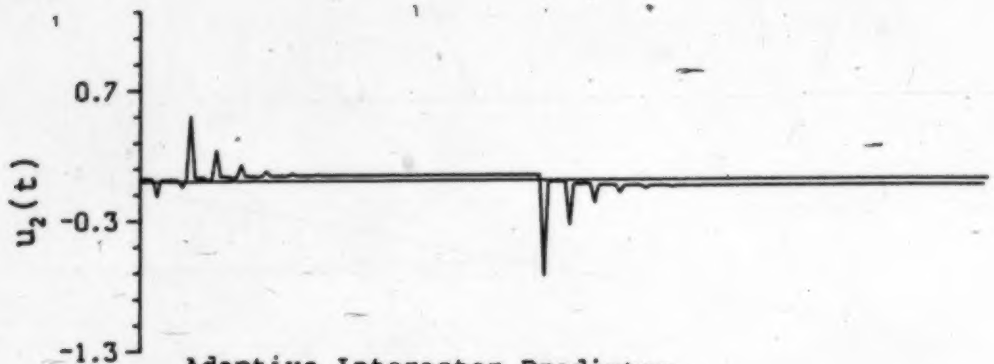


Adaptive Interactor Predictor

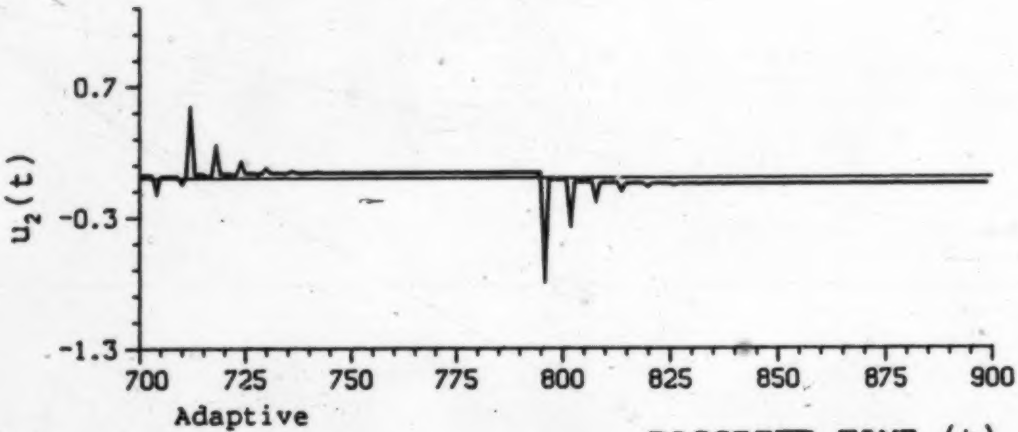


Adaptive Kalman Filter Predictor

Figure 5.5c Adaptive Predictive Control of the Top Composition Using AKFP and AIP (Deterministic Case).



Adaptive Interactor Predictor



Adaptive

Kalman Filter Predictor

Figure 5.5d Adaptive Predictive Control of the Bottom Composition Using AKFP and AIP (Deterministic Csae).

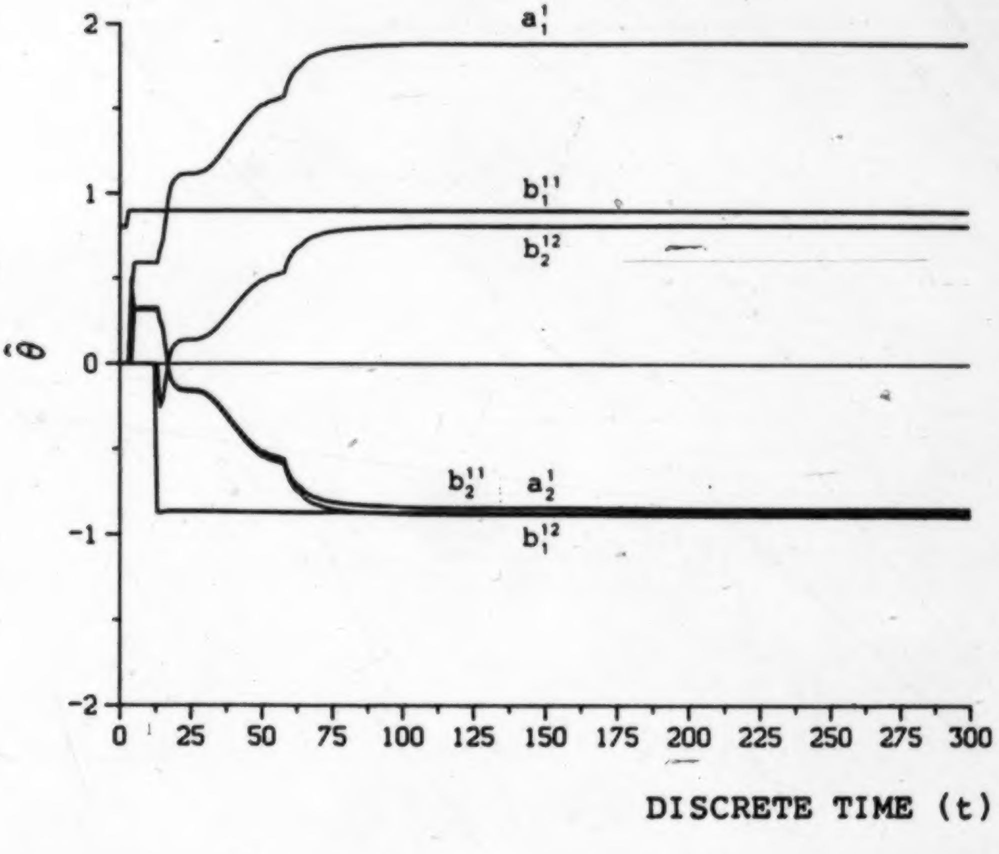


Figure 5.5e Parameter Estimation in AKFP for the First MISO Subsystem (using Extended Least Squares Identification).

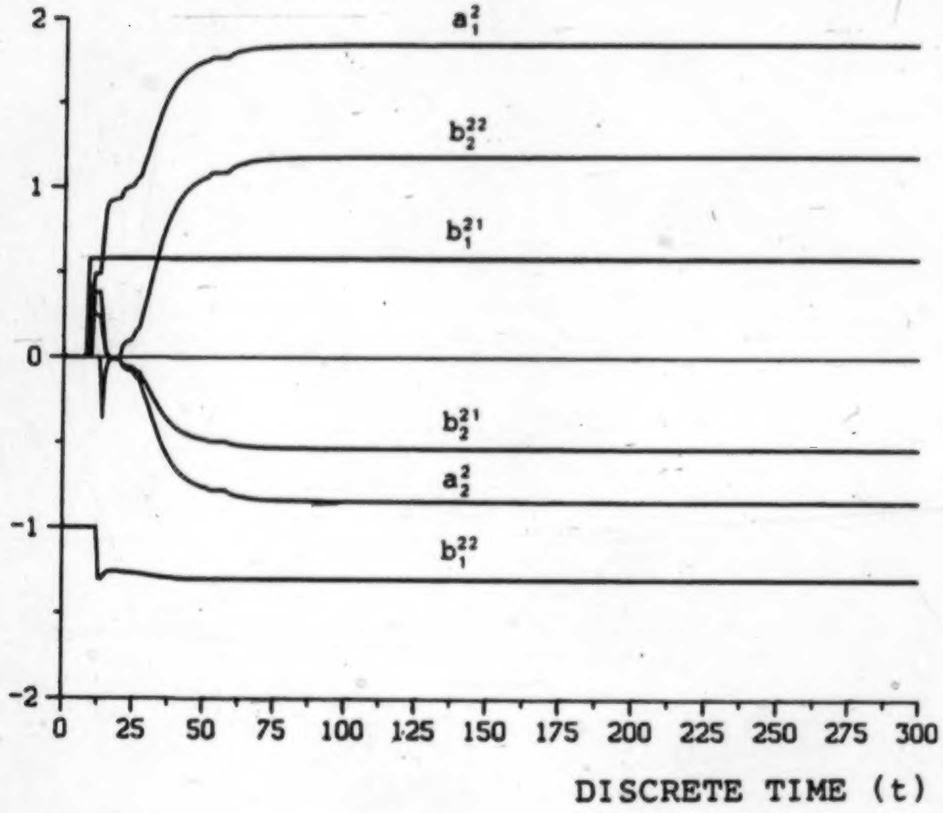
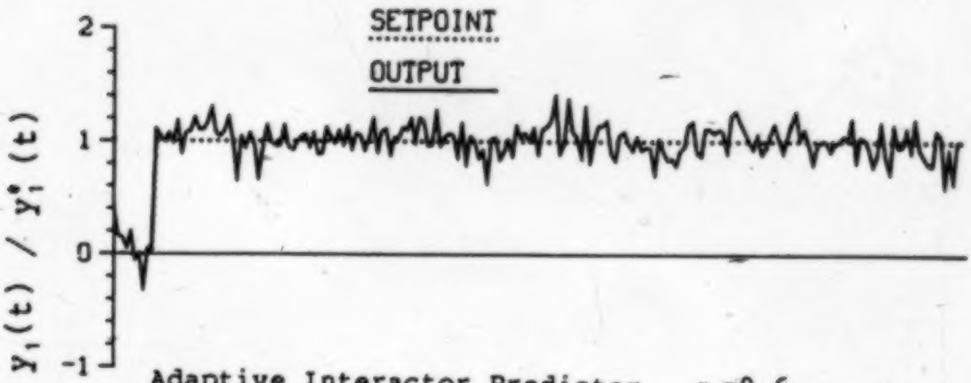
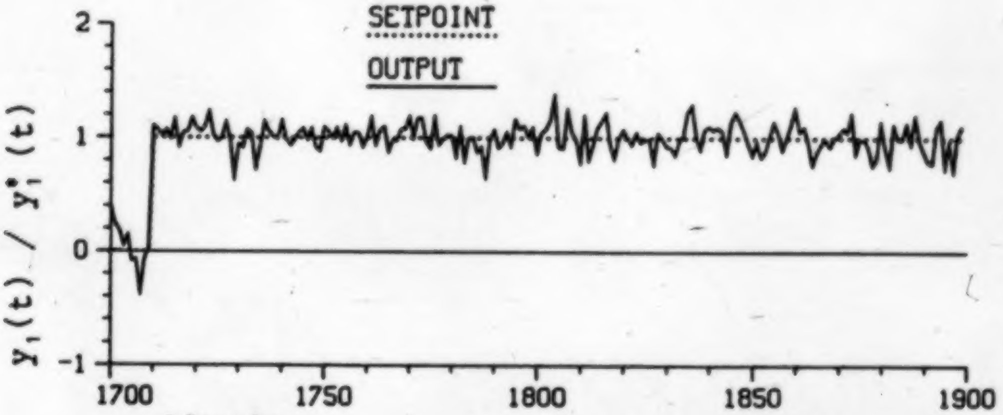


Figure 5.5f Parameter Estimation in AKFP for the Second MISO Subsystem (using Extended Least Squares Identification).

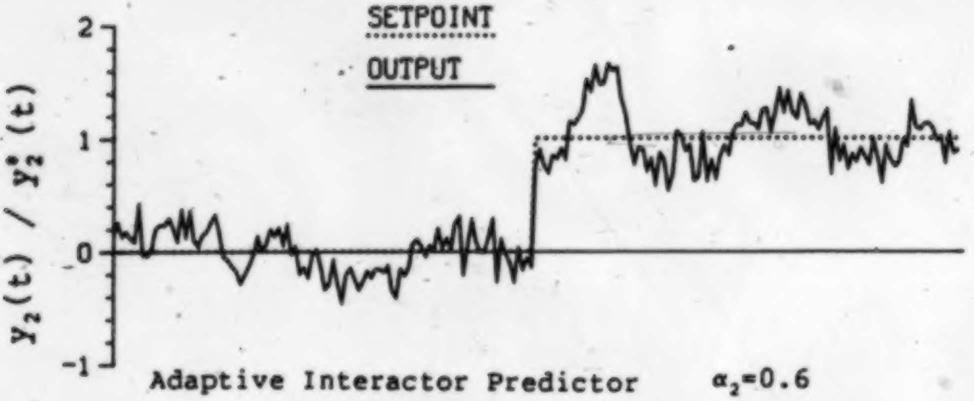


Adaptive Interactor Predictor $\alpha_1=0.6$

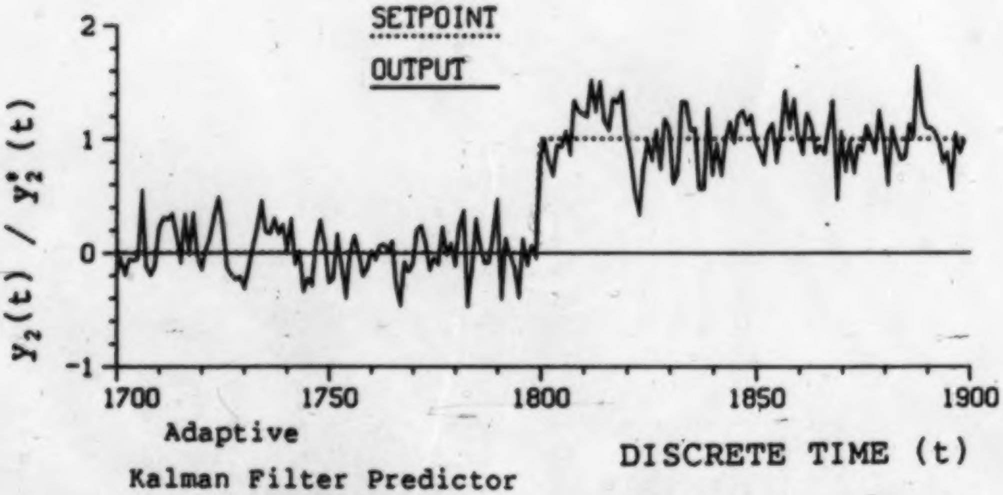


Adaptive
Kalman Filter Predictor
$R_{v_1}/R_{v_1}=0.05$

Figure 5.6a Adaptive Predictive Control of the Top Composition Using AKFP and AIP (Stochastic Case).

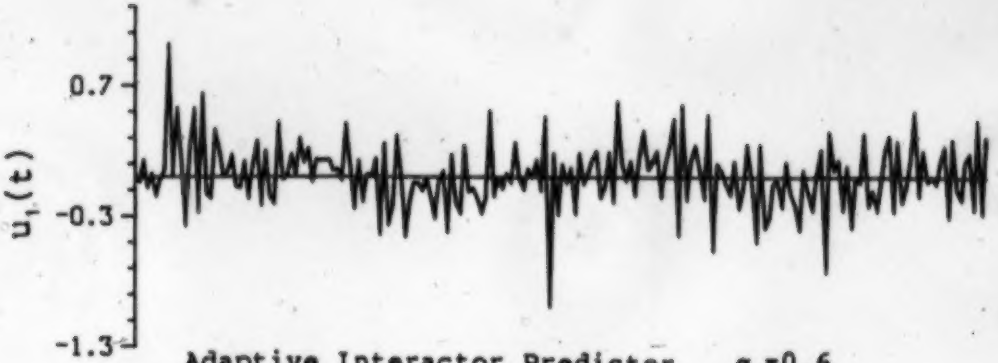


Adaptive Interactor Predictor $\alpha_2=0.6$

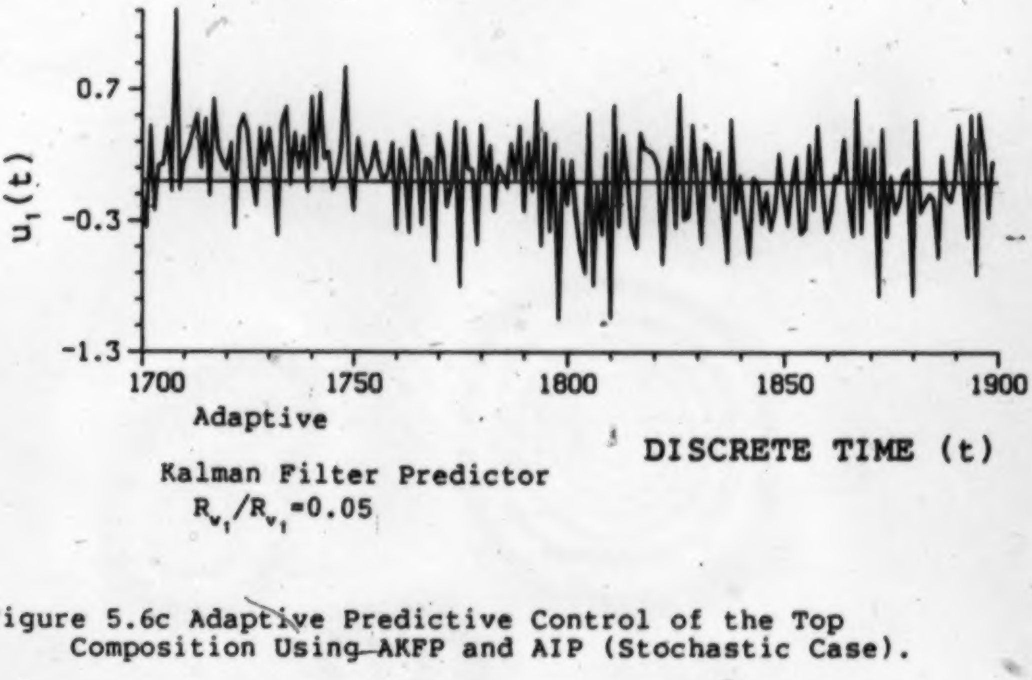


Adaptive
Kalman Filter Predictor
$R_{v_2}/R_{v_2}=0.05$

Figure 5.6b Adaptive Predictive Control of the Bottom Composition Using AKFP and AIP (Stochastic Case).



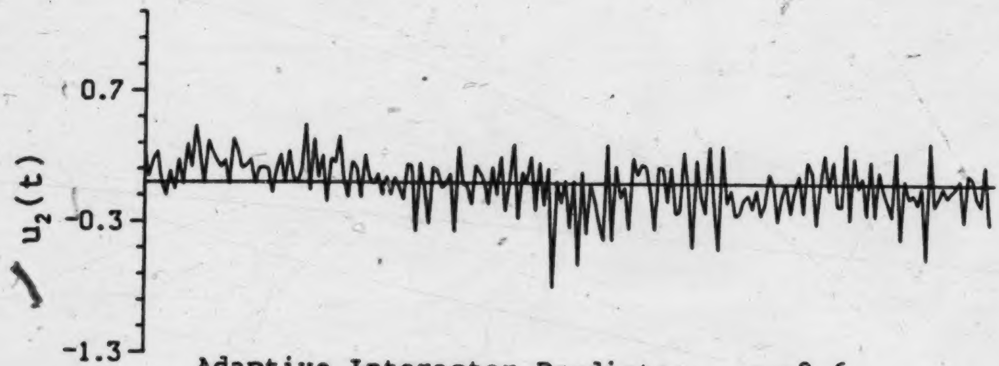
Adaptive Interactor Predictor α_1=0.6



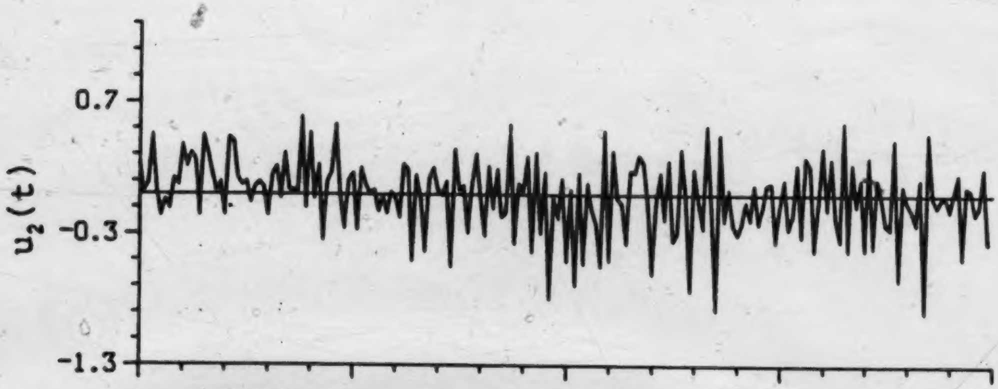
Adaptive

Kalman Filter Predictor
R_{v_1}/R_{v_1}=0.05

Figure 5.6c Adaptive Predictive Control of the Top Composition Using AKFP and AIP (Stochastic Case).



Adaptive Interactor Predictor $\alpha_2=0.6$



Adaptive

Kalman Filter Predictor
$R_{v_2}/R_{v_2}=0.05$

Figure 5.6d Adaptive Predictive Control of the Bottom Composition Using AKFP and AIP (Stochastic Case).

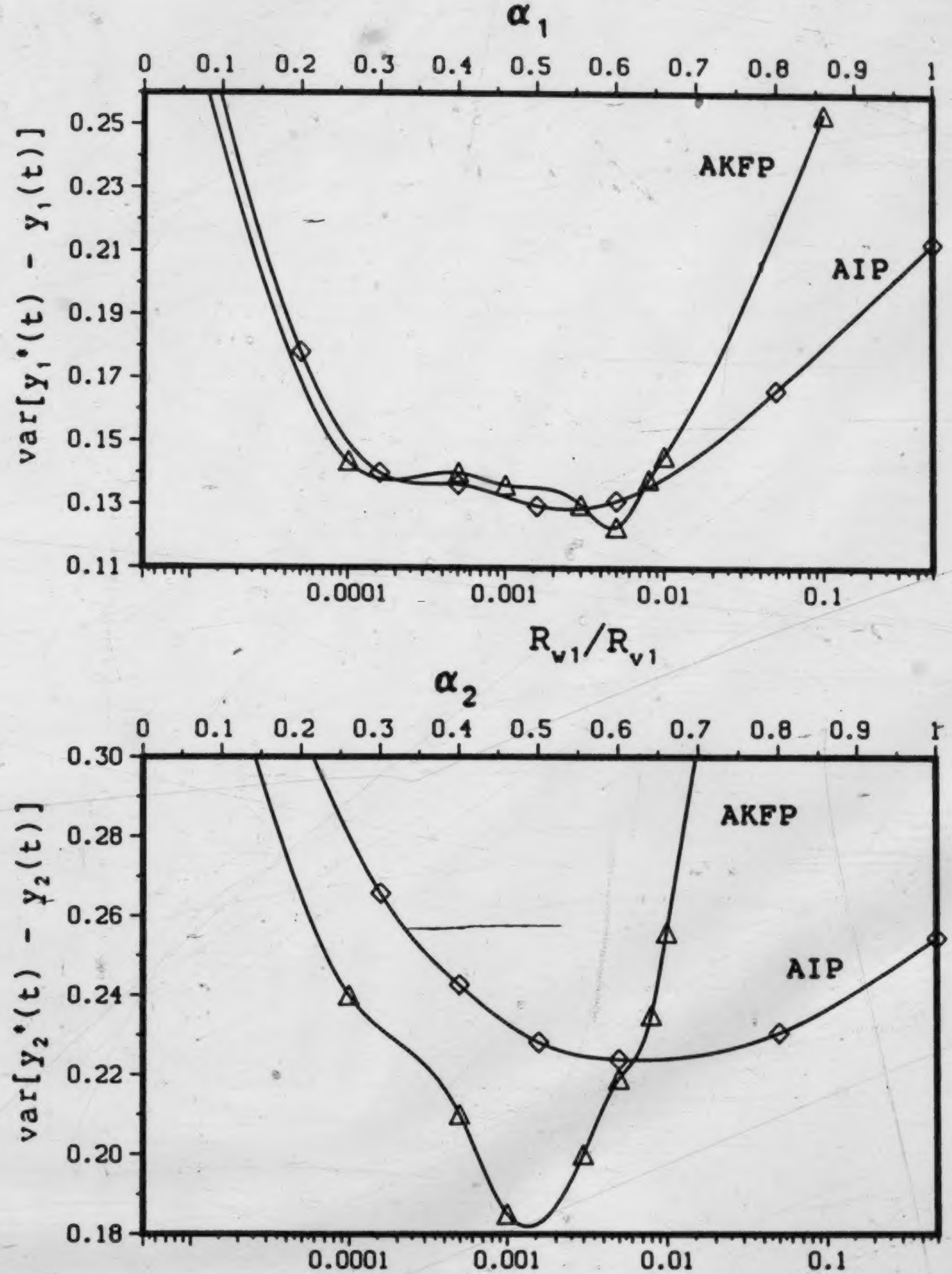


Figure 5.6e Variance of Tracking Error for AKFP and AIP.

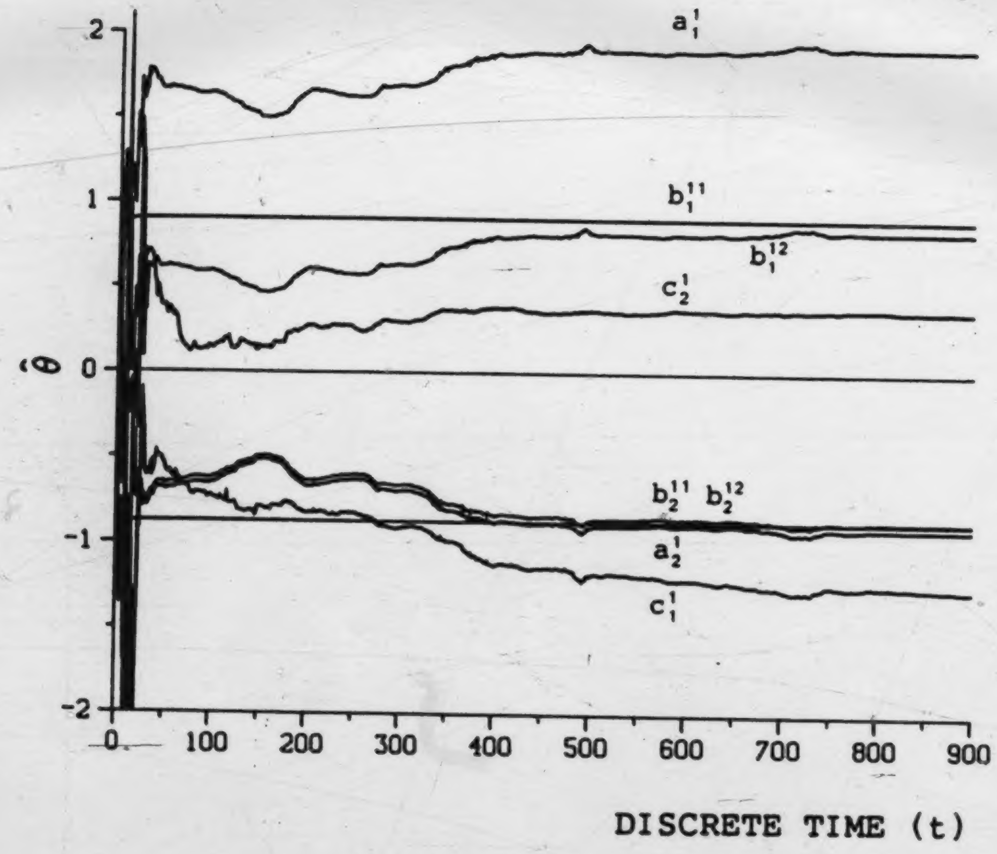


Figure 5.6f Parameter Estimation in AKFP for the First MISO Subsystem (using Extended Least Squares Identification).

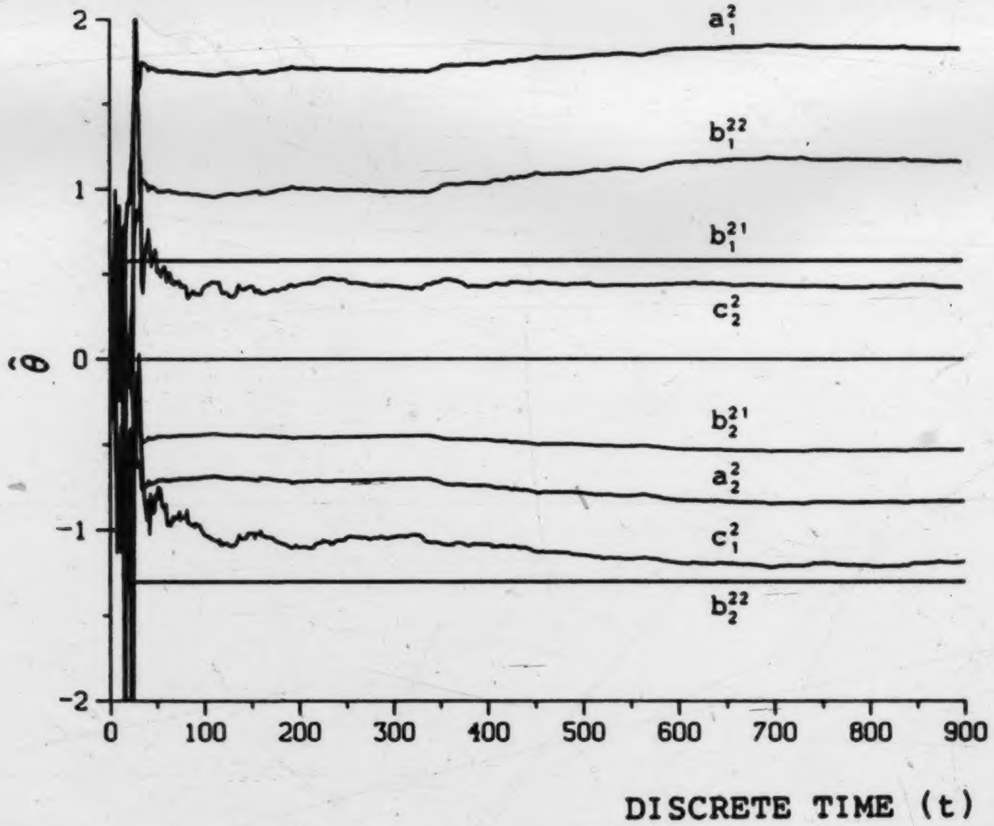
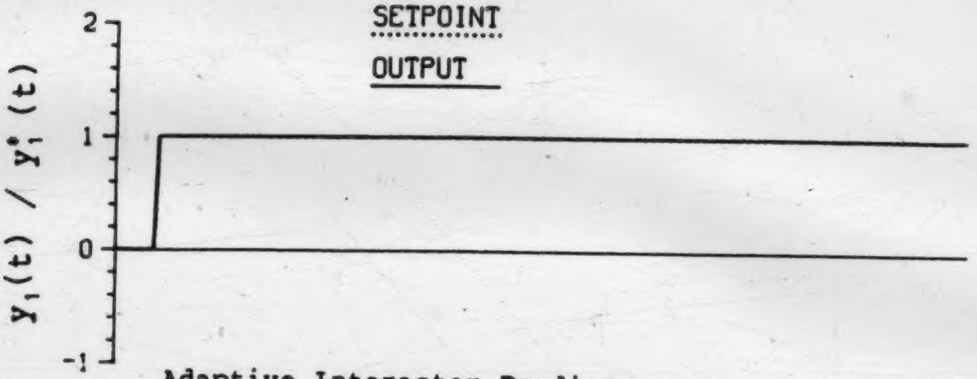
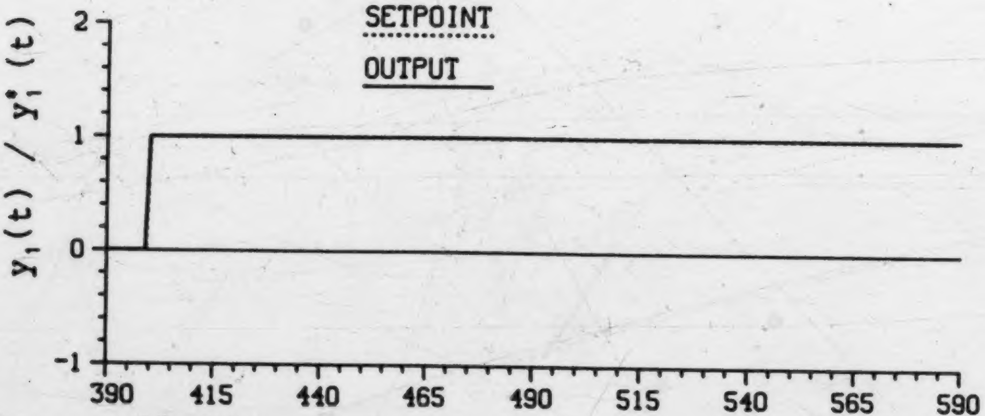


Figure 5.6g Parameter Estimation in AKFP for the Second MISO Subsystem (using Extended Least Squares Identification).

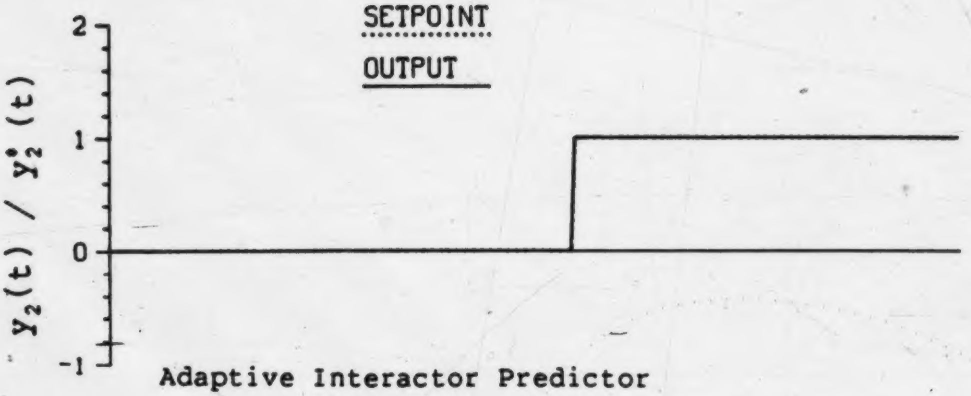


Adaptive Interactor Predictor

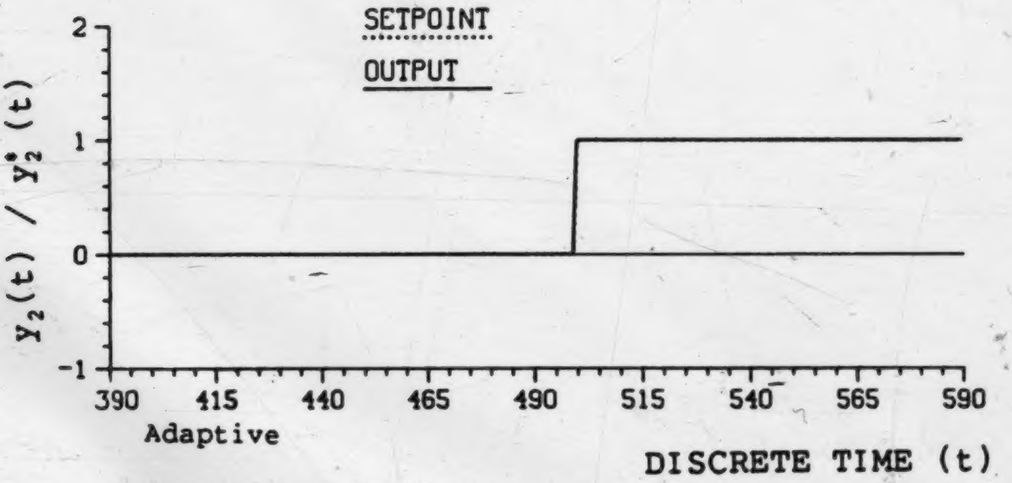


Adaptive Kalman Filter Predictor

Figure 5.7a Adaptive Predictive Control of y_1 using AKFP and AIP (Deterministic Process with a Precompensator).



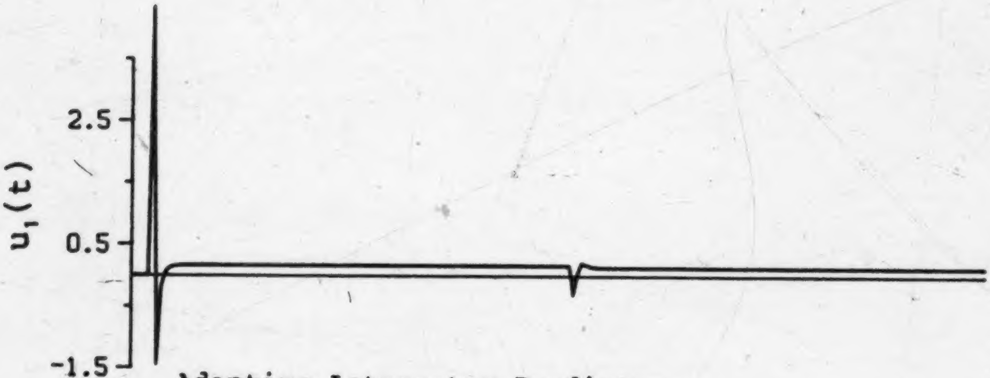
Adaptive Interactor Predictor



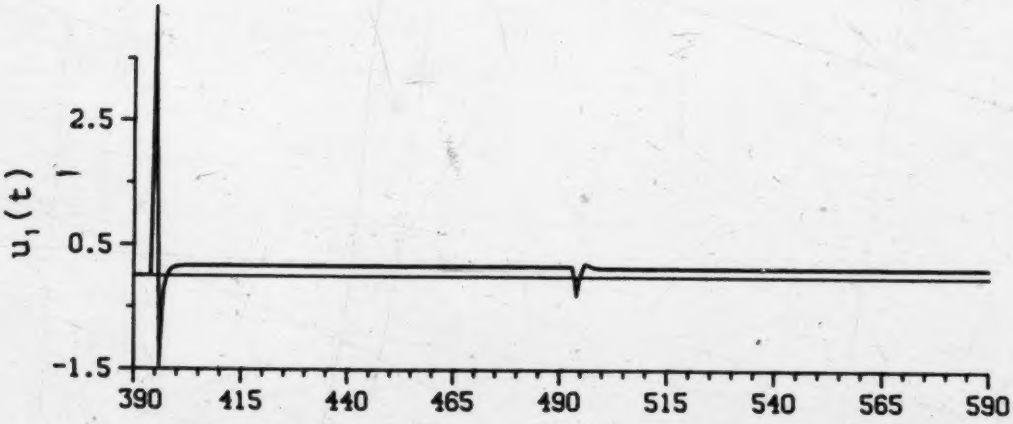
Adaptive

Kalman Filter Predictor

Figure 5.7b Adaptive Predictive Control of y_2 using AKFP and AIP (Deterministic Process with a Precompensator).



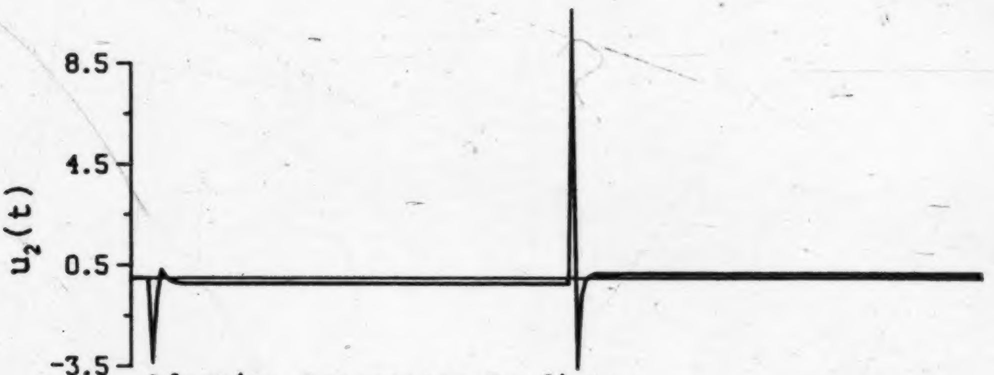
Adaptive Interactor Predictor



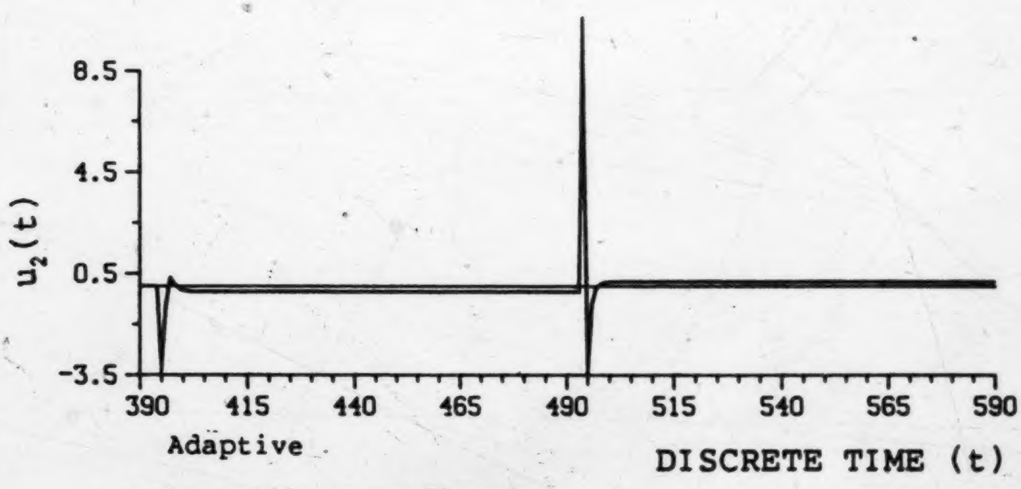
Adaptive

Kalman Filter Predictor

Figure 5.7c Adaptive Predictive Control of y_1 using AKFP and AIP (Deterministic Process with a Precompensator).



Adaptive Interactor Predictor



Adaptive Kalman Filter Predictor

Figure 5.7d Adaptive Predictive Control of y_2 using AKFP and AIP (Deterministic Process with a Precompensator).

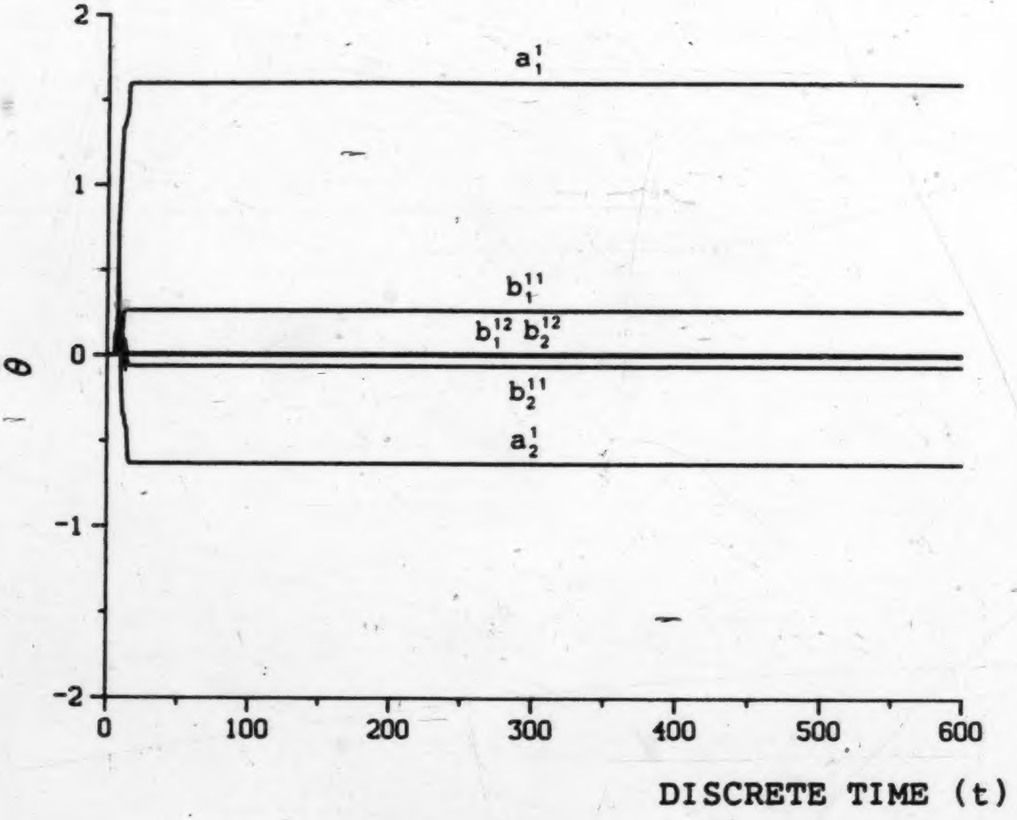


Figure 5.7e Parameter Estimation in AKFP for the First MISO Subsystem (using Extended Least Squares Identification).

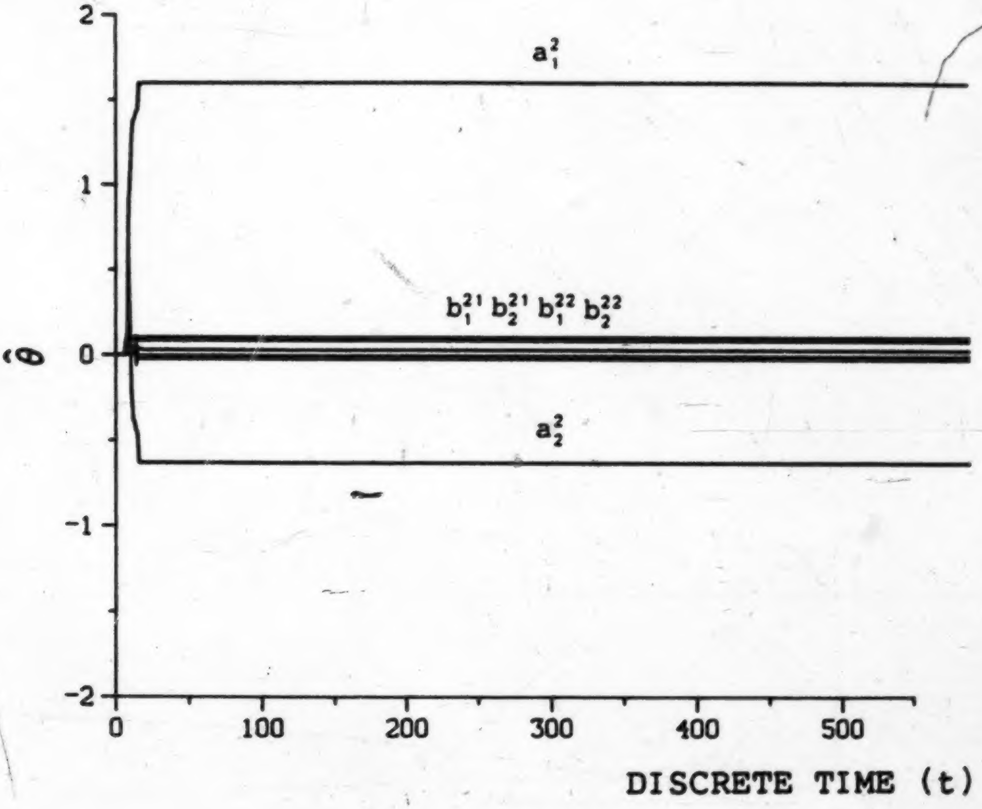
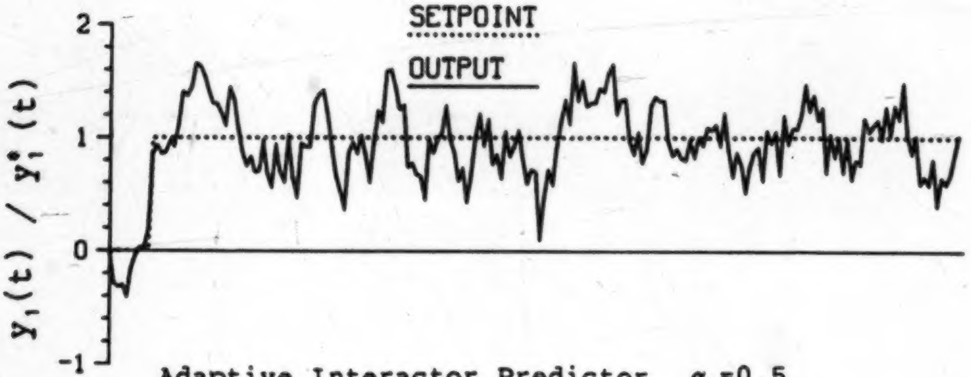
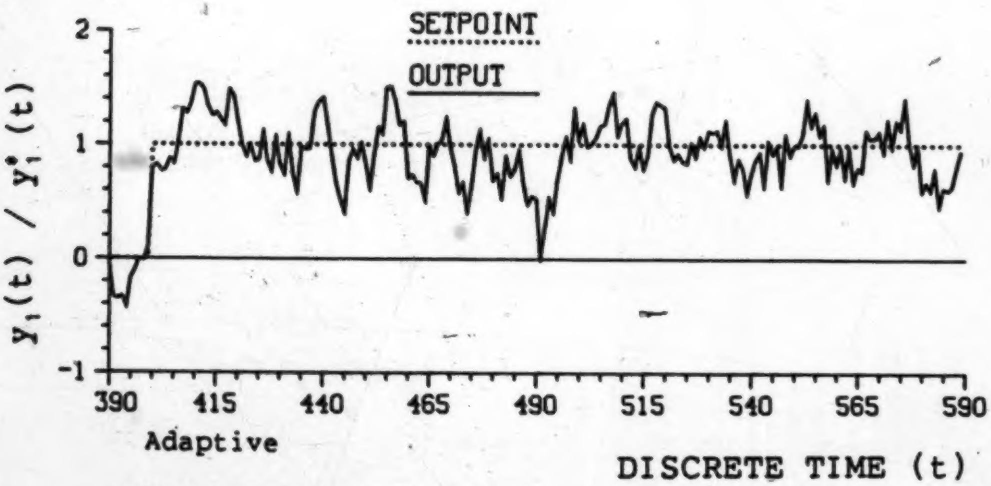


Figure 5.7f Parameter Estimation in AKFP for the Second MISO Subsystem (using Extended Least Squares Identification).

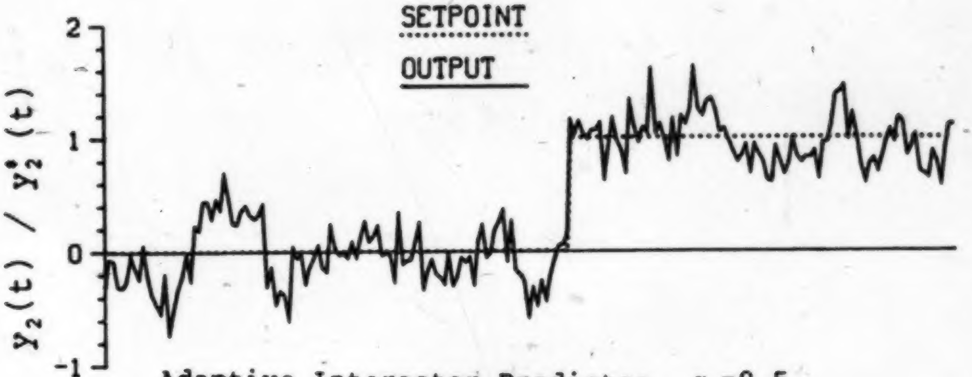


Adaptive Interactor Predictor $\alpha_1=0.5$

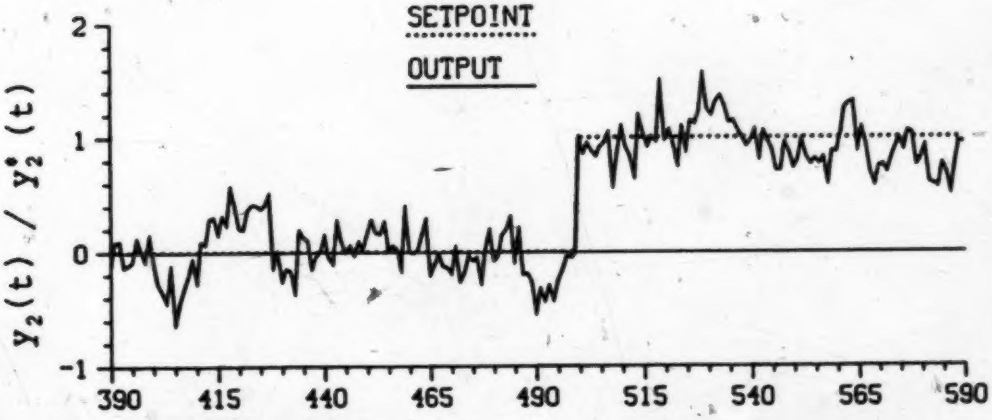


Adaptive
Kalman Filter Predictor
$R_{v_1}/R_{v_1}=0.03$

Figure 5.8a Adaptive Predictive Control of y_1 using AKFP and AIP (Stochastic Process with a Precompensator).



Adaptive Interactor Predictor α_2=0.5



Adaptive

Kalman Filter Predictor
R_{v_2}/R_{v_2}=0.03

Figure 5.8b Adaptive Predictive Control of y_2 using AKFP and AIP (Stochastic Process with a Precompensator).

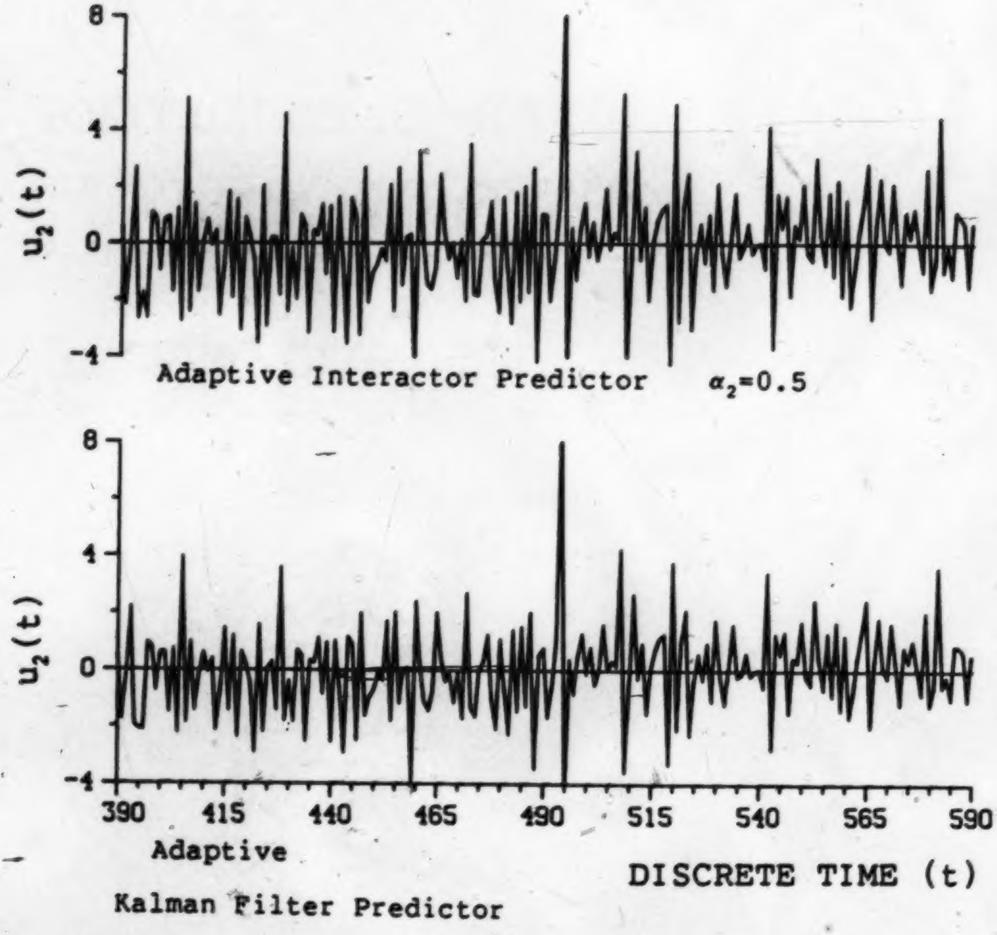
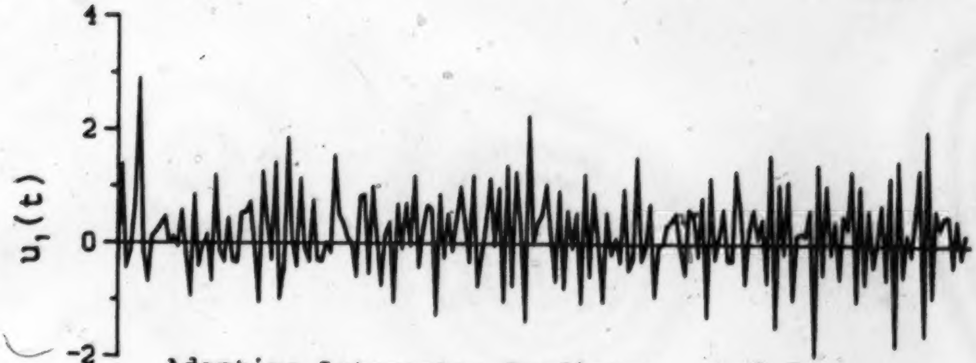
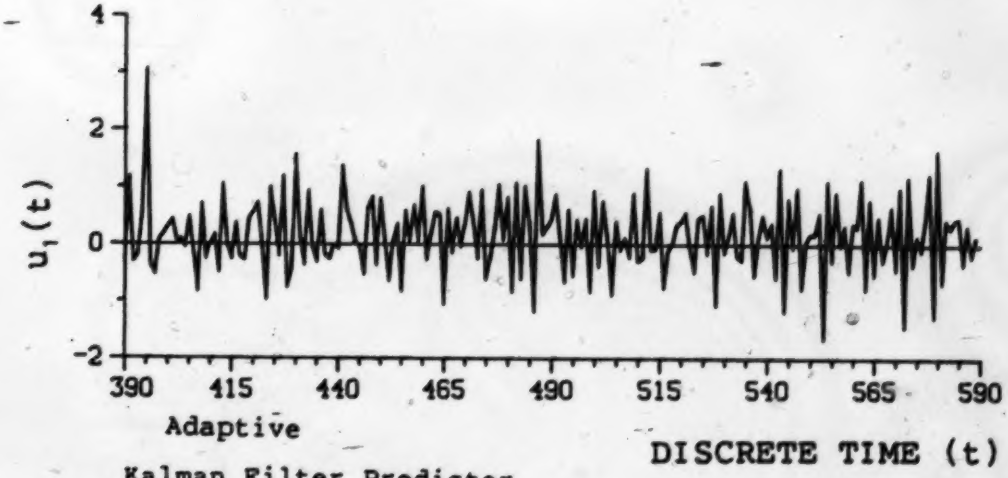


Figure 5.8d Adaptive Predictive Control of y_2 using AKFP and AIP (Stochastic Process with a Precompensator).

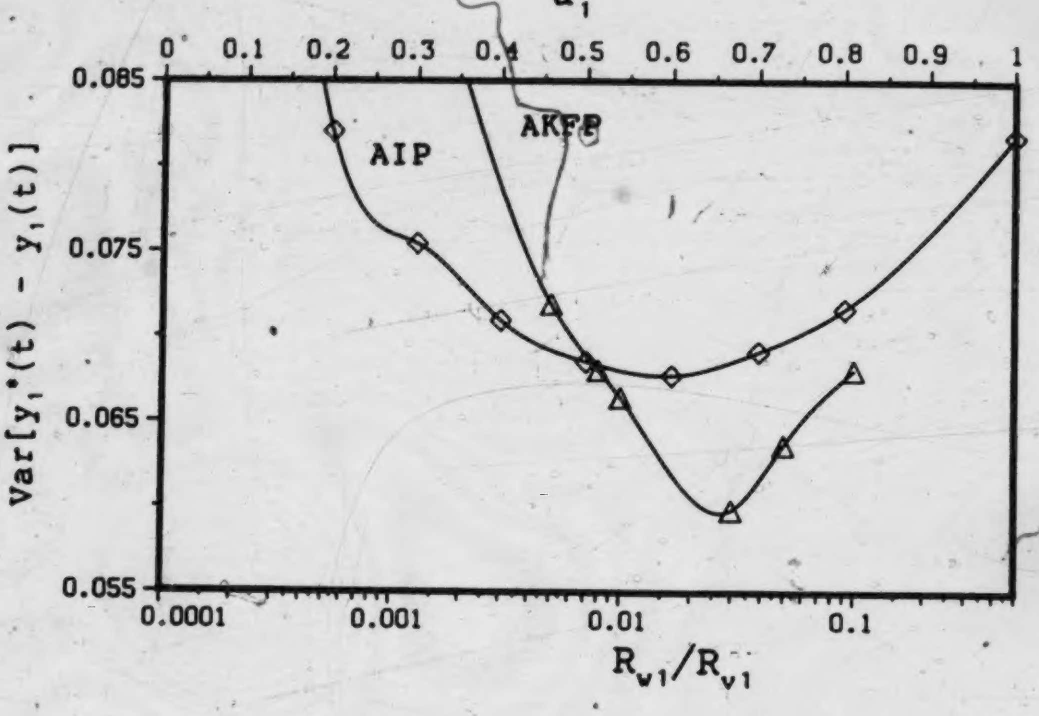


Adaptive Interactor Predictor $\alpha_1=0.5$



Adaptive
Kalman Filter Predictor

Figure 5.8c Adaptive Predictive Control of y_1 using AKFP and AIP (Stochastic Process with a Precompensator).



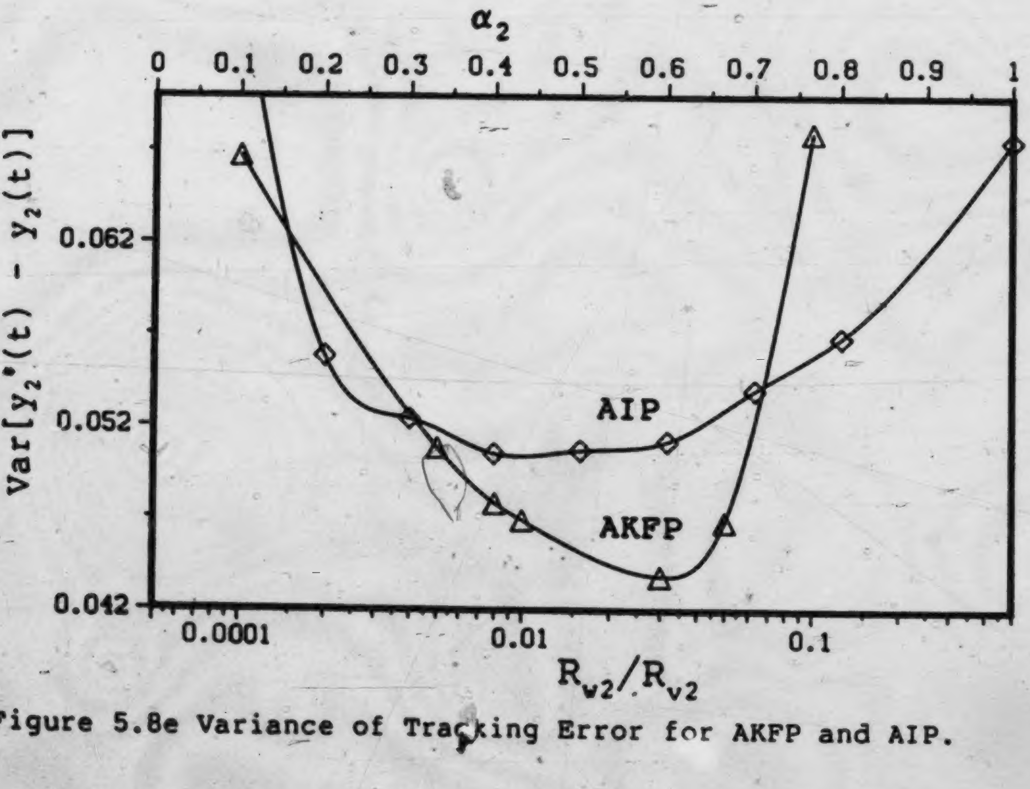


Figure 5.8e Variance of Tracking Error for AKFP and AIP.

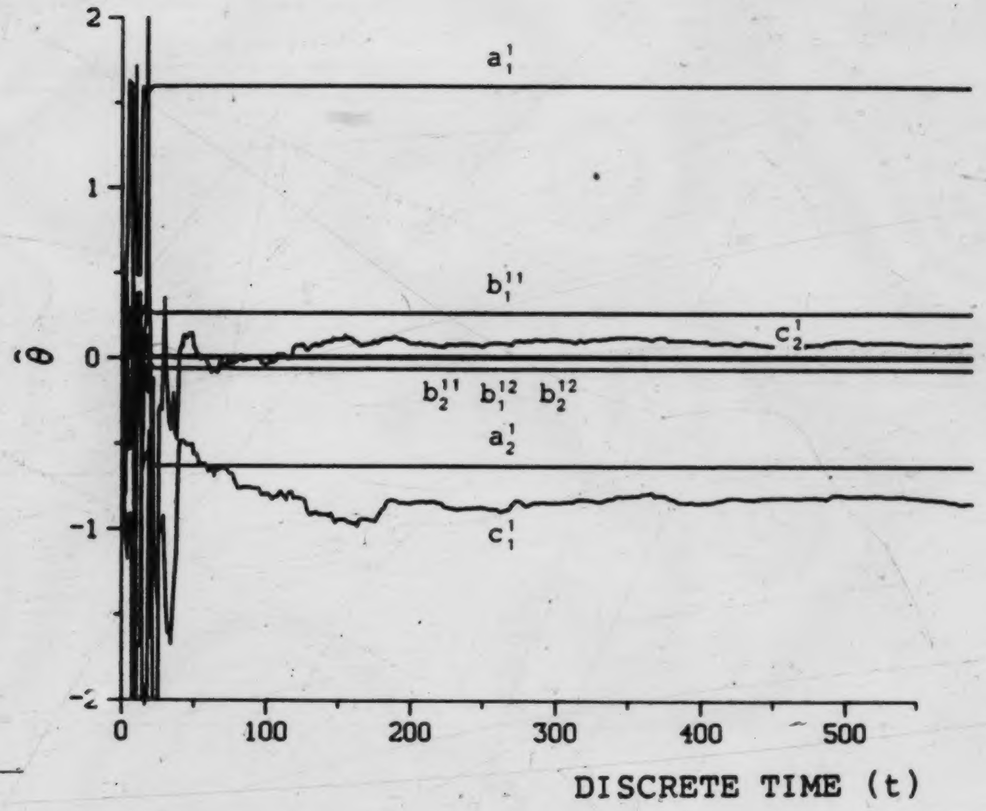


Figure 5.8f Parameter Estimation in AKFP for the First MISO Subsystem (using Extended Least Squares Identification).

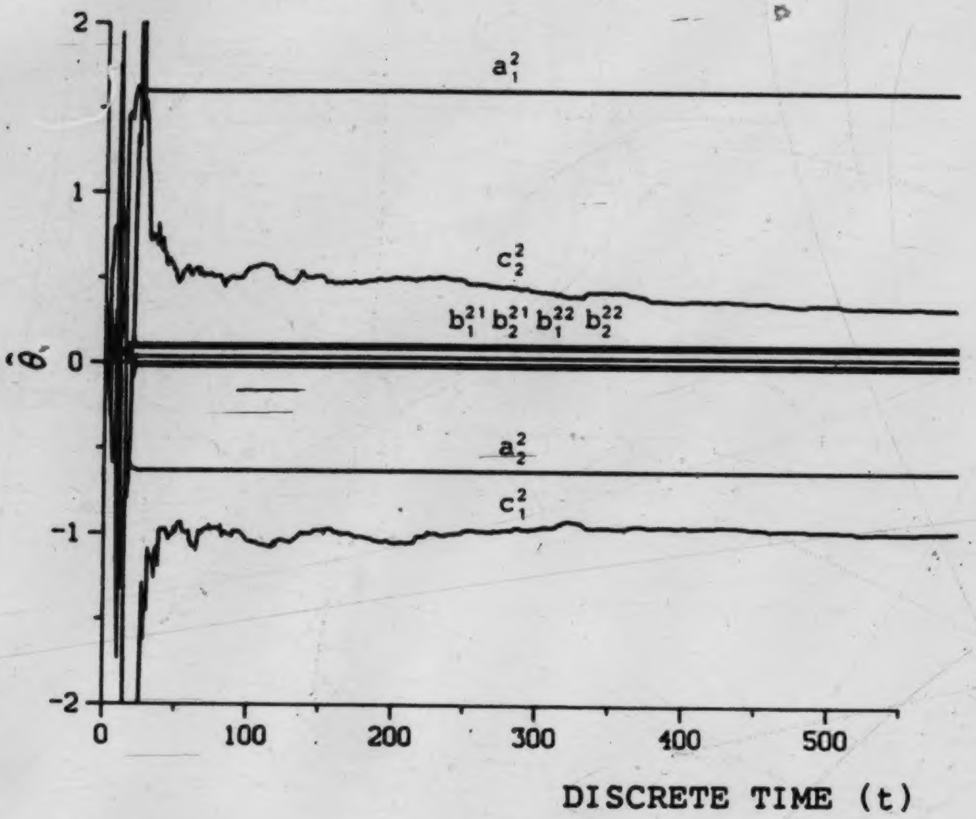


Figure 5.8g Parameter Estimation in AKFP for the Second MISO Subsystem (using Extended Least Squares Identification).

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6. Overall Conclusions and Future Work

6.1 Conclusions

The most significant contribution of this work is a asymptotically stable, MIMO, adaptive predictive control system, using the state space approach, for processes with time-delays, noise and/or sustained disturbances. The control scheme is practical and efficient to implement, and represents an alternative to the common 'self-tuning controller', especially for stochastic applications.

Other contributions and some of the steps leading to the MIMO adaptive controller are:

1. Formulation of an observable, state space representation for processes with time delays. In this representation, the current output and the future outputs i.e. $y(t),..,y(t+d)$ (where d= time delay) are expressed as state variables, and the only non unity, non zero elements of the state space matrices are the coefficients of the ARMA model. Therefore in adaptive applications, process identification requries the estimation of exactly the same parameters as in ARMA input-output formulations.
2. A Kalman filter was employed to obtain minimum variance estimates of the output $y(t), . . ,y(t+d)$ of a SISO process. It was proven that the steady state Kalman filter used to estimate the states of this special formulation was stable, and that the time varying

Kalman filter converged to the steady state Kalman filter. The Kalman Filter Predictor(KFP) can be tuned to obtain minimum variance performance by using the ratio of process and measurement noise covariances (R_w/R_v) as a tuning parameter. A computationally efficient algorithm is developed for the KFP by taking advantage of the sparseness of the state space matrices.

3. The state space model and the Kalman filter formulation were extended to MIMO systems with time delays by using the interactor factorization of Wolovich and Falb (1976), to factor out the 'natural' time delays. The state space model, in general, consists of p serially connected MISO subsystems (p=number of outputs), and their states are estimated using p serially connected MISO Kalman filters (figure 3.2). This decomposition provides an efficient implementation of the multivariable KFP.

For the case when the process parameters are known apriori it was possible to formulate a generalized Kalman filter predictor which can be applied to procsses with either diagonal or triangular interactor matrices.

For the case when the process parameters are unknown, they are estimated on-line. It is shown that, when the interactor of the process is diagonal the MIMO system can be decomposed in to p independent MISO

Kalman filter predictors, and that the non zero, non unity elements of the state space formulation are exactly the same as that of the ARMA parameters of the MISO systems. This decomposition, reduces the number of parameters that must be estimated in adaptive systems, i.e. avoids the dimensionality problem often encountered when applying state space techniques to large multivariable systems, and allows the direct extension of the results obtained from the SISO AKFP to multivariable systems. However when the interactor of the process is triangular the adaptive extension is complicated. This difficulty is overcome by employing a precompensator (Singh and Narendra,1984) to diagonalize the triangular interactor.

4. The output from the fixed and/or adaptive Kalman filter predictors can be used in a number of applications such as feedback control and time-delay compensation. Predictive controllers were developed for the SISO and MIMO cases, which used the entire predicted output vector from the Kalman filter in the controller formulation. This avoids the need of the Diophantine identity as usually ~~used~~, to formulate the (adaptive) predictive controllers. It is proven that the predictive controller based on the KFP performs as a minimum variance controller, if the process parameters and the noise covariances are known apriori. When the covariances are not known the KFP can be tuned by using

the ratio of the covariances as a tuning parameter. The adaptive predictive controller will give minimum variance performance after the parameters have converged.

The number of parameters that has to be estimated for the AKFP is the same as that of the explicit adaptive minimum variance controllers,but is less than that of the implicit adaptive minimum variance controllers and implicit self-tuning controllers.

5. A better understanding of the structure and performance of the KFP was obtained by using the innovation model as a means of converting the KFP into an input-output form that could be represented by a block diagram and/or a familiar input-output equation. The important results of this analysis were,

* the SISO KFP was shown to be structurally equivalent to the Smith predictor except for a filter transfer function which filtered the output estimation error (which represents the noise, disturbances or modelling errors), before it was added to the feedback control loop (cf. figures 2.4 and 2.6).

* The MIMO KFP was shown to be structurally equivalent to the Interactor Predictor(IP)(Sripada et al,1985), except for a filter transfer function matrix in the KFP (cf. figures 3.5 and H.2).

* The biasing problem of the KFP, in the presence of deterministic disturbances was analysed, and shown

to be due to the Proportional and Derivative(PD) nature of the state estimation in the KFP (figure 2.7).

* The Modified KFP (MKFP) which was designed to handle deterministic disturbances, was shown to introduce an integral term to the existing PD estimation of the KFP (figure 2.7).

* The Kalman filter was expressed in the familier ARMAX input-output form. This provided insight into the structure and the significance of the noise term in input-output models (equation 2.4.18). A 'self-tuning' KFP was implemented by estimating the parameters of the ARMAX model and using the coefficients of the C polynomial to calculate the Kalman gains (Sections 4.4 and 5.7).

6. The Kalman filter predictor is the natural extension of the Smith predictor time-delay compensation scheme, to stochastic, multivariable processes with time delays. It was shown that the Smith predictor and the KFP were structurally similar, except for an additional filter in the KFP, which can be tuned by using the noise ratio R_w/R_v to give minimum variance performance. It was also shown that the SISO KFP reduced to the SP, and the MIMO KFP reduced to the IP when there was no noise and perfect modelling. Both SP and IP can be improved to handle noise by introducing an adhoc (e.g. exponential)

filter in the same location as in the KFP.

7. The KFP was modified to handle deterministic disturbances and offset by augmenting the state space formulation with the states due to the disturbances. An incremental predictive controller was developed using the MKFP, which showed very good disturbance rejection (section 2.7).

6.2 Future Work

1. It is important to test the performance of the KFP or AKFP in real systems.
2. The predictive control scheme developed using the KFP, does not impose any constraints on the control force. For practical applications, a predictive control law that imposes a weighting on the control force may be desirable.
3. The multivariable adaptive Kalman filter for processes with triangular interactor was developed by introducing a precompensator to diagonalize the interactor. Since this precompensator introduces time delays into the process, the prediction error in the AKFP is higher. Hence it is important to pursue the possibility of using the triangular interactor to develop the multivariable adaptive Kalman filter predictor.
4. The predictive control scheme developed in this report is based on one step ahead prediction, where the control force is calculated to achive a single setpoint

in future. Instead of using a single point the predictive control law could be developed to follow a trajectory of setpoint values.

5. It is shown by simulations that the modified Kalman filter predictor does disturbance prediction after an initial transient. However this behaviour has not been proved analytically and needs further investigation.
6. Instead of the incremental predictive control scheme, the estimated disturbance (state x_ρ) can be used to develop a positional predictive controller or a feedforward controlller.
7. In the present studies the MKFP has been investigated only for deterministic disturbances. A further investigation is necessary regarding the behaviour of the MKFP, when there is process and measurement noise and model mismatch present in the system, in addition to the deterministic disturbances.
8. The AKFP was developed by incorporating an extended least squares algorithm to estimate the parameters. the convergence of the AKFP depends on the convergence of the parameter estimation scheme. A more numerically stable, parameter estimation scheme that guarantees the assymptotic convergence is the improved least squares algorithm due to Sripada and Fisher(1986). The performance of the AKFP could be improved by incorporating this parameter estimation algorithm.

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Appendix A - Kalman Filter (KF)

The Kalman filter was first presented by Kalman (1960, 1963), and Kalman and Bucy (1961). The Kalman filter can be used to estimate the states of a process given by a stochastic state space model. Interesting discussion of the theory and properties of the Kalman filter is also given in Astrom (1970), Astrom and Wittenmark (1984), and Goodwin and Sin (1984). Kalman filter Theory presented in this section is based on the approach given in Astrom (1970).

Theory:

Theorem A.1:(see Astrom, 1970)

Let x and y be two vectors which are jointly Gaussian. The conditional distribution of x and y is normal with mean,

$$E\,[x/y] = m_x + R_{xy}R_y^{-1}\,(y-m_y)$$

and covariance

$$E\,\{\,[x - E(x/y)\,]\,[x - E(x/y)\,]^T/y\,\} =$$

$$R_x - R_{xy}R_y^{-1}\,R_{yx} = R_z$$

The stochastic variables y and x-E[x/y] are independent.

Let the stochastic state space equation of the process be given by,

$$x(t+1) = \Phi x(t) + \Lambda u(t) + \omega(t) \qquad (A.1)$$

$$y(t) = \Theta x(t) + v(t) \qquad (A.2)$$

where {v(t)} and {ω(t)} are random white noise with mean zero and covariances given by,

$$E\{\omega(t)\omega^T(t)\} = R_1$$

$$E\{v(t)v^T(t)\} = R_2 \qquad (A.3)$$

$$E\{\omega(t)v^T(t)\} = 0$$

The matrices Φ, Θ, R_1 and R_2 may vary with time. It is also assumed that R_2 is positive definite and that the initial state $x(t_0)$ is independent of v and ω and normal with mean m and covariance R_0.

Now the formulation of the Kalman filter is~~,~~ to solve the problem of estimating $x(t+1)$ based on the observations of the output $y(t), y(t-1), \cdots, y(t_0)$
in such a way that the criterion,

$$E\ \{g(a^T(x(t+1) - \hat{x}(t+1/t+1)))\} \qquad (A.4)$$

is minimized.

The function g is assumed to be symmetric and non decreasing for positive arguments.

The following notations are used in the derivation:

$$\hat{x}(t|t) = E\ [x(t)|Y_t] \qquad (A.5)$$

= estimate of $x(t)$ using the data up to time t,

where,

$$Y_t^T = [\ y^T(t_0), y^T(t_0+1), \cdots, y^T(t)] \qquad (A.6)$$

$$\overline{x}(t|t-1) = E\ [\ x(t)|Y_{t-1}\ ] \qquad (A.7)$$

= estimation of $x(t)$ using the data up to time t-1.

$$\tilde{x}(t|t) = x(t) - \hat{x}(t|t) \qquad (A.8)$$

= estimation error due to the estimation $\hat{x}(t|t)$.

$$\tilde{x}(t|t-1) = x(t) - \overline{x}(t|t-1) \qquad (A.9)$$

= estimation error due to the estimation

$\bar{x}(t|t-1)$.

$$P(t) = E\,[\,\tilde{x}(t|t)\tilde{x}(t|t)^T\,] \tag{A.10}$$

= covariance of the estimation error due to the estimation $\tilde{x}(t|t)$.

$$M(t) = E\,[\,\tilde{x}(t|t-1)\tilde{x}(t|t-1)^T\,] \tag{A.11}$$

= covariance of the estimation error due to the estimation $\bar{x}(t|t-1)$.

To obtain a recursive formula for the estimation, it is assumed that $\hat{x}(t|t)$ is known, and a formula for $\hat{x}(t+1/t+1)$ is developed.

From equation (A.5),

$$\begin{aligned} \hat{x}(t+1/t+1) &= E\,[x(t+1)|Y_{t+1}\,] \\ &= E\,[x(t+1)|Y_t,\ y(t+1)\,] \end{aligned} \tag{A.12}$$

To evaluate the conditional expectation for given Y_t and $y(t)$, variables are changed to obtain independent variables.

From theorem (A.1) it follows that Y_t and,

$$\begin{aligned} \tilde{y}(t+1|t) &= y(t+1) - E\,[y(t+1)|Y_t\,] \\ &= y(t+1) - \theta\,E\,[x(t+1|t)\,] \\ &= y(t+1) - \theta\,\bar{x}(t+1|t) \\ &= \theta\,x(t+1) - \theta\bar{x}(t+1|t) + v(t+1) \\ &= \theta\,\tilde{x}(t+1|t) + v(t+1) \end{aligned} \tag{A.13}$$

are independent.

The quantity $\tilde{y}(t)$ is sometimes referred to as the innovation at time t+1, because it is the part of the measured output signal which contains some information which was not previously available. Thus instead of evaluating the conditional expectation of $x(t+1)$ given Y_t and $y(t+1)$, the conditional expectation given the transformed variables Y_t and $\tilde{y}(t+1|t)$ is evaluated.

Thus (A.12) becomes,

$$\hat{x}(t+1|t+1) = E\,[x(t+1)|Y_t,\ \tilde{y}(t+1|t)\,] \tag{A.14}$$

Since Y_t and $\tilde{y}(t+1|t)$ are independent, from Theorem A.1,

$$\begin{aligned}\hat{x}(t+1|t+1) = {} & E\,[x(t+1)|Y_t] + E\,[x(t+1)|\tilde{y}(t+1|t)\,] \\ & - E\,x(t+1)\end{aligned} \tag{A.15}$$

The different terms of the right member of (A.15) are evaluated now.

$$\begin{aligned}E\,[x(t+1)|Y_t\,] &= E\,[\Phi x(t) + \Lambda u(t) + \omega(t)|Y_t\,] \\ &= E\,[\,\Phi x(t)|Y_t\,] + \Lambda u(t) \\ &= \Phi\hat{x}(t|t) + \Lambda u(t)\end{aligned} \tag{A.16}$$

To evaluate $E\,[x(t+1)/\tilde{y}(t+1|t)\,]$ use Theorem A.1.

Then,

$$E\,[x(t+1)/\tilde{y}(t+1|t)\,] = E\,x(t+1) + R_{xy_1}{}^{-}R_y^{1}\ \tilde{y}(t+1) \tag{A.17}$$

where

$$\begin{aligned}R_{xy} &= \mathrm{Cov}\,[x(t+1),\ \tilde{y}(t+1|t)\,] \\ &= E\,\{[x(t+1) - E\,x(t+1)\,]\,[\Theta\tilde{x}(t+1|t) + v(t+1)\,]\}^T\end{aligned} \tag{A.18}$$

Since $x(t)$ and $v(t)$ are independent with zero mean,

$$R_{xy_1} = E\,[\bar{x}(t+1|t) + \tilde{x}(t+1|t)\,]\,[\tilde{x}(t+1|t)^T\,\Theta^T] \tag{A.19}$$

Since $\bar{x}(t|t-1)$ and $\tilde{x}(t|t-1)$ are independent from Theorem A.1,

$$R_{xy_1} = E\,(\tilde{x}(t+1|t)\tilde{x}(t+1|t)^T)\Theta^T \tag{A.20}$$

$$R_{xy_1} = M(t+1)\Theta^T \tag{A.21}$$

$$\begin{aligned} R_{y_1} &= \mathrm{Cov}\,[\tilde{y}(t+1|t),\ \tilde{y}(t+1|t)\ ] \\ &= E\ [\ \Theta\tilde{x}(t+1|t) + v(t+1)\ ]\ [\Theta\tilde{x}(t+1|t) + v(t+1)\ ]^T \\ &= \Theta\ [\ E\tilde{x}(t+1|t)\tilde{x}^T(t+1)\ ]\ \Theta^T + R_2 \\ &= \Theta\ M(t+1)\Theta^T + R_2 \end{aligned} \tag{A.22}$$

Substituting (A.21), (A.22) in (A.17),

$$\begin{aligned} E\ [x(t+1)|\tilde{y}(t+1|t)\ ] = Ex(t+1) &+ M(t+1)\ \Theta^T \\ &[\Theta\ M(t+1)\Theta^T + R_2\ ]^{-1}\tilde{y}(t+1|t)\ ] \end{aligned} \tag{A.23}$$

Now defining the Kalman gain L,

$$L(t+1) = M(t+1)\Theta^T\ [\ \Theta\ M(t+1)\Theta^T + R_2\ ]^{-1} \tag{A.24}$$

Equation (A.23) is written as,

$$\hat{x}(t+1|t+1) = \overline{x}(t+1|t) + L(t+1)\ [y(t+1) - \Theta\overline{x}(t+1|t)] \tag{A.25}$$

$$\begin{aligned} \overline{x}(t+1|t) &= E(x(t+1|t)) \\ &= \Phi\hat{x}(t|t) + \Lambda u(t) \end{aligned} \tag{A.26}$$

To obtain a formula for the covariance M(t),

$$\begin{aligned} M(t+1) &= E\ [\ \tilde{x}(t+1|t)\tilde{x}(t+1|t)^T\ ] \\ &= E\ [\ (x(t+1) - E(x(t+1|t))\ (x(t+1) \\ &\quad - E(x(t+1|t))^T\ ] \end{aligned} \tag{A.27}$$

Substituting from equation (A.26) and (A.1),

$$\begin{aligned} M(t+1) &= E\ \{\ [\Phi x(t) - \Phi\hat{x}(t|t) + w(t)\ ] \\ &\quad [\Phi x(t) - \Phi\hat{x}(t|t) + w(t)]^T\} \\ &= \Phi\ E\ [\tilde{x}(t|t)\ ]\ [\tilde{x}(t|t)\ ]^T\ \Phi^T + R_1 \end{aligned} \tag{A.28}$$

From (A.10),

$$P(t) = E\ \{[\ \tilde{x}(t|t)\ ]\ [\ \tilde{x}(t|t)\ ]^T\}$$

hence,

$$M(t+1) = \Phi \, P(t) \, \Phi^T + R_1 \qquad (A.29)$$

To obtain an equation for P(t),

$$\begin{aligned}
P(t) &= E\,\{[\tilde{x}(t|t)\,]\,[\tilde{x}(t|t)\,]^T\} \\
&= E\,\{[x(t) - \hat{x}(t|t)\,]\,[x(t) - \hat{x}(t|t)\,]^T\} \\
&= E\,\{[x(t) - \bar{x}(t|t-1) - L(t)\,\tilde{y}(t|t-1)\,] \\
&\quad [x(t) - \bar{x}(t|t-1) - L(t)\tilde{y}(t|t-1)\,]^T\} \\
&= E\,\{[\tilde{x}(t|t-1) - L(t)\,\Theta\tilde{x}(t|t-1) - L(t)v(t+1)\,] \\
&\quad [\tilde{x}(t|t-1) - L(t)\Theta\tilde{x}(t|t-1) - L(t)v(t+1)\,]^T\} \\
&= E\,\{[\,[I - L(t)\Theta\,]\,\tilde{x}(t|t-1) - L(t)v(t+1)\,] \\
&\quad [\,[I - L(t)\Theta\,]\,\tilde{x}(t|t-1) - L(t)v(t+1)\,]^T\} \\
&= [I - L(t)\Theta\,]\,E\,\tilde{x}(t|t-1)\tilde{x}(t|t-1)^T \\
&\quad [I - \Theta^T L(t)^T\,] + L(t)R_2L(t)^T \\
&= [I - L(t)\Theta\,]\,M(t)\,[I - \Theta^T\,] + L(t)R_2L(t)^T \\
&= M(t) - L(t)\Theta M(t) - M\Theta^T L(t)^T + \\
&\quad L(t)\Theta M(t)\Theta^T L(t)^T + L(t)R_2L(t)^T
\end{aligned}$$

$$\begin{aligned}
P(t) &= M(t) - L(t)\Theta M(t) - L(t)\,[\Theta M(t)\,\Theta^T + R_2\,] \\
&\quad L(t)^T + L(t)\Theta M(t)\Theta^T L(t)^T + L(t)R_2L(t)^T
\end{aligned}$$

$$P(t) = M(t) - L(t)\Theta M(t) \qquad (A.30)$$

To determine the initial conditions of equations (A.25),

$$\begin{aligned}
\hat{x}(t_0+1/t_0+1) &= \bar{x}(t_0+1/t_0) + L(t_0+1)\,\tilde{y}(t_0+1/t_0) \\
&= \bar{x}(t_0+1/t_0) + L(t_0+1)\,[y(t_0+1) \\
&\quad - \Theta\bar{x}(t_0+1/t_0)\,] \\
&= m + L(t_0+1)\,[y(t_0+1) - \Theta m\,] \qquad (A.31)
\end{aligned}$$

From Theorem A.1,

$$P(t_0+1) = M(t_0+1) - L(t_0+1)\Theta M(t_0+1)$$

If $M(t_0) = R_0$ (A.32)

then comparing (A.30) and (A.32), it is clear that $M(t_0)=R_0$ is the initial condition for (A.30).

From the above derivations the Kalman filter algorithm can be summarized as follows:

1. Gain Calculation (from equation A.24)

   $L(t) = M(t)\Theta^T [\Theta M(t)\Theta^T + R_2]^{-1}$ with $M(t_0) = R_0$.

   Measurement update (at the time of the measurement)

   State update (from equation A.25)

   $\hat{x}(t) = \bar{x}(t) + L(t) [y(t) - \Theta\bar{x}(t)]$

   Covariance update (from equation A.30)

   $P(t) = M(t) - L(t)\Theta M(t)$

2. Time update (between measurements)

   State update (from equation A.26)

   $\bar{x}(t+1) = \Phi\hat{x}(t) + \Lambda u(t)$

   Covariance update (from equation A.29)

   $M(t+1) = \Phi P(t)\Theta^T + R_1$

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Appendix B - Stability & Convergence of Kalman Filter

The algorithm given in Appendix A, recursively calculates the time varying Kalman gains L(t), and error covariances M(t) & P(t). In many cases the covariances M(t) and P(t) and hence the Kalman gains L(t) converge to steady state values as t-->∞.

If M(t) and P(t) converge to M and P, the limiting solution of M will satisfy the following Algebraic Riccati Equation (ARE), obtained by combining the equations (A.30) and (A.29) and substituting M(t+1)=M.

The ARE is given by,

$$M - \Phi M \Phi^T + \Phi L \Theta M \Phi^T - R_1 = 0 \qquad \text{(B.1)}$$

where

L - steady state Kalman filter gain

$$L = M\Theta^T[\Theta M \Theta^T + R_2]^{-1} \qquad \text{(B.2)}$$

Also define the steady-state filter state transition matrix $\overline{\Phi}$, and $\overline{\theta}$,

$$\overline{\Phi} = \Phi - L\Theta\Phi \qquad \text{(B.3)}$$

$$\overline{\Theta} = R_2^{-1/2}\,\Theta \qquad \text{(B.4)}$$

and factor R_1 and R_2 as,

$$R_1 = EE^T \qquad \text{(B.5)}$$

$$R_2 = R_2^{1/2}\,R_2^{1/2} \qquad \text{(B.6)}$$

Then the ARE is given by,

$$M - \Phi M \Phi^T + \Phi M \overline{\Theta}^T(\,\overline{\Theta} M \overline{\Theta}^T + I)^{-1}\,\overline{\Theta} M \Phi^T - EE^T = 0 \qquad \text{(B.7)}$$

For the stability of the KF we are interested in the solutions of ARE which are real, symmetric, positive semidefinite, and which give a steady-state filter having roots on or inside the unit circle.

For convergence of the KF, we are interested in the convergence of the time varying KF, given in appendix A, to the solution of ARE.

A good summary of these properties, and necessary definitions, theorems and Lemmas are given in Goodwin & Sin (1984). More details and proofs of the theorems are given in Martensson (1971), Kucera (1972a, 1972b), Chau, Goodwin & Sin (1983) and Anderson and Moore (1979). Theorems & Lemmas are given here without proof, as given in Goodwin & Sin (1984).

Definition 1.

A real symmetric positive semidefinite solution of ARE is said to be a stabilizing solution if the corresponding filter state transition matrix $\bar{\Phi}$, has all its eigenvalues inside the unit circle.

Definition 2.

A real symmetric positive semidefinite solution of the ARE is said to be a strong solution if the corresponding filter state transition matrix, $\bar{\Phi}$, has all its eigenvalues inside or on the unit circle.

Some of the key properties of the ARE are summarized in the following Lemma:

Lemma B.1

Provided that (Θ,Φ) is detectable, the strong solution of the ARE exists and is unique.

1. If (Φ,E) is stabilizable, the strong solution is the only positive semidefinite solution of the ARE.
2. If (Φ,E) has no uncontrollable modes on the unit circle, the strong solution coincides with the stabilizing solution.
3. If (Φ,E) has an uncontrollable mode on the unit circle, then, although the strong solution exists, there is no stabilizing solution.
4. If (Φ,E) has an uncontrollable mode inside, or on the unit circle, the strong solution is not positive definite.
5. If (Φ,E) has an uncontrollable mode outside the unit circle, then as well as the strong solution, there is at least one other positive semidefinite solution of the ARE.

The above Lemma gives the conditions for the existence and uniqueness of both stabilizing and strong solutions.

There are three theorems that define the convergence of the solution of the time varying matrix Riccati difference equation to the stabilizing or strong solution of the ARE.

Theorem B.1

Subject to, (Φ,E) is stabilizable.

1. (Φ,E) is detectable.
2. $M_0 \geq 0$

then,

$$\lim_{t\to\infty} M(t) = M_s \quad \text{(exponentially fast)}$$
$$\lim_{t\to\infty} L(t) = L_s \quad \text{(exponentially fast)}$$
$$\lim_{t\to\infty} \bar{\Phi}(t) = \bar{\Phi}_s \quad \text{(exponentially fast)}$$

M_s, L_s, $\bar{\Phi}_s$ are steady state values of $M(t)$, $L(t)$ and $\bar{\Phi}(t)$.

Theorem B.2

Subject to, no uncontrollable modes of (Φ,E) on the unit circle

1. (Θ,Φ) are detectable.
2. $M_0>0$,

then,

$$\lim_{t\to\infty} M(t) = M_s \quad \text{(exponentially fast)}$$
$$\lim_{t\to\infty} L(t) = L_s \quad \text{(exponentially fast)}$$
$$\lim_{t\to\infty} \underline{\bar{\Phi}(t) = \Phi_s} \quad \text{(exponentially fast)}$$

Theorem B.3

Subject to, (Θ,Φ) being observable.

~~1.~~ ~~$(M_0$~~ $- M_s)>0$ or $M_0=M_s$

then,

$$\lim_{t\to\infty} M(t) = M_s$$
$$\lim_{t\to\infty} L(t) = L_s$$
$$\lim_{t\to\infty} \bar{\Phi}(t) = \bar{\Phi}_s$$

In this case M_s is the (unique) strong solution of the ARE and L_s and $\bar{\Phi}_s$ are the corresponding steady state filter gain and state transition matrix. $\bar{\Phi}_s$ will have roots inside the unit circle unless (Φ,E) has uncontrollable modes on the unit circle, in which $\bar{\Phi}_s$ will also have the same roots on the unit circle.

The above theorems give the sufficient conditions for the asymptotic time invariance and stability of the filter.

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Appendix C - Innovation Model of the Kalman Filter

There are number of different ways one could express the Kalman filter algorithm, e.g. Astrom (1970). Depending on the structure of the state space representation and the definition of the conditional mean, i.e. $\hat{x}(t) = E\ [\ x(t)|Y_t]$ or $\hat{x}(t) = E\ [\ x(t)|Y_{t+1}]$, one could derive a number of different Kalman filter Algorithms. For each algorithm there is a corresponding innovation model (Kailath, 1970) representation. The innovation model corresponding to the algorithm in appendix A is given below.

From equation (A.29) and (A.30) we can write,

$$\hat{x}(t+1) = \Phi\hat{x}(t) + \Lambda u(t) + L(t+1)\ [y(t+1) - \Theta\Phi\hat{x}(t) - \Theta\Lambda u(t)] \qquad (C.1)$$

$$\hat{y}(t+1) = \Theta\Phi\hat{x}(t) + \Theta\Lambda u(t) \qquad (C.2)$$

Define,

$$\omega(t) = y(t) - \Theta\Phi\hat{x}(t-1) - \Phi\Lambda u(t-1) \qquad (C.3)$$

$[\omega(t)]$ is called the innovation sequence.

Since,

$$y(t) = \Theta x(t) + v(t) = \Theta\Phi x(t-1) - \Theta\Lambda u(t-1) + v(t) \qquad (C.4)$$

then

$$\omega(t) = \Theta\Phi\ [x(t-1) - \hat{x}(t-1)] + v(t) \qquad (C.5)$$

Hence,

$$E\ [\omega(t)|Y_{t-1}] = 0 \qquad (C.6)$$

From (C.3)

$$y(t) = \omega(t) + \hat{Y}(t)$$
$$= \omega(t) + E\ [y(t)|Y_{t-1}] \qquad (C.7)$$

thus $\omega(t)$ represents the new information contained in $y(t)$

which was not in Y_{t-1}. Thus $\omega(t)$ is given the name innovation sequence.

Consider,

$$E\,[\omega(t)\omega(t)^T] = E\,\{\,[\Theta\Phi\tilde{x}(t-1) + v(t)\,]\,[\Theta\Phi\tilde{x}(t-1) + v(t)]^T\,\}$$

$$= \Theta\Phi\tilde{x}(t-1)\tilde{x}(t-1)^T\,\Phi^T\Theta^T + R_1$$

$$= \Theta\Phi P(t-1)\Phi^T\Theta^T + R_1 \tag{C.8}$$

From (C.1), (C.2) and (C.3),

$$\hat{x}(t+1) = \Phi\hat{x}(t) + \Lambda u(t) + L(t+1)\omega(t+1) \tag{C.9}$$

$$y(t) = \Theta\Phi\hat{x}(t-1) + \Theta\Lambda u(t-1) + \omega(t) \tag{C.10}$$

(C.9) and (C.10) are called the innovation model.

References

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Appendix D - A Lemma for KFP

Lemma D.1

If a state space formulation has the following form

$$x(t+1) = Ax(t) + B\,u(t) \tag{D.1}$$

where

$$A = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & 1 & 0 \end{bmatrix}_{nxn} \qquad B = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix}_{nx1}$$

then any element x_r, $r \leq n$ of the state vector x is given by,

$$x_r(t) = [b_r q^{-1} + b_{r-1} q^{-2} + \cdots + b_1 q^{-r}]u(t) \tag{D.2}$$

Proof:-

Expanding equation (D.1) for each state and by successive substitution (D.2) can be obtained.

Another way is to express (1) as,

$$[qI-A]\,x(t) = B\,u(t) \tag{D.3}$$

$$\mathrm{Det}[qI-A] = q^n \tag{D.4}$$

Hence,

$$[qI-A]^{-1} = \begin{bmatrix} z^{-1} & & & & \\ z^{-2} & z^{-1} & & 0 & \\ \cdot & & & & \\ \cdot & & & & \\ z^{-r} & z^{-r+1} & \cdot & z^{-1} & \\ \cdot & & & & \\ \cdot & & & & \\ z^{n} & z^{-n+1} & \cdot\ \cdot\ \cdot & z^{-2} & z^{-1} \end{bmatrix} \tag{D.5}$$

Since,

$$x(t) = [qI-A]^{-1}B\ u(t) \tag{D.6}$$

$$\therefore x_r(t) = [q^{-r}q^{-r+1}\cdots q^{-1}0\cdots 0]B\ u(t)$$

$$=[\ b_1q^{-r} + b_2q^{-r+1}\cdots b_rq^{-1}\ ]u(t) \tag{D.7}$$

Thus the Lemma is proved.

Appendix E - Simplified implementation of the Kalman Filter Algorithm for KFP

The dimension of the state space model used in the KFP increases with increasing time delays. If the KFP is implemented using the KF algorithm given in appendix A, and the calculations are done in matrix operations in the computer, these will be an excessive computational effort. However, considering the sparse nature of the Φ matrix, a set of equations are obtained to implement the same KFP but now with less calculations.

For a multi-input,single-output system,

a. Gain Calculation

$$L_j(t) = M_{j,n+d}(t)/[M_{\text{[illegible]}+d,n+d}(t) + R_2] \tag{E.1}$$

where,

$L_j(t)$ = j^{th} component of Kalman Gain $L(t)$.

$M_{ij}(t)$ = ij^{th} component of covariance $M(t)$

b. Measurement Update

b1. State Update

$$\hat{x}_j(t) = \bar{x}_j(t) + L_j(t)[y(t) - \bar{x}_{n+d}(t)] \tag{E.2}$$

b2. Covariance Update

$$P_{ij}(t) = M_{ij}(t) - L_i(t)M_{n+d,j}(t) \tag{E.3}$$

$P_{ij}(t)$ = ij^{th} component of covariance $P(t)$.

c. Time Update

c1. State Update

$j = 1$

$\bar{x}_j(t+1) = -a_n\hat{x}_n(t) + \Lambda_n u(t)$

$1<j\le n$

$$\bar{x}_j(t+1) = \hat{x}_{j-1}(t) - a_{n-j+1}\hat{x}_n(t) + \Lambda_{n-j+1}u(t)$$

$n+d \geq j > n$

$$\bar{x}_j(t+1) = \hat{x}_{j-1}(t) \tag{E.4}$$

where, Λ_j = j^{th} row of matrix Λ

c2. Covariance Update

$$M_{ji}(t+1) = -a_n Q_{jn} + R^1_{ji}$$

$n \geq i > 1$

$$M_{ji}(t+1) = Q_{j,i-1} - a_{n-i+1}Q_{jn} + R^1_{ji}$$

$n+d \geq i > n$

$$M_{ji}(t+1) = Q_{j,i-1} + R^1_{ji} \tag{E.5}$$

where

$$Q_{1j} = -a_n P_{nj}(t)$$

$$Q_{ij} = P_{i-1,j}(t) - a_{n-i+1}P_{nj}(t) \quad \text{for} \quad n \geq i > 1$$

$$Q_{ij} = P_{i-1,j}(t) \quad \text{for} \quad n+d \geq i > n$$

The table E.1 shows the number of multiplications and divisions in the direct and simplified KF implementations. The number of inputs is assumed to be m.

If n=1, d=3, and m=1, the direct implementation needs 208 multiplications, whereas the simplified implementation needs only 12 multiplications.

Table E.1 Computation Load in Direct and Simplified Kalman Filter Algorithms

Implementation	Number of divisions	Number of multiplications
direct	$n+d$	$2(n+d)^3 + 4(n+d)^2 + (m+3)(n+d)$
simplified	$n+d$	$(5+m)n + 2d$

Appendix F - Procedure to Obtain the Interactor Matrix

The interactor factorization procedure for a continuous process is given in Wolovich and Falb(1976). The dis crete time version of this procedure as presented in Goodwin and Sin(1984) is given below. For a mxm proper transfer function matrix T(q), there exists a unique interactor matrix $\xi_T(q)$ such that,

$$\lim_{q \longrightarrow \infty} \xi_T(q)T(q) = K_T \qquad (F.1)$$

is finite and non-singular,

$$\xi_T^*(q) = H_T(q) \text{ diag } (q^{\lambda 1}, q^{\lambda 2}, \cdot \cdot q^{\lambda m}) \qquad (F.2)$$

and

$$H_T(q) = \begin{bmatrix} 1 & & & \\ h_{21}(q) & 1 & & 0 \\ \cdot & & \cdot & \vdots \\ \cdot & & & \cdot \\ h_{m1}(q) & h_{m2}(q) & \cdot \cdot & 1 \end{bmatrix} \qquad (F.3)$$

It is possible to write $T(q) = R(q)P(q)^{-1}$ where R(q) and P(q) are relatively right prime polynomial matrices, and P(q) column proper.

Let,

$\partial_i(P) = r_i$, $i=1 \cdot \cdot m$

= the column degress of P(q) and $\sum_{i=1}^{n} r_i = n$.

det R(q) - a non zero polynomial of degree q.

Step 1:

There are unique integers l_i, $i=1,\cdots m$, such that,

$$\lim_{q \longrightarrow \infty} q^{l_i}\, T_i(q) = d_i \qquad i=1,\cdots m \tag{F.4}$$

where $T_i(q)$ is the i^{th} row of $T(q)$ and d_i is both finite and nonzero.

Define the first row $\xi_T(q)$, of $\xi_T(q)$ by,

$$\xi_T(q)_1 = (q^{l_1}, 0, \cdot \cdot 0) \tag{F.5}$$

so that,

$$\lim_{q \longrightarrow \infty} \xi_T(q)_1\, T(q) = \xi_1 = d_1 \tag{F.6}$$

Step 2:

If d_1 is linearly independent of ξ_1, then set,

$$\xi_T(q)_2 = (0, q^{l_2}, 0, \cdot \cdot 0) \tag{F.7}$$

so that,

$$\lim_{q \longrightarrow \infty} \xi_T(q)_2\, T(q) = \xi_2 = d_2 \tag{F.8}$$

On the other hand, if d_2 and ξ_1 are linearly dependent so that $d_2=\alpha_1^1\, \xi_1$ with $\alpha_1^1 \neq 0$ then we let,

$$\bar{\xi}^1(z)_2 = q^{l_2^1}\, [(0, q^{l_2}, 0, \cdot \cdot 0) - \alpha_1^1\, \xi_T(q)_1] \tag{F.9}$$

where l_2^1 is the unique integer for which

$$\lim_{q \longrightarrow \infty} \bar{\xi}_T^1(q)_2 T(q) = \bar{\xi}_2^1$$

is both finite and non zero.

If $\bar{\xi}_2^1$ is linearly independent of ξ_1, then we get,

$$\xi_T(q)_2 = \bar{\xi}_2^1(q)_2 \tag{F.10}$$

and note that,

$$\lim_{q \to \infty} \xi_T(q)_2 T(q) = \overline{\xi}_2^1 \qquad (F.11)$$

is linearly independent of ξ_1.

If not then, $\overline{\xi}_2^1 = \alpha_1^2 \xi_1$ and we let,

$$\overline{\xi}_T^2 (q)_2 = q^{l_2^2} [\overline{\xi}_T(q)_2 - \alpha_1^2 \xi_1(q)_1] \qquad (F.12)$$

where l_2^2 is the unique integer for which,

$$\lim_{q \to \infty} \overline{\xi}_T^2 (q) T(q) = \overline{\xi}_2^2$$

is both finite and non zero.

If $\overline{\xi}_2^2$ and ξ_1 are linearly independent, then we get $\xi_T(q)_2 = \overline{\xi}_T^2 (q)_2$, if not, we repeat the procedure until either linear independence is obtained or,

$$l_1 + l_2^k = n-q \qquad (F.13)$$

In case $l_1 + l_2^k = n-q$, set $f_3=0, \cdot \cdot f_m=0$ and the corresponding $h_{ij}=0$.

The remaining rows of $\xi_T(q)$ are defined recursively in an entirely analogous manner.

Finally we get either,

1. a matrix $\xi_T(q)$ of the form (F.2) such that F.1 is satisfied or
2. $\xi_T(q)_1, \cdot \cdot \xi_T(q)_r$, $r \leq m$, such that,

$$\lim_{q \to \infty} \xi_T(q)_j T(q) = \xi_j \qquad (F.14)$$

with $\xi_1, \cdot \cdot \cdot, \xi_r$ linearly independent and

$$\sum_{1}^{r} f_i = n-q \qquad (F.15)$$

In this case we get $f_{r+1} = 0, \cdot \cdot, f_m = 0$ and the corresponding $h_{ij} = 0$ to obtain $\xi_T(q)$.

If r=m, then

$$\lim_{q \to \infty} \xi_T(q)T(q) = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \cdot \\ \cdot \\ \xi_m \end{bmatrix} = K_T \qquad (F.16)$$

is finite and non-singular.

Examples:

1) Consider the TFM

$$G(q) = \begin{bmatrix} \dfrac{q^{-1}}{1-0.1q^{-1}} & \dfrac{q^{-1}}{1-0.1q^{-1}} \\[2ex] \dfrac{2q^{-1}}{1-0.3q^{-1}} & \dfrac{2q^{-1}}{1-0.4q^{-1}} \end{bmatrix}$$

clearly $l_1=1$, $l_2=1$, so that $d_1=(1\ 1)$, $d_2=(2\ 2)$.

$\therefore\ \xi_T(q)_1 = (q^{l_1}\ 0) = (q\ 0)$

and $\lim_{q \to \infty} \xi_T(q)_1 T(q) = \xi_1 = d_1 = (1\ 1)$.

Since ξ_1 and d_2 are linearly dependent, we have,

$d_2 = \alpha_1^1\, \xi_1 = (2\ 2)$ with $\alpha_1^1 = 2$

$\therefore\ \bar{\xi}_T^1(q)_2 = q^{l_2^1}\ [(0,q) - 2(q\ 0)]$; here $l_2^1 = 1$.

So,

$\bar{\xi}_T^1(q)_2 = q(-2q\ q) = (-2q^2\ q^2)$

$\lim_{q \to \infty} \bar{\xi}_T^1(q)_2\, T(q) = \bar{\xi}_2^1 = (.4\ .4)$

Once again, $\bar{\xi}_2^1$ and ξ_1 are linearly independent.

$$\therefore\ \bar{\xi}_2^1 = \alpha_1^2\,\xi_1 \quad \text{with} \quad \alpha_1^2=0.4$$

$$\begin{aligned}\therefore\ \bar{\xi}_T^2(q)_2 &= q^{l_2^2}\ [(-2q^2\ \ q^2) - 0.4(q\ \ 0)],\ \bar{l}^2=1\\ &= q[-2q^2-0.4q\ \ q^2]\\ &= (-2q^3-0.4q^2\ \ q^3)\end{aligned}$$

$$\lim_{q\longrightarrow\infty} \bar{\xi}_T^2(q)_2\,T(q) = (0.12\ \ 0.16) = \bar{\xi}_2^2$$

is linearly independent of ξ_1.

$$\therefore\ \xi_T(q)_2 = \bar{\xi}_T^2(q)_2 = (-2q^3-0.4q^2\ \ q^3)$$

so that,

$$\xi_T(q) = \begin{bmatrix} q & 0 \\ -2q^3-0.4q^2 & q^3 \end{bmatrix}.$$

and

is finite and non-singular.

2) Consider the TFM

$$G(q) = \begin{bmatrix} \dfrac{q^{-3}}{1+0.4q^{-1}} & \dfrac{2q^{-4}}{1-0.5q^{-1}} \\[2ex] \dfrac{3q^{-4}}{1+0.1q^{-1}} & \dfrac{q^{-5}}{1+0.8q^{-1}} \end{bmatrix}$$

clearly $l_1=3$, $l_2=4$ and $d_1=(1\ \ 0)$, $d_2=(3\ \ 0)$

$$\therefore\ \xi_T(q)_1 = (q^{l_1}\ \ 0) = (q^3\ \ 0).$$

So that $\lim_{q \to \infty} \xi_T(q)_1 = \xi_1 = d_1 = (1\ 0)$.

Since ξ_1 and d_2 are linearly dependent, $d_2 = \alpha_1^1 \xi_1$ where $\alpha_1^1 = 3$.

We have,

$\bar{\xi}_T^1(q)_2 = q^{l_2^1}\,[(0\ q^4) - 3(q^3\ 0)] = q^{l_2^1}(-3q^3\ q^4)$.

We find $l_2^1 = 1$ so $\bar{\xi}_T^1(q)_2 = (-3q^4\ q^5)$

$\therefore \lim_{q \to \infty} \bar{\xi}_T^1(q)_2\, T(q) = (.9\ -7)$ is linearly independent of ξ_1.

$\therefore \xi_T(q)_2 = \bar{\xi}_T^1(q)_2 = (-3q^4\ q^5)$

$$\therefore \xi_T(q) = \begin{bmatrix} \xi_T(q)_1 \\ \xi_T(q)_2 \end{bmatrix} = \begin{bmatrix} q^3 & 0 \\ -3q^4 & q^5 \end{bmatrix}$$

$$\lim_{q \to \infty} \xi_T(q)T(q) = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ .9 & -7 \end{bmatrix} = K_T$$

is finite and non-singular.

References

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Appendix G - Ogunnaike and Ray Multivariable Predictor

A schematic block diagram of the Ogunaike and Ray (1979) Predictor (ORP) is shown in figure G.1.

Consider a MIMO process with m inputs and p outputs.

Let the TFM of the model of the process including the time delays be represented by,

$$T(q) = \begin{bmatrix} q^{-d11}t_{11}(q) & q^{-d12}t_{12}(q) & \cdots & q^{-d1m}t_{1m}(q) \\ q^{-d21}t_{21}(q) & q^{-d22}t_{22}(q) & \cdots & q^{-d2m}t_{2m}(q) \\ \cdot & \cdot & \cdot & \cdot \\ q^{-dp1}t_{p1}(q) & q^{-dp2}t_{p2}(q) & \cdots & q^{-dpm}t_{pm}(q) \end{bmatrix} \qquad (G.1)$$

where t_{ij} is a proper rational function in the operator q, and d_{ij} is the delay between i^{th} output and the j^{th} input. The unit delay due to discretization is included in t_{ij}.

The process model G_p without time delay is obtained by removing all the time delays d_{ij}, i=1,p, j=1,m from $G_m(q)$.

Hence,

$$G_p(q) = \begin{bmatrix} t_{11}(q) & t_{12}(q) & \cdots & t_{1m}(q) \\ t_{21}(q) & t_{22}(q) & \cdots & t_{2m}(q) \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ t_{p1}(q) & t_{p2}(q) & \cdots & t_{pm}(q) \end{bmatrix} \qquad (G.2)$$

The structure of this multivariable predictor is similar to the Smith predictor.

The controller G_c in figure G.1 is tuned to G_p instead of the actual process G. Since the time delays have been removed in

G_p the controller can be designed using any conventional design technique.

If there is no mismatch between the process and the model, this predictor dead time compensator will guarantee the steady state set point tracking, but it is difficult to say anything about the dynamic tracking.

For the feedback control system given in figure G.1 the controller output u(t) is given by,

$$u(t) = \frac{G_c}{1+G_pG_c+G_c(G-G_m)}\, y^*(t) - \frac{G_c}{1+G_pG_c+G_c(G-G_m)}\, \eta(t)$$

and the closed loop equation is given by,

$$y(t) = \frac{G_cG}{1+G_pG_c+G_c(G-G_m)}\, y^*(t) - \frac{G_cG}{1+G_pG_c+G_c(G-G_m)}\, \eta(t) + \eta(t)$$

References

OGUNNAIKE, B. A. and RAY, W. H., (1979), "Multivariable Controller Design for Linear Systems Having Multiple Time Delays" AIChE Journal ,Vol.25, No.6, 1043.

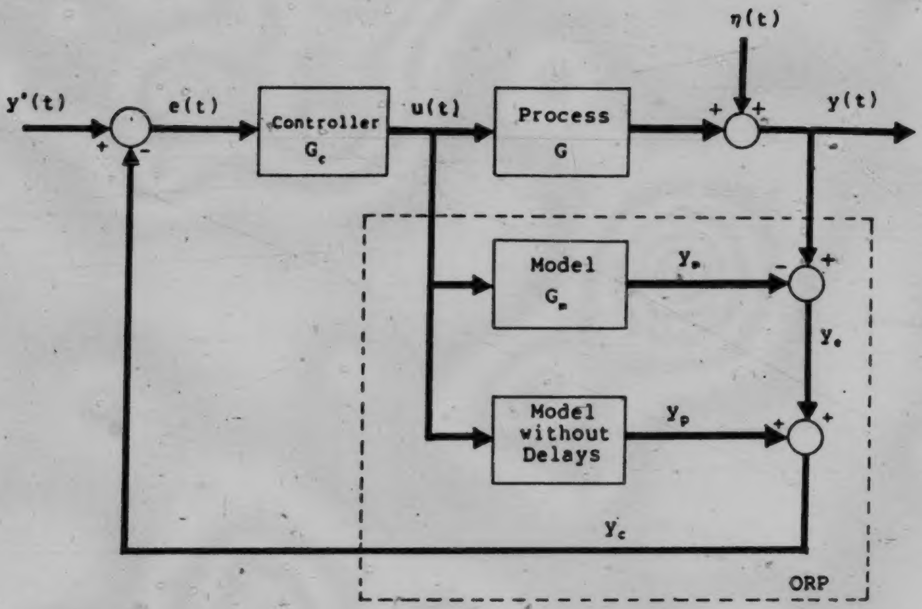
η(t)
y'(t)
e(t)
Controller
G_c
u(t)
Process
G
y(t)
+
-
Model
G_m
y_m
y_e
Model
without
Delays
y_p
y_c
ORP

Figure G.1 Schematic Block Diagram of the Ogunnaike and Ray Predictor.

Appendix H - Interactor Predictor

The schematic block diagram of the Interactor predictor due to Sripada, Fisher and Shah (1985) is shown in figure H.1.

If T(q) is the transfer function of the MIMO system having m ultiple time delays, then from the interactor factorization due to Wolovich and Falb (1976).

$$T(q) = \xi_T^{-1}(q)\, R_T(q) \qquad (H.1)$$

$$= \xi^{-1}(q)\, R(q) \qquad (H.2)$$

where

$$\xi^{-1}(q) = q\, \xi_T^{-1}(q) \qquad (H.3)$$

$$R(q) = q^{-1} R_T(q) \qquad (H.4)$$

$$\xi^{-1}(q) = \begin{bmatrix} q^{-\lambda_1} & & & \\ g_{21}(q^{-1}) & q^{-\lambda_2} & O & \\ \vdots & & \ddots & \\ g_{p1}(q^{-1}) & & & q^{-\lambda_p} \end{bmatrix}$$

Define

$$R(q) = q^{-1} A^{-1}(q^{-1})\, B(q^{-1}) \qquad (H.6)$$

$$\xi^{-1}(q) = D(q^{-1}) + G_H(q^{-1}) \qquad (H.7)$$

where

$$A(q^{-1}) = I + A_1 q^{-1} + A_2 q^{-2} + \cdot\ \cdot + A_n . q^{n'}$$

with A_1, A_2 · · A_n, are diagonal.

$$B(q^{-1}) = B_1 + B_2 q^{-1} + \cdot\ \cdot + B_n q^{-n}$$

$$D(q^{-1}) = \mathrm{diag}\, [q^{-\lambda_1}\ q^{-\lambda_2} \cdot\ \cdot\ q^{-\lambda_p}]$$

and

$$G_H(q^{-1}) = \begin{bmatrix} 0 & & & \\ g_{21} & 0 & & \text{O} \\ g_{31} & g_{32} & 0 & \\ \cdot & & \cdot & \\ \cdot & & & \cdot \\ g_{p1} & g_{p2} & & 0 \end{bmatrix}$$

In the interactor predictor the model G_m is defined as,

$$G_m(q^{-1}) = \xi^{-1}(q)\ R(q) = \xi^{-1}q^{-1}A^{-1}B$$

$$= Dq^{-1}A^{-1}B + G_H q^{-1}A^{-1}B \qquad \text{(H.8)}$$

the model output $y_m(t)$ is then given by,

$$y_m(t) = Dq^{-1}A^{-1}B\ u(t) + G_H q^{-1}A^{-1}B\ u(t) \qquad \text{(H.9)}$$

The predictor G_p of the interactor is obtained by factoring out the interactor from the model G_m:

$$G_p(q^{-1}) = \xi_s^{-1}\ R(q) = \xi_s^{-1}\ q^{-1}A^{-1}B \qquad \text{(H.10)}$$

The term

$$\xi_s^{-1} = \lim_{q \longrightarrow \infty} \xi^{-1}(q) \qquad \text{(H.11)}$$

is introduced to assure the zero steady state error in the output, if ~~the~~ interactor predictor is used with the PID control, and called the steady state compensation.

The output of G_p is given by,

$$y_p(t) = \xi_s^{-1}\ q^{-1}A^{-1}B\ u(t) \qquad \text{(H.12)}$$

Define:

$$y_1(t) = q^{-1}A^{-1}B\ u(t) \qquad \text{(H.13)}$$

Then from figure H.1 the predicted output $\hat{y}_c(t)$ is given by,

$$\hat{y}_c(t) = y_p(t) + y(t) - y_m(t) \tag{H.14}$$

$$= \xi_s^{-1}\ q^{-1}A^{-1}B\ u(t) + [y(t) - D(g^{-1})q^{-1}A^{-1}B\ u(t) - G_H y_1(t)] \tag{H.15}$$

The schematic block diagram of the IP can now be drawn in a different form using the equation (H.15) as shown in figure H.2. This configuration is obtained to compare with the configuration of the Kalman filter perdictor (Chapter 3).

References

SRIPADA, N. S., FISHER, D. G. and SHAH, S. L., (1985), "Control of MIMO Time Delay Systems Using an Interactor Matrix", Internal Report, Dept. of Chem. Eng. University of Alberta, Edmonton, AB.

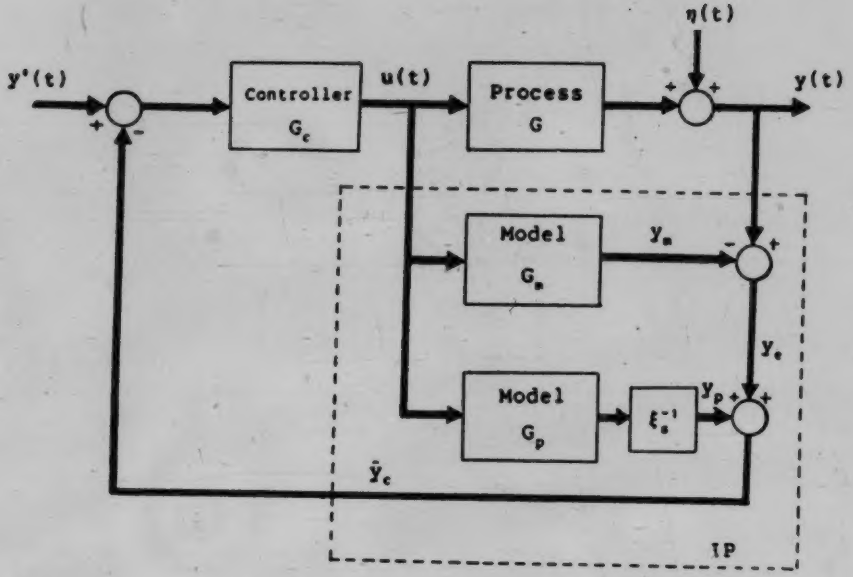
η(t)
y'(t)
+
−
Controller
G_c
u(t)
Process
G
+
+
y(t)
Model
G_m
y_m
−
+
y_e
Model
G_p
ξ_a^{-1}
y_p
+
+
ŷ_c
IP

Figure H.1 Schematic Block Diagram of the Interactor Predictor.

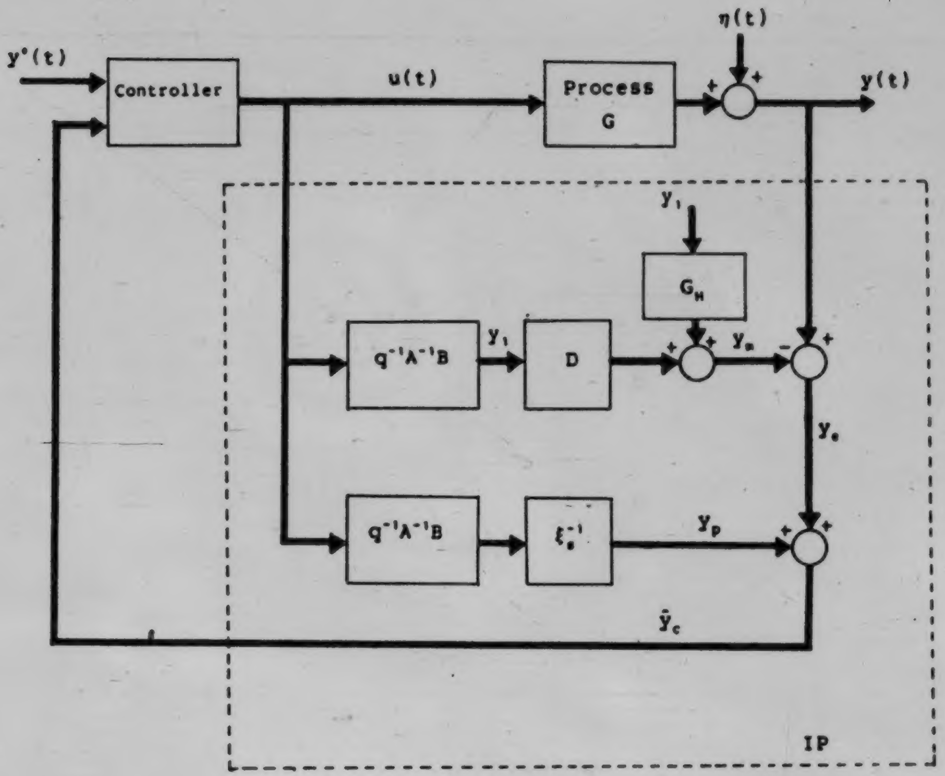
$\eta(t)$
$y^*(t)$
Controller
$u(t)$
Process
G
+
+
$y(t)$
y_1
G_M
$q^{-1}A^{-1}B$
y_1
D
+
+
y_m
−
+
y_e
$q^{-1}A^{-1}B$
ξ_e^{-1}
y_p
+
+
$\hat{y}_c$
IP

Figure H.2 Schematic Block Diagram of the Interactor Predictor Based on Equation H.15.

Appendix I

Lemma I.1:

If the ARMA representation of the residual matrix R(q) due to interactor factorization is given by,

$$R(q) = q^{-1}A^{-1}(q^{-1})\ B(q^{-1}) \tag{I.1}$$

where

$$A(q^{-1}) = I+A_1q^{-1}+A_2q^{-2}+\cdot\cdot+A_nq^{-n} \tag{I.2}$$

and

$$B(q^{-1}) = B_1 + B_2q^{-1} + \cdot \cdot + B_{n'}q^{-n'} \tag{I.3}$$

then, B_1 is non-singular.

Proof:

From the properties of the interactor factorization

$$\lim_{q\rightarrow\infty} \xi(q)T(q) = K \tag{I.4}$$

where K is non-singular.

Since,

$$\xi(q)T(q) = R_T(q) = qR(q) = qq^{-1}A^{-1}(q^{-1})B(q^{-1}) \tag{I.5}$$

and $A(q^{-1})$ is monic, then,

$$\lim_{q\rightarrow\infty} A(q^{-1}) = \lim_{q\rightarrow\infty} A(q^{-1})^{-1} = I \tag{I.6}$$

From (I.4) and (I.5),

$$\lim_{q\rightarrow\infty} A^{-1}(q^{-1})B(q^{-1}) = K$$

$$\lim_{q \longrightarrow \infty} A^{-1}(q^{-1}) \, [B_1 + B_2 q^{-1} + \cdot \cdot + B_{n'} . q^{-n'}] = K$$

$$\therefore \lim_{q \longrightarrow \infty} IB_1 = K \tag{I.7}$$

Since IB_1 is independent of q,

$B_1 = K \neq 0$

Hence, B_1 is non-singular.

Appendix J - A Predictive Controller Using the Interactor Predictor

A predictive control scheme based on the predictions from the interactor predictor is derived assuming a deterministic system. Let the deterministic process model be given by,

$$y(t) = \xi^{-1}Ru(t) = \xi_T^{-1} A^{-1}B\, u(t) \qquad (J.1)$$

where ξ_T^{-1} , A^{-1}, B are as given by equations (3.2.18), (3.3.17 and 19).

Also define a new variable $\bar{y}(t)$ the filtered output,

$$\bar{y}(t) = \xi_T\, y(t) = q\, \xi\, y(t) \qquad (J.2)$$

and the filtered desired output $\bar{y}^*(t)$ as,

$$\bar{y}^*(t) = \xi y^*(t) \qquad (J.3)$$

From (J.1),

$$\bar{y}(t) + A_1\, \bar{y}(t-1) + \cdot \cdot + A_n\, \bar{y}(t-n) = Bu(t) \qquad (J.4)$$

Since

$$\bar{y}(t-1) = \xi y(t) = y^0(t) \qquad (J.5)$$

from (J.4) and (J.5) we get,

$$\bar{y}(t) + A_1 y^0(t) + \cdot \cdot + A_n y^0(t-n+1) = Bu(t) \qquad (J.6)$$

In equation (J.6) the L.H.S. terms are all predictions. Thus we can write a predictor equation as follows:

$$\bar{y}(t) + A_1\hat{y}^0(t) + A_2\hat{y}_0(t-1) \cdot \cdot \cdot + A_n\hat{y}_0(t-n+1) = B\, u(t) \qquad (J.7)$$

The predictions $\hat{y}_0(t) \cdot \cdot \hat{y}_0(t-n+1)$ can be obtained from the IP for deterministic systems.

If,

$$B(q^{-1}) = B_1 + B_2 q^{-2} + \cdot \cdot + B_{n'} q^{n'} \tag{J.8}$$

then the control law that would make $\bar{y}(t)$ or the filtered output to reach the filtered desired set point $\bar{y}^*(t)$ is obtained by making,

$$\bar{y}(t) = \bar{y}^*(t) \tag{J.9}$$

The corresponding predictive control law is given by,

$$u(t) = B_1^{-1} \; [\bar{y}^*(t) + (A-I)\tilde{y}^0(t) - (B-B_1)u(t)] \tag{J.10}$$

where B_1 is non-singular as shown in Appendix I.

Appendix K - Procedure for Precompensator Design

In adaptive control of a MIMO system it is necessary to know apriori, or to estimate the real coffecients of the interactor matrix. But the explicit estimation of the real coefficients leads to difficulties. When the interactor is diagonal since there are no real coefficients in the interactor, the formulation of the adaptive control schemes becomes easier. It was shown by Singha and Narendra(1985), that by using a suitable precompensator it is possible to obtain a diagonal interactor for a process that has a triangular interactor.

The precompensator design procedure is presented below using an example.

Design Procedure

The equation for the i^{th} iteration of this recursive procedure is given by

$$W_p(q)M_{i-1}(q) = G_i(q)[K_{0i} + K_{1i}q_{-1} + \cdot \cdot \cdot]$$

$$G_i(q) = \mathrm{diag}[\min_{j=1,p}(q^{-d1j}), \min(q^{-d2j}), \cdot \cdot, \min(q^{-dpj})]$$

of $W_p(q)M_{i-1}(q)$

K_{0i} - is the first term of the expansion.

K_0^i - is the permutation of K_{0i}

At each iteration the rank of K_{0i} is checked by counting the number of nonzero elements along the diagonal

K_0^i

Let

$$W_p(q) = \begin{bmatrix} K_1q^{-2} & K_2q^{-4} & K_3q^{-1} \\ K_4q^{-1} & K_5q^{-3} & K_6q^{-2} \\ K_7q^{-1} & K_8q^{-8} & K_9q^{-9} \end{bmatrix}$$

For each iteration i we have to specify the following.

(a) A suitable matrix L(q), which multiplies $M_{i-2}(q)$ to yield $M_{i-1}(q)$

(b) Matrix $G_i(q)$

(c> r_i - the rank of K_{0i} or K_0^i .

Note:- L(q), $M_{i-1}(q)$, $G_i(q)$ and K_{0i} are defined with respect to the original matrix $W_p(q)$ and not its permutation.

Step 1 Initialization

$i=1$

$L(q) = M_0(q) = I$

$G_1(q) = \text{diag}[q_{-1},q_{-1},q_{-1}]$

then

$$K_{01} = \begin{bmatrix} 0 & 0 & K_3 \\ K_4 & 0 & 0 \\ K_7 & 0 & 0 \end{bmatrix}$$

by column operations

$$K_0^1 = \begin{bmatrix} K_3 & 0 & 0 \\ 0 & K_4 & 0 \\ 0 & K_7 & 0 \end{bmatrix}$$

then

$r_1=2$

(a) First determine the linearly dependent rows of K_0'(or K_{01}),e.g. col 2 and 3.

(b) Determine the columns of K_0' (or correspondingly those of K_{01}) with nonzero elements which pertain to the linearly dependent rows of K_0',e.g. 2nd column.

(c) Define $L(q)$ on following criteria.

1. $L(q)$ is diagonal.
2. jth element of $L(q)$ is

$= q^{-1}$ - if jth column of the K_{01} has non zero elements at the rows that are linearly dependent.

$= 1$ if jth column of the K_{01} has no non zero elements at the rows that are linearly dependent.

Thus

$$L(q) = \mathrm{diag}(q^{-1}, 1, 1)$$

Step 2 $i = 2$

define

$$M_1(q) = M_0(q)L(q)$$

then

$$W_p(q)M_1(q) = \begin{bmatrix} K_1q^{-3} & K_2q^{-4} & K_3q^{-1} \\ K_4q^{-2} & K_5q^{-3} & K_6q^{-2} \\ K_7q^{-2} & K_8q^{-8} & K_9q^{-9} \end{bmatrix}$$

then

$$G_2(q) = \mathrm{diag}\,(\,q^{-1},\, q^{-2},\, q^{-2}\,)$$

$$K_{02} = \begin{bmatrix} 0 & 0 & K_3 \\ K_4 & 0 & K_6 \\ K_7 & 0 & 0 \end{bmatrix}$$

$$K_0^2 = \begin{bmatrix} K_3 & 0 & 0 \\ K_6 & K_4 & 0 \\ 0 & K_7 & 0 \end{bmatrix}$$

rank $r_2 = 2$ and we can show

$$L(q) = \mathrm{diag}\,[\, q^{-1},\ 1,\ q^{-1} \,]$$

Step 3 $i=3$

$$M_2(q) = M_1(q)L(q)$$

$$=\mathrm{diag}\,[\, q^{-1},\ 1,\ q^{-1} \,]$$

then

$$W_p(q)M_2(q) = \begin{bmatrix} K_1q^{-3} & K_2q^{-4} & K_3q^{-2} \\ K_4q^{-2} & K_5q^{-3} & K_6q^{-3} \\ K_7q^{-2} & K_8q^{-8} & K_9q^{-10} \end{bmatrix}$$

then

$$G_3(q) = \mathrm{diag}\,(\, q^{-2},\ q^{-2},\ q^{-2} \,)$$

$$K_{03} = \begin{bmatrix} 0 & 0 & K_3 \\ K_4 & 0 & 0 \\ K_7 & 0 & 0 \end{bmatrix}$$

$$K_0^2 = \begin{bmatrix} K_3 & 0 & 0 \\ 0 & K_4 & 0 \\ 0 & K_7 & 0 \end{bmatrix}$$

rank $r_3 = 2$ and we can show

$$L(q) = \mathrm{diag}\,[\, q^{-1},\ 1,\ 1 \,]$$

Step 4 i=4

$$M_3(q) = \mathrm{diag}\ [\ q^{-2},\ 1,\ q^{-1}\ ]$$

$$W_p(q)M_3(q) = \begin{bmatrix} K_1q^{-4} & K_2q^{-4} & K_3q^{-2} \\ K_4q^{-3} & K_5q^{-3} & K_6q^{-3} \\ K_7q^{-3} & K_8q^{-8} & K_9q^{-10} \end{bmatrix}$$

then

$$G_4(q) = \mathrm{diag}\ (\ q^{-2},\ q^{-3},\ q^{-3}\ )$$

$$K_{04} = \begin{bmatrix} K_1 & K_2 & 0 \\ K_4 & K_5 & K_6 \\ K_7 & 0 & 0 \end{bmatrix}$$

$$K_0^4 = \begin{bmatrix} K_7 & 0 & 0 \\ K_1 & K_2 & 0 \\ K_4 & K_5 & K_6 \end{bmatrix}$$

rank $r_4 = 3$

Since rank is 3,

the precompensator

$$W_{pc}(q) = M_3(q) = \mathrm{diag}[\ q^{-2},\ 1,\ q^{-1}\ ]$$

It is clear that the precompensator introduces time delays to the process.

References

SINGH, R.P. and NARENDRA, K.S., (1984), "Prior Information in Design of Multivariable Adaptive Controllers", IEEE Trans. A.C., Vol.29, No.12, pp 1108-1111.

Appendix L - Comments on Simulation Results

The simulation results in this thesis are to illustrate the results of theoretical or mathematical analysis and are not used as raw data form which to deduce conclusions. Therefore the details of the programs were not considered to be critical and were not included in the thesis. However, the following comments concern some of the more obvious features of the simulation results.

1. The simulation programs were written in Fortran 77 (IBM Fortranvs compiler) and were executed on an Amdahl 5870/V8 computer operating under the Michigan Terminal System (MTS).
2. The International Mathematical Scientific Library (IMSL) subroutine GGNQF was used to generate the white noise sequences for the simulations.
3. (a) It is important to note that although the setpoint changes plotted in the figures have a slight slope they were true step changes.

   (b) The graphs for output error variance were plotted using the spline fitting routine in the Tellagraph plotting package. Only the points shown represent data. The lines connecting the points are for convenience only.
4. The CPU execution time for a typical simulation run for a SISO adaptive KFP with predictive control (800 sampling instants) is 4.5 seconds. However the execution times do not give a realistic indication of

the computational requirements for implementation because the programs are not optimized, and include the process simulation plus data and plot files.

5. The output error variances (e.g. figure 2.12c) were calculated only for a single run. No rigorous statistical evaluations were made.